A novel extended Tsypkin criterion for discrete-time descriptor systems with ferromagnetic hysteresis nonlinear physical phenomenon*

Zhang Hua-Guang(张化光)[a)b)†], Song Zheng(宋 铮)[a)b)], and Zhang Qing-Ling(张庆灵)[c)]

[a)] *School of Information Science and Engineering, Northeastern University, Shenyang* 110819, *China*

[b)] *Key Laboratory of Integrated Automation of Process Industry (Northeastern University) of National Education Ministry, Shenyang* 110819, *China*

[c)] *Institute of Systems Science, Northeastern University, Shenyang* 110819, *China*

(Received 7 March 2010; revised manuscript received 10 May 2010)

This paper concerns the absolute stability problem of discrete-time descriptor systems with feedback connected ferromagnetic hysteresis nonlinearities. The ferromagnetic hysteresis model satisfies the passivity conditions of hysteresis operator, that is the input–output relation of the transformed operator is passive. The bound condition of the solution of the ferromagnetic hysteresis model is given. Through the framework of loop transformation, an augmented discrete-time descriptor system model is established for the stability analysis. A new extended Tsypkin criterion for the absolute stability of discrete-time descriptor systems with hysteresis is presented based on the linear matrix inequalities technique. A numerical example is given to illustrate the effectiveness of the extended criterion.

Keywords: ferromagnetic hysteresis nonlinearity, discrete-time descriptor systems, extended Tsypkin criterion

PACC: 0547

1. Introduction

Hysteresis phenomenon widely occurs in electronic relay circuits, shape memory alloys, smart materials actuators, mechanical gears and electromagnetic devices. Hysteresis nonlinearity, which is one of the typical nonlinearities, has a nonlinear loop that depends on the amplitude of the system input.[1] There are various types of hysteresis models, such as the Preisach model,[2] Prandtl–Ishlinskii model,[3] Duhem model,[4] and ferromagnetic model.[5,6] Some researchers studied the physical characteristics of the hysteresis nonlinear phenomenon, such as adhesion elastic hysteresis,[7] the hysteresis of subharmonic in liquid containing microbubbles.[8] The parameter identification problem was studied for metal rubber isolator with hysteresis nonlinearities.[9] Generally speaking, unlike those sector bounded memoryless nonlinearities such as deadzone or saturation, the output of hysteresis nonlinearity at any given time is a function of the entire past history of the input, and thus hysteresis nonlinearity has memory.[10] The loop and memory characteristics of hysteresis complicate the system analysis. A differential equation form of the ferromagnetic hysteresis model was presented to describe the relationship between the magnetic field strength and the magnetic flux density in ferromagnetic materials.[5,6] An adaptive backstepping control method was presented to solve the tracking problem of uncertain nonlinear system with ferromagnetic hysteresis nonlinearity.[11]

On the other hand, descriptor systems have been one of the major research fields of control theory due to the comprehensive applications in electrical and mechanical models.[12−15] The applications for descriptor systems in describing large-scale systems, power systems and interconnected networked systems are well known. Some kind of fuzzy descriptor systems were studied through guaranteed cost control or neural network control methods.[16−18] Generalized quadratic stability and stabilization for singular systems were presented in the viewpoint of system families.[19] The absolute stability problem was proposed by Lur'e and Postnikov as a condition for stability of feedback connected systems consisting of a

*Project supported by the National Natural Science Foundation of China (Grant Nos. 50977008, 60821063 and 61034005), and National Basic Research Program of China (Grant No. 2009CB32060).

†Corresponding author. E-mail: hgzhang@ieee.org, skarlz3000@yahoo.com.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

linear part with nonlinear feedback constrained by a sector condition.[20−23] Extended strict positive realness (ESPR) was developed for descriptor systems,[13] where ESPR was studied for both continuous and discrete time singular systems by using generalized algebraic Riccati equations and inequalities. An LMI (linear matrix inequality)-based strictly positive real lemma was given for discrete time singular systems, and the LMI-based result was applied to investigate the absolute stability of discrete time systems.[15] The ESPR and strongly absolute stability were investigated for descriptor systems with output feedback nonlinearities.[12] However, the nonlinearities considered in Ref. [12] are sector bounded memoryless nonlinearities, not the hysteresis type nonlinearities. As is stated, hysteresis is a kind of important inherent nonlinear phenomenon, and widely occurs in practical system devices. There have been few works on descriptor systems with hysteresis nonlinearities in the existing literatures so far. The KYP lemma and invariance principle were applied to achieve absolute stability criterion for standard linear systems with hysteresis nonlinearities.[10] Passivity properties were investigated for Preisach type hysteresis nonlinearities.[24] A multivariable extension of the Tsypkin criterion was presented for a standard discrete system with sector bounded nonlinearities.[25] However, whether such kind of control method is suitable for discrete-time descriptor systems with hysteresis nonlinearities is still an open problem.

In this paper, absolute stability problem of discrete-time descriptor system with feedback connected ferromagnetic hysteresis nonlinearities is studied. To the best of our knowledge, this is the first time in the literature that absolute stability problem of discrete-time descriptor systems with hysteresis feedback is investigated. The circulation orientation of the ferromagnetic hysteresis loop is counterclockwise, thus the hysteresis model satisfies the passivity conditions of the hysteresis operator, besides, the input–output relation of the transformed operator is passive. The bound condition of the solution of the ferromagnetic hysteresis model is given to establish a local Lipschitz condition. Through the framework of loop transformation, an augmented discrete-time descriptor system model is established for the stability analysis. A new extended Tsypkin criterion for the absolute stability analysis of the discrete-time descriptor system with hysteresis feedback is presented based on the LMI technique. A numerical example is given to demonstrate the effectiveness of the method.

Throughout the paper, if not explicitly stated, all matrices are assumed to have compatible dimensions. Let M be a complex matrix, and let M^{T} and M^* stand for the transpose and the conjugate transpose of M, respectively. The inequality $M > 0$ ($M \geq 0$) means that M is a positive-definite (semidefinite) matrix, while $M < 0$ ($M \leq 0$) means that M is a negative-definite (semidefinite) matrix.

2. Ferromagnetic hysteresis

2.1. Ferromagnetic hysteresis model

Consider the following ferromagnetic hysteresis model[11]

$$\dot{B}_m = \alpha|\dot{H}_m|\,[\,f(H_m) - B_m] + \dot{H}_m g(H_m), \quad (1)$$

where the magnetic flux density B_m and the magnetic field strength H_m are real-valued functions of time with piecewise continuous derivatives $\dot{B}_m$ and $\dot{H}_m$, α is a positive constant, $|\dot{H}_m|$ is the absolute value of $\dot{H}_m$, $f(\cdot)$ and $g(\cdot)$ are real-valued functions on $(-\infty, +\infty)$ and must meet the following assumptive restrictions:

Assumption 1 The function $f(\cdot)$ must be a piecewise smooth, monotonously increasing odd function of H_m, with a derivative $f'(\cdot)$ which obtains a finite limit $f'(\infty)$ while $H_m \to \infty$.

Assumption 2 The function $g(\cdot)$ must be a piecewise continuous, even function of H_m, with a limit obeying $g(H_m) \to f'(H_m), H_m \to \infty$.

Assumption 3 For all finite H_m, the functions $f'(\cdot)$ and $g(\cdot)$ must satisfy the inequalities

$$\begin{aligned} f'(H_m) &\geq g(H_m) \\ &\geq \alpha\,\mathrm{e}^{\alpha H_m} \int_{H_m}^{\infty} [\,f'(\xi) - g(\xi)]\,\mathrm{e}^{-\alpha\xi}\mathrm{d}\xi. \end{aligned}$$

According to the above model and assumptions, the relationship between B_m and H_m can be obtained as a function $B_m(H_m)$, named hysteresis loop curve. As an illustration, the functions and parameters are selected as follows: $f(H_m) = \beta\tanh(H_m)$, $g(H_m) = f'(H_m)\left(1 - \lambda\,\mathrm{e}^{-\rho|H_m|}\right)$, $H_m = l\sin(\omega t)$, where β, λ, ρ, α, l and ω are arbitrary parameters. Select $\beta = 1$, $\lambda = 0.85$, $\rho = 0.1$, $\alpha = 5$, $l = 2$, $\omega = \pi, 0.5\pi, 0.2\pi$, and 0.1π. The hysteresis loop is shown in Fig. 1. Note that the traversed orientation of the hysteresis loops is counter-clockwise.

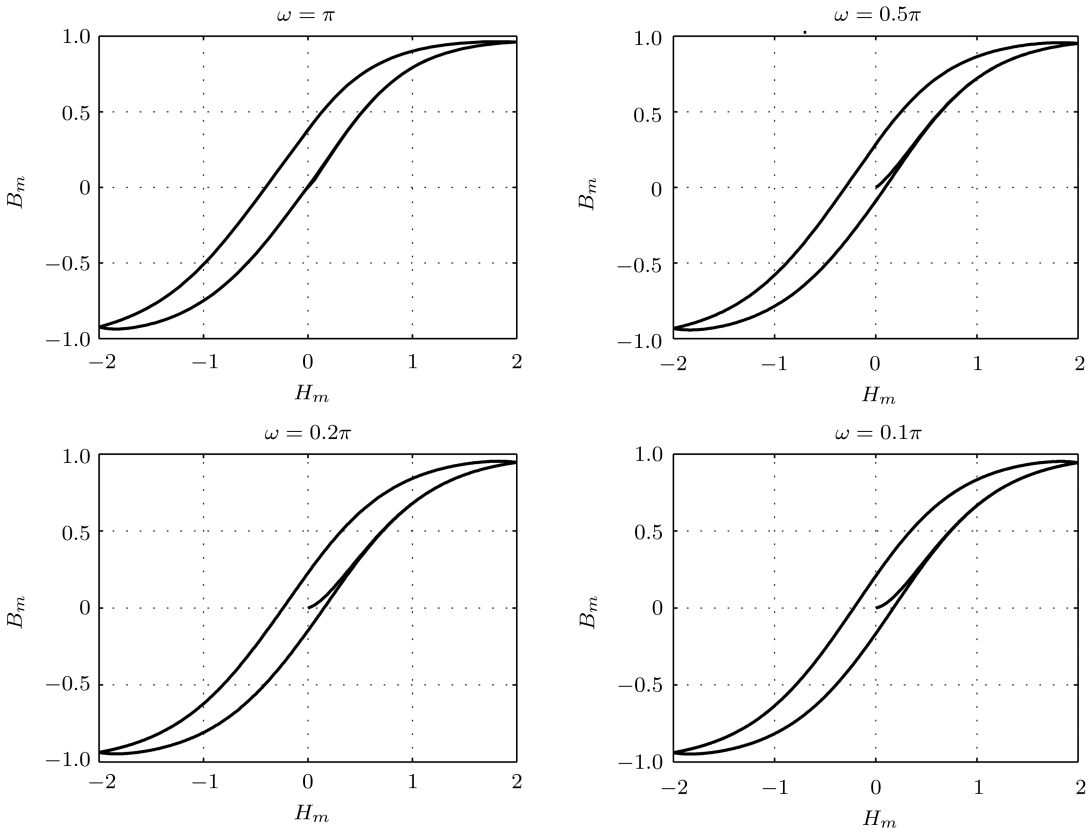


Fig. 1. Ferromagnetic hysteresis loop with $\omega = \pi, 0.5\pi, 0.2\pi$, and 0.1π.

In physical systems, hysteresis nonlinearity is usually denoted as $w(t) = \Phi[v](t)$, where Φ denote the hysteresis operator with $v(t)$ as the hysteresis input and $w(t)$ as the hysteresis output. The hysteresis operator $w(t) = \Phi[v](t)$ typically refers to the input–output relation between two time-dependent quantities which cannot be expressed as a single-valued function. The relationship usually takes the form of loops that are traversed either in clockwise or counterclockwise direction. Typically, hysteresis can be classified as rate independent hysteresis and rate dependent hysteresis, respectively. For the rate independent hysteresis, the shape of the input–output relation is unchanged for arbitrary time rate of input function; while for the rate-dependent hysteresis, the shape is changed with the frequency of the input signal. The details of rate dependence/independence of hysteresis can be referred to Ref. [4]. As shown in Fig. 1, the shapes of the response for different input frequencies are the same. Thus such a kind of hysteresis models are called rate-independent hysteresis models.

To unify the expression of the aforementioned ferromagnetic hysteresis model in the controller design, let $H_m = v(t)$, $B_m = w(t)$, then equation (1) translates into

$$\dot{w}(t) = \alpha|\dot{v}(t)|\,[\,f(v(t)) - w(t)] + \dot{v}(t)g(v(t)). \quad (2)$$

Considering the initial value point (v_0, w_0), where $v(0) = v_0$, $w(0) = w_0$, we denote the solution of Eq. (2) as

$$\begin{aligned} & w(t) \\ &= f(v(t)) + [w_0 - f(v_0)]\ \mathrm{e}^{-\alpha \mathrm{sgn}(\dot{v})(v - v_0)} \\ &\quad - \mathrm{e}^{-\alpha \mathrm{sgn}(\dot{v})v} \int_{v_0}^{v} (f'(\xi) - g(\xi))\ \mathrm{e}^{\alpha \xi \mathrm{sgn}(\dot{v})} \mathrm{d}\xi, \end{aligned} \quad (3)$$

when $v_0 = 0$, $w_0 = 0$, solution (3) changes to

$$\begin{aligned} w(t) \ = \ & f(v(t)) - \mathrm{e}^{-\alpha \mathrm{sgn}(\dot{v})v} \int_0^v (\,f'(\xi) - g(\xi)) \\ & \times \mathrm{e}^{\alpha \xi \mathrm{sgn}(\dot{v})} \mathrm{d}\xi. \end{aligned} \quad (4)$$

Define

$$d[v](t) \triangleq -\mathrm{e}^{-\alpha \mathrm{sgn}(\dot{v})v} \int_0^v (\,f'(\xi) - g(\xi))\ \mathrm{e}^{\alpha \xi \mathrm{sgn}(\dot{v})} \mathrm{d}\xi,$$

then equation (4) changes to

$$w(t) = \Phi[v](t) = f(v(t)) + d[v](t). \quad (5)$$

The following proposition gives the bound condition of the solution of the ferromagnetic hysteresis model.

Proposition 1 For arbitrary input function $v(t)$, $d[v](t)$ is bounded in a finite bound $\mathcal{B}_d$, $\mathcal{B}_d \in R^+$, R^+ denotes the set of positive integers.

Proof When $\dot{v} > 0$ and $v \to +\infty$,

$$\lim_{v\to+\infty} d[v](t) = -\lim_{v\to+\infty} \frac{\int_0^v (f'(\xi) - g(\xi))\, \mathrm{e}^{\alpha\xi}\mathrm{d}\xi}{\mathrm{e}^{\alpha v}} = \frac{g(+\infty) - f'(+\infty)}{\alpha}, \tag{6}$$

when $\dot{v} < 0$ and $v \to -\infty$,

$$\lim_{v\to-\infty} d[v](t) = -\lim_{v\to-\infty} \frac{\int_0^v (f'(\xi) - g(\xi))\, \mathrm{e}^{-\alpha\xi}\mathrm{d}\xi}{\mathrm{e}^{-\alpha v}} = \frac{g(-\infty) - f'(-\infty)}{-\alpha}. \tag{7}$$

From Assumptions 1, 2 and 3 we know that $g(\infty)$ and $f'(\infty)$ are limited, so $g(\infty) - f'(\infty)$ must be limited, too. Thus, $\lim_{v\to\infty} |(g(v) - f'(v))/\alpha| \le \mathcal{B}_d$. Besides, it is obvious that when $v = 0$, $d[v](t) = 0$, hence for arbitrary $v(t)$, $d[v](t)$ is bounded in $\mathcal{B}_d$. □

Remark 1 The bound condition of the considered ferromagnetic hysteresis model shows that the disturbance term cannot be arbitrarily large, this fulfills the demand of the practical physical systems.

2.2. Passivity of hysteresis operator

In order to analyse the absolute stability of systems with hysteresis, the passivity property of hysteresis model should be studied and a passive hysteresis operator is needed to be established. From the viewpoint of energy loss, for a hysteresis nonlinearity $\Phi[\cdot]$ as shown in Fig. 2, it is not passive from μ to η. The associated energy pair is not (μ, η), but $(\mu, \dot{\eta})$. This agrees with the well-known result that the hysteretic energy loss in a magnetic circuit due to periodic excitation is equal to $\oint H_m \mathrm{d}B_m$, the area of the hysteresis loop.[24] Hence we will demonstrate a proposition that it is passive from μ to $\dot{\eta}$.

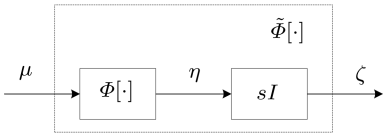


Fig. 2. Passive hysteresis operator.

Proposition 2 Consider a vector hysteresis nonlinearity $\Phi[\cdot]$ as in Eq. (5), where $\phi_i[\cdot]$ is differentiable a.e. in R, $0 \le \phi_i'[\cdot] < k_i$, $i = 1, \ldots, m$, $K = \mathrm{diag}(1/k_1, \ldots, 1/k_m)$, k_i is the maximum slope of $\phi_i[\cdot]$, and the traversed orientation of the hysteresis loops is counter-clockwise. Then the input–output relation of the operator $\tilde{\Phi}[\cdot]$ shown in Fig. 2 is passive, i.e., for $\tilde{\Phi}[\cdot]$ it is passive from μ to ζ.

Proof From the general passivity theory we have

$$\int_0^T \mu^T \zeta \,\mathrm{d}t = \int_0^T \mu^T \frac{\mathrm{d}}{\mathrm{d}\mu}\Phi[\mu](t)\frac{\mathrm{d}\mu}{\mathrm{d}t}\,\mathrm{d}t = \oint_{\Phi[\mu](0)}^{\Phi[\mu](T)} \mu^T \,\mathrm{d}\Phi[\mu] \ge 0. \tag{8}$$

As the circulation of the hysteresis loops is counter-clockwise, from Ref. [10], the last inequality of the integral (8) holds. For the closed path, the value of the integral (8) equals the area enclosed by the hysteresis loop. Hence, the operator $\tilde{\Phi}[\cdot]$ is passive from μ to ζ. □

Remark 2 A hysteresis with counter-clockwise loops is considered as a passive hysteresis.[10] Note that only the counter-clockwise circulation of the passive hysteresis allows it to be converted to a passive operator by transformation. As shown in Fig. 1, the orientation of the loops is counter-clockwise, thus the considered ferromagnetic hysteresis is passive hysteresis. A consequence of Proposition 2 is the conclusion that the transformed passive hysteresis operator $\tilde{\Phi}[\cdot]$ fulfills the following sector condition:

$$\tilde{\Phi}^T[\mu](K\tilde{\Phi}[\mu] - \mu) \le 0 \quad \forall\, \mu \in R^m. \tag{9}$$

In this paper, the ferromagnetic hysteresis model (1) with Assumptions 1–3 satisfies the conditions mentioned in Proposition 2, and the bound condition is presented through the Proposition 1. This allows us to establish a local Lipschitz condition for the ferromagnetic hysteresis nonlinearity as follows: If the input $v : R^m \to C[0, t]$, where $v(t_2)$ is a sufficiently small perturbation of $v(t_1)$ on an interval $\|v(t_1) - v(t)\| < \delta$, $\forall t \in [t_1, t_2]$, then the local Lipschitz condition $|\Phi[v](t_1) - \Phi[v](t_2)| \le \Phi'_{\max}\|v(t_1) - v(t_2)\|$ will hold, where $0 \le \Phi'[v] \le \Phi'_{\max} < \infty$, $\Phi'_{\max}$ is the maximum slope appearing in the hysteresis nonlinearity. This Lipschitz condition ensures that the transformed operator is a bounded operator on $C[0, t]$ and is required for a differentiable Lyapunov function.

3. Discrete descriptor systems

Consider a normal discrete descriptor system as follows:

$$\begin{cases} Ex(k+1) = Ax(k) + Bu(k), \\ y(k) = Cx(k) + Du(k), \end{cases} \tag{10}$$

where $x(k) \in R^n$ is the state, $u(k) \in R^m$ is the exogenous input, $y(k) \in R^m$ is the output. The matrix $E \in R^{n\times n}$ may be singular, $\text{rank}(E) = r \leq n$. The A, B, C, D are known real constant matrices with appropriate dimensions. Let (E, A, B, C, D) denote the descriptor system of the form (10). The transfer function of the above descriptor system is as follows:

$$G(z) = C(zE - A)^{-1}B + D. \tag{11}$$

Definition 1 The pair (E, A) is said to be regular if $\det(zE - A)$ is not identically zero; the pair (E, A) is said to be causal if $\deg(\det(zE - A)) = \text{rank}(E)$; the pair (E, A) is said to be stable if its generalized spectral radius $\rho(E, A) \triangleq \max\limits_{\lambda\in\{z|\,\det(zE-A)=0\}} |\lambda| < 1$; the pair (E, A) is said to be admissible if it is regular, causal and stable.

Suppose that the pair (E, A) is regular, then there exist two nonsingular matrices M and N such that (E, A, B, C, D) can be transformed to the Weierstrass canonical form

$$(\bar{E}, \bar{A}, \bar{B}, \bar{C}, \bar{D}) \equiv (MEN, MAN, MB, CN, D) \tag{12}$$

with

$$\bar{E} = \begin{bmatrix} I_r & 0 \\ 0 & J \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix},$$
$$\bar{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C_1 & C_2 \end{bmatrix},$$

where $J \in R^{(n-r)\times(n-r)}$ is a nilpotent matrix, i.e., $J^l = 0$ for some integer $l \geq 1$, $A_1 \in R^{r\times r}$, and B_1, B_2, C_1, C_2 are in compatible dimensions. The pair (E, A) is causal if and only if $J = 0$.

Definition 2 (i) The descriptor system (10) is said to be positive real (PR) if its transfer function $G(z)$ is analytic in $|z| > 1$ and satisfies $G(z)+G^*(z) \geq 0$ for $|z| > 1$.

(ii) The descriptor system (10) is said to be strictly PR (SPR) if its transfer function $G(z)$ is analytic in $|z| \geq 1$ and satisfies $G(\mathrm{e}^{j\theta}) + G^*(\mathrm{e}^{j\theta}) > 0$ for $\theta \in [0, 2\pi)$.

(iii) The descriptor system (10) is said to be extended SPR (ESPR) if it is SPR and $G(\infty)+G^{\mathrm{T}}(\infty) > 0$.

Lemma 1[21] The discrete descriptor system (10) is admissible and ESPR if and only if there exists a matrix $P = P^{\mathrm{T}}$ such that the following LMIs hold:

$$E^{\mathrm{T}}PE \geq 0,$$

$$\begin{bmatrix} A^{\mathrm{T}}PA - E^{\mathrm{T}}PE & C^{\mathrm{T}} - A^{\mathrm{T}}PB \\ C - B^{\mathrm{T}}PA & -(D^{\mathrm{T}} + D - B^{\mathrm{T}}PB) \end{bmatrix} < 0. \tag{13}$$

4. Main results

In this section we will extend the Tsypkin criterion to the discrete-time descriptor systems with ferromagnetic hysteresis nonlinearities. Consider the absolute stability problem for the following system:

$$\begin{cases} Ex(k+1) = Ax(k) + Bu(k), \\ y(k) = Cx(k), \\ u(k) = -\Phi[y](k), \end{cases} \tag{14}$$

where $\Phi[\cdot] = (\phi_1[\cdot], \ldots, \phi_m[\cdot])^{\mathrm{T}}$ is the hysteresis nonlinearity discussed in Section 2. The system is shown in Fig. 3. The transfer function is $G(z) = C(zE - A)^{-1}B$. The following theorem is the extended Tsypkin criterion for the absolute stability of the discrete-time descriptor systems with ferromagnetic hysteresis nonlinearities.

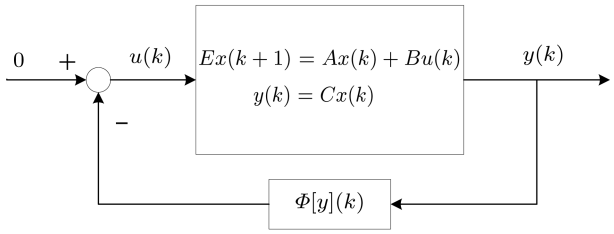


Fig. 3. Discrete-time descriptor systems with ferromagnetic hysteresis nonlinearities.

Theorem 1 Let $\Gamma \triangleq \text{diag}(\gamma_1, \ldots, \gamma_m) > 0$, and the ferromagnetic hysteresis nonlinearity $\Phi[\cdot]$ satisfies the condition (9). The discrete descriptor system (14) is absolutely stable, if there exist matrices P, L, and W with $P = P^{\mathrm{T}}$ satisfying

$$\begin{cases} \hat{A}^{\mathrm{T}}P\hat{A} - \hat{E}^{\mathrm{T}}P\hat{E} + L^{\mathrm{T}}L = 0, \\ \hat{B}^{\mathrm{T}}P\hat{A} - \hat{C} + W^{\mathrm{T}}L = 0, \\ 2K - \hat{B}^{\mathrm{T}}P\hat{B} - W^{\mathrm{T}}W = 0, \\ \hat{E}^{\mathrm{T}}P\hat{E} \geq 0, \end{cases} \tag{15}$$

where the transformed system matrices are as follows:

$$\hat{E} = \begin{bmatrix} E & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & 0 \\ 0 & I & 0 \end{bmatrix},$$
$$\hat{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} 0 & I + \Gamma & -\Gamma \end{bmatrix}, \quad \hat{D} = K.$$

Proof The proof is given in the Appendix. □

Remark 3 Theorem 1 is a novel extension of traditional Tsypkin criterion for the discrete-time descriptor systems with ferromagnetic hysteresis nonlinearities. The LMIs (A5) conditions in the proof are the equivalence of the equations conditions in Theorem 1.

5. Numerical example

Consider a discrete-time descriptor system (14) as follows:

$$E = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad K = 1, \quad \Gamma = 1.$$

The ferromagnetic hysteresis nonlinearities $\Phi[\cdot]$ have the same parameters as those in Fig. 1. $H_1(z) = (20z^3 - 18z^2 + 12z - 1)/(20z^3 + 2z^2)$ can be verified to be ESPR. The LMIs (A5) is solved, and have the feasible solution as

$$P = \begin{bmatrix} 0.2122 & -0.3546 & 0.4296 & -0.2674 \\ -0.3546 & -1.2627 & 1.8894 & -0.8711 \\ 0.4296 & 1.8894 & 5.5150 & -1.6434 \\ -0.2674 & -0.8711 & -1.6434 & 1.9793 \end{bmatrix}.$$

The norm of the system state is shown in Fig. 4.

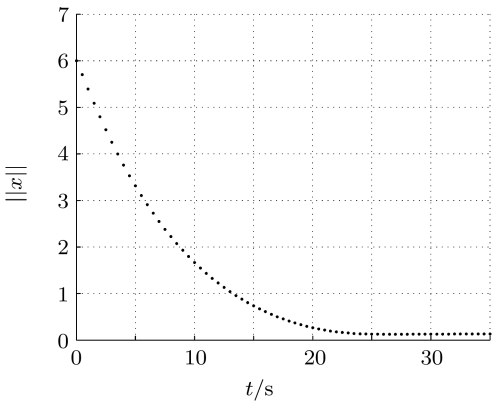


Fig. 4. The state norm $\|x\|$ with the initial value $x(0) = [6, 0]^{\mathrm{T}}$.

6. Conclusion

This work investigates the properties of ferromagnetic hysteresis nonlinearity. The absolute stability problem of discrete descriptor systems with feedback connected ferromagnetic hysteresis nonlinearities is studied for the first time. An extended Tsypkin criterion is presented within the framework of loop transformation. Numerical example is given to demonstrate the effectiveness of the stability criterion.

Appendix: Proof of Theorem 1

Consider a loop transformation shown in Fig. 5, we have:

$$\tilde{G}(z) = z^{-1}G(z).$$

For a discrete descriptor system (10), the transfer function is Eq. (11). Then

$$\begin{aligned} \tilde{G}(z) &= \begin{bmatrix} z^{-1}C(zE - A)^{-1} & z^{-1}I \end{bmatrix} \begin{bmatrix} B \\ D \end{bmatrix} \\ &= \begin{bmatrix} 0 & I \end{bmatrix} \left(z \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} B \\ D \end{bmatrix} \\ &\triangleq \tilde{C}(z\tilde{E} - \tilde{A})^{-1}\tilde{B}. \end{aligned} \tag{A1}$$

Thus the transformed system can be described in the augmented state-space model as

$$\begin{cases} \tilde{E}\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k), \\ \tilde{y}(k) = \tilde{C}\tilde{x}(k) + \tilde{D}\tilde{u}(k), \end{cases} \tag{A2}$$

where the augmented system state is $\tilde{x}(k) \triangleq [x^{\mathrm{T}}(k+1), y^{\mathrm{T}}(k)]^{\mathrm{T}} \in R^{n+m}$, the augmented system output is $\tilde{y}(k) \triangleq y(k) \in R^m$, the augmented system input is $\tilde{u}(k) \triangleq u(k+1) \in R^m$, and the augmented system matrices are

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ D \end{bmatrix},$$
$$\tilde{C} = \begin{bmatrix} 0 & I \end{bmatrix}, \quad \tilde{D} = 0.$$

For the system (14) considered in this paper, the augmented system matrix $\tilde{B}$ is $\tilde{B} = \begin{bmatrix} B^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}}$, hence the transformed system is as follows:

$$\begin{cases} \tilde{E}\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k), \\ \tilde{y}(k) = \tilde{C}\tilde{x}(k), \\ \tilde{u}(k) = -\tilde{\Phi}[\tilde{y}](k). \end{cases} \tag{A3}$$

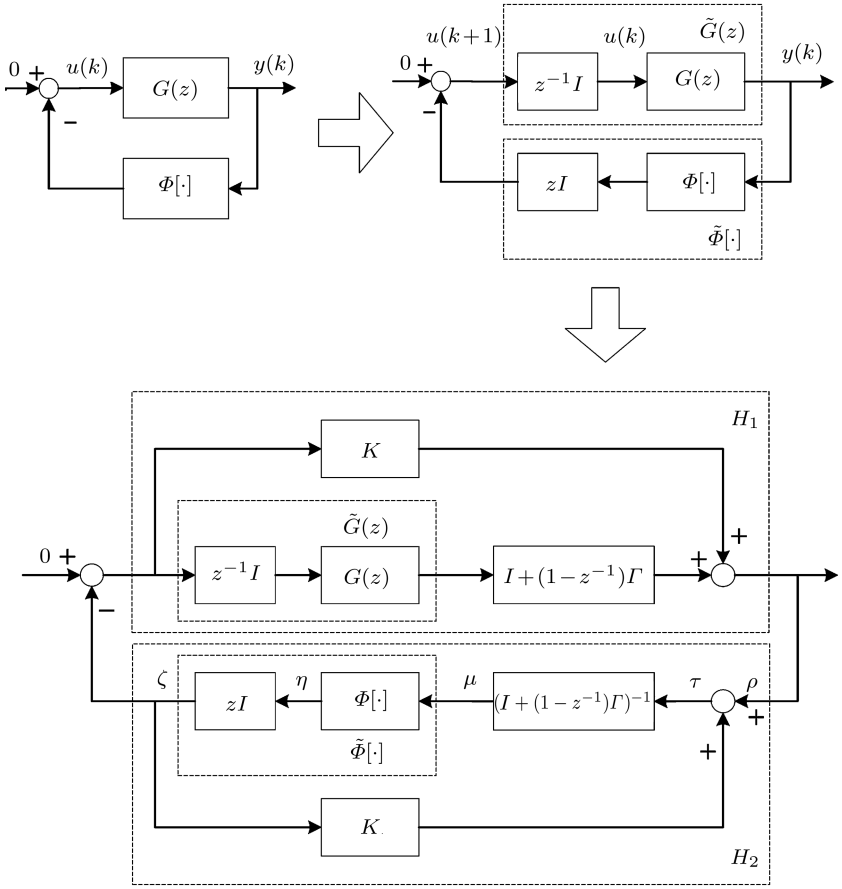


Fig. 5. Loop transformation.

First we should verify that the transfer function $H_1(z)$ is ESPR. From the loop transformation shown in Fig. 5, we transform the augmented system as follows:

$$\begin{aligned} H_1(z) &= K + (I + (1 - z^{-1})\Gamma)\tilde{G}(z) \\ &= K + \begin{bmatrix} (I+\Gamma)\tilde{C} & -\Gamma \end{bmatrix} \begin{bmatrix} (z\tilde{E} - \tilde{A})^{-1} & 0 \\ (zI)^{-1}\tilde{C}(z\tilde{E} - \tilde{A})^{-1} & (zI)^{-1} \end{bmatrix} \begin{bmatrix} \tilde{B} \\ 0 \end{bmatrix} \\ &= K + \begin{bmatrix} \tilde{C} + \Gamma\tilde{C} & -\Gamma \end{bmatrix} \left(z \begin{bmatrix} \tilde{E} & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C} & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} \tilde{B} \\ 0 \end{bmatrix} \\ &\triangleq \hat{D} + \hat{C}(z\hat{E} - \hat{A})^{-1}\hat{B}, \end{aligned} \tag{A4}$$

where the transformed augmented system matrices are

$$\hat{E} = \begin{bmatrix} E & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & 0 \\ 0 & I & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} 0 & I + \Gamma & -\Gamma \end{bmatrix}, \quad \hat{D} = K.$$

Define the transformed augmented system state as $\hat{x}(k) \triangleq [\tilde{x}^{\mathrm{T}}(k+1), \tilde{y}^{\mathrm{T}}(k)]^{\mathrm{T}} = [x^{\mathrm{T}}(k+2), y^{\mathrm{T}}(k+1), y^{\mathrm{T}}(k)]^{\mathrm{T}}$, the output as $\hat{y}(k) \triangleq \tilde{y}(k) = y(k)$, the input as $\hat{u}(k) \triangleq \tilde{u}(k+1) = u(k+2)$.

Note that $\hat{E}\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}\hat{u}(k) = \hat{A}\hat{x}(k) - \hat{B}\tilde{\Phi}[\tilde{y}](k+1)$, where the hysteresis nonlinearities are $\tilde{\Phi}[\cdot] = (\tilde{\phi}_1[\cdot], \ldots, \tilde{\phi}_m[\cdot])^{\mathrm{T}}$, the transformed system matrix $\hat{C}$ is $\hat{C} = \begin{bmatrix} 0 & I + \Gamma & -\Gamma \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \end{bmatrix} + \Gamma \begin{bmatrix} 0 & I & -I \end{bmatrix} \triangleq \hat{C}_1 + \Gamma\hat{C}_2$, then the output can be recognized as $y(k+1) = \hat{C}_1\hat{x}(k)$, and $y(k) = (\hat{C}_1 - \hat{C}_2)\hat{x}(k)$.

The equations and inequality in expression (15) are equivalent to the LMIs as follows:

$$\begin{aligned}&\hat{E}^{\mathrm{T}}P\hat{E}\geq 0,\\&\begin{bmatrix}\hat{A}^{\mathrm{T}}P\hat{A}-\hat{E}^{\mathrm{T}}P\hat{E} & \hat{C}^{\mathrm{T}}-\hat{A}^{\mathrm{T}}P\hat{B}\\ \hat{C}-\hat{B}^{\mathrm{T}}P\hat{A} & -2K+\hat{B}^{\mathrm{T}}P\hat{B}\end{bmatrix}<0.\end{aligned}\tag{A5}$$

According to Lemma 1, $H_1(z)$ is ESPR.

Next, we prove that the system is absolutely stable. Consider the Lyapunov function candidate as follows:

$$V(\hat{x}(k))=\hat{x}^{\mathrm{T}}(k)\hat{E}^{\mathrm{T}}P\hat{E}\hat{x}(k)+2\sum_{i=1}^{m}\int_{0}^{\hat{y}_i(k)}\gamma_i\tilde{\phi}_i[\sigma]\,\mathrm{d}\sigma.\tag{A6}$$

Since $\hat{E}^{\mathrm{T}}P\hat{E}$ is positive semi-definite, the ferromagnetic hysteresis nonlinearity Φ satisfies the properties mentioned in Section 2 and Proposition 2, and $\Gamma>0$, it follows that $V(\hat{x}(k))$ given by Eq. (A6) is positive-definite. The Lyapunov function difference is given as follows:

$$\begin{aligned}\Delta V(\hat{x}(k))=&V(\hat{x}(k+1))-V(\hat{x}(k))\\=&\hat{x}^{\mathrm{T}}(k)\hat{A}^{\mathrm{T}}P\hat{A}\hat{x}(k)-\hat{x}^{\mathrm{T}}(k)\hat{E}^{\mathrm{T}}P\hat{E}\hat{x}(k)-\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)\hat{B}^{\mathrm{T}}P\hat{A}\hat{x}(k)\\&-\hat{x}^{\mathrm{T}}(k)\hat{A}^{\mathrm{T}}P\hat{B}\tilde{\Phi}[\tilde{y}](k+1)+\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)\hat{B}^{\mathrm{T}}P\hat{B}\tilde{\Phi}[\tilde{y}](k+1)+2\sum_{i=1}^{m}\int_{\hat{y}_i(k)}^{\hat{y}_i(k+1)}\gamma_i\tilde{\phi}_i[\sigma]\,\mathrm{d}\sigma.\end{aligned}\tag{A7}$$

Note that the integral term in Eq. (A7) can be bounded by

$$\sum_{i=1}^{m}\int_{\hat{y}_i(k)}^{\hat{y}_i(k+1)}\gamma_i\tilde{\phi}_i[\sigma]\,\mathrm{d}\sigma\leq\sum_{i=1}^{m}\tilde{\phi}_i[\hat{y}_i](k+1)\gamma_i(\hat{y}_i(k+1)-\hat{y}_i(k)),$$

thus

$$\begin{aligned}\Delta V(\hat{x}(k))\leq&\hat{x}^{\mathrm{T}}(k)\hat{A}^{\mathrm{T}}P\hat{A}\hat{x}(k)-\hat{x}^{\mathrm{T}}(k)\hat{E}^{\mathrm{T}}P\hat{E}\hat{x}(k)-\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)\hat{B}^{\mathrm{T}}P\hat{A}\hat{x}(k)\\&-\hat{x}^{\mathrm{T}}(k)\hat{A}^{\mathrm{T}}P\hat{B}\tilde{\Phi}[\tilde{y}](k+1)+\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)\hat{B}^{\mathrm{T}}P\hat{B}\tilde{\Phi}[\tilde{y}](k+1)\\&+2\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)\Gamma[\hat{y}(k+1)-\hat{y}(k)].\end{aligned}\tag{A8}$$

Note that $\Gamma(\hat{y}(k+1)-\hat{y}(k))=\Gamma(y(k+1)-y(k))=\Gamma(\hat{C}_1\hat{x}(k)-(\hat{C}_1-\hat{C}_2)\hat{x}(k))=\Gamma\hat{C}_2\hat{x}(k)=(\hat{C}-\hat{C}_1)\hat{x}(k)=\hat{C}\hat{x}(k)-\tilde{y}(k+1)$, thus,

$$\begin{aligned}\Delta V(\hat{x}(k))\leq&\hat{x}^{\mathrm{T}}(k)(\hat{A}^{\mathrm{T}}P\hat{A}-\hat{E}^{\mathrm{T}}P\hat{E})\hat{x}(k)+\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)(\hat{C}-\hat{B}^{\mathrm{T}}P\hat{A})\hat{x}(k)\\&+\hat{x}^{\mathrm{T}}(k)(\hat{C}-\hat{B}^{\mathrm{T}}P\hat{A})^{\mathrm{T}}\tilde{\Phi}[\tilde{y}](k+1)-\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)(2K-\hat{B}^{\mathrm{T}}P\hat{B})\tilde{\Phi}[\tilde{y}](k+1)\\&+2\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)(K\tilde{\Phi}[\tilde{y}](k+1)-\tilde{y}(k+1)).\end{aligned}\tag{A9}$$

Using expression (15) yields

$$\begin{aligned}\Delta V(\hat{x}(k))\leq&-(L\hat{x}(k)-W\tilde{\Phi}[\tilde{y}](k+1))^{\mathrm{T}}(L\hat{x}(k)-W\tilde{\Phi}[\tilde{y}](k+1))\\&+2\tilde{\Phi}[\tilde{y}]^{\mathrm{T}}(k+1)(K\tilde{\Phi}[\tilde{y}](k+1)-\tilde{y}(k+1)).\end{aligned}\tag{A10}$$

From inequality (9) we obtain $\Delta V(\hat{x}(k))\leq 0$, thus the absolute stability of the considered systems is proved.

References

[1] Khalil H K 2001 *Nonlinear Systems* 3rd edn. (New York: Prentice Hall) p. 18

[2] Visintin A 1994 *Differential Models of Hysteresis* (New York: Springer-Verlag) p. 12

[3] Brokate M and Sprekels J 1996 *Hysteresis and Phase Transitions* (New York: Springer-Verlag) p. 22

[4] Oh J and Bernstein D S 2005 *IEEE T Automat. Control* **50** 631
[5] Hodgdon M L 1988 *IEEE T Magn.* **24** 218
[6] Sabir M 1995 *Int. J. Eng. Sci.* **33** 1233
[7] Wei Z and Zhao Y P 2004 *Chin. Phys.* **13** 1320
[8] Gong Y J, Zhang D, Gong X F, Tan K B and Liu Z 2006 *Chin. Phys.* **15** 1526
[9] Yan H, Jing H Y, Liu W J and Ulannov A M 2009 *Acta Phys. Sin.* **58** 5238 (in Chinese)
[10] Paré T, Hassibi A and How J 2001 *Int. J. Control* **74** 1140
[11] Song Z, Zhang H G and Liu D R 2008 *Proceedings of 27th Chinese Control Conference* Kunming, Yunnan, China, July 16–18, p. 24
[12] Yang C Y, Zhang Q L, Lin Y P and Zhou L N 2007 *IEEE T Circuits-I* **54** 1142
[13] Zhang L Q, Lam J and Xu S Y 2002 *IEEE T Circuits-I* **49** 401
[14] Zhang L Q, Huang B and Lam J 2003 *IEEE T Circuits-II* **50** 615
[15] Lee L and Chen J L 2003 *IEEE T Circuits-I* **50** 788
[16] Tian W H and Zhang H G 2008 *J. Syst. Eng. Elec.* **19** 584
[17] Tian W H and Zhang H G 2008 *2008 IEEE International Conference on Networking, Sensing and Control* Sanya, China, April 6–8, p. 864
[18] Luo Y H, Zhang H G, Lun S X and Wang Y C 2008 *Chinese Control and Decision Conferences* China, July 1–4, p. 2437
[19] Wu J R 2004 *Acta Phys. Sin.* **53** 325 (in Chinese)
[20] Ma T D, Zhang H G and Fu J 2008 *Chin. Phys.* B **17** 4407
[21] Xu S Y and Lam J 2006 *Robust Control and Filtering of Singular Systems* (Berlin: Springer-Verlag) p. 119
[22] Liao X X 2006 *Absolute Stability of Nonlinear Control Systems* (Beijing: Science Press) p. 221
[23] Brogliato B, Lozano R, Maschke B and Egeland O 2007 *Dissipative Systems Analysis and Control Theory and Applications:* 2nd edn. (London: Springer) p. 69
[24] Gorbet R B, Morris K A and Wang D W L 2001 *IEEE T Control. Syst. T.* **9** 5
[25] Kapila V and Haddad W M 1996 *IEEE T Automat. Control* **41** 149