Function projective synchronization in partially linear drive–response chaotic systems*

Zhang Rong(张 荣)† and Xu Zhen-Yuan(徐振源)

School of Science, Jiangnan University, Wuxi 214122, *China*

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This paper gives the definition of function projective synchronization with less conservative demand for a scaling function, and investigates the function projective synchronization in partially linear drive–response chaotic systems. Based on the Lyapunov stability theory, it has been shown that the function projective synchronization with desired scaling function can be realized by simple control law. Moreover it does not need scaling function to be differentiable, bounded and non-vanished. The numerical simulations are provided to verify the theoretical result.

Keywords: function projective synchronization, projective synchronization, scaling function, partially linear system

PACC: 0545, 0547

1. Introduction

Since chaotic synchronization has been found to have potential applications in vast areas of physics and engineering science, in particular in secure communication,[1−3] it has become an active research subject in nonlinear science.[4,5] Various types of chaotic synchronization have been investigated.[6−8] Amongst all kinds of these synchronizations, projective synchronization (PS) has been extensively investigated in recent years,[9−16] because it can obtain faster communication with its proportional feature. The PS has been first reported by Mainieri and Rehacek[17] in partially linear chaotic systems, in which the responses of two identical systems synchronize up to a constant scaling factor. Function PS (FPS) is a more general type of PS, because in the FPS the scaling factor is changed from constant to function. As compared with PS, FPS means that the drive and response systems could be synchronized up to a desired scaling function. The FPS could be used to get more secure application to communications, due to the unpredictability of its scaling functions which can additionally enhance the security of communication.

The FPS was first introduced by Tang *et al.*,[18] who gave the FPS of the coupled Lü systems by the backstepping method. Very recently FPS has been investigated in two-cell quantum-CNN chaotic oscillators,[19−21] in coupled partially linear chaotic systems,[22] and in some non-partially-linear system.[23−26] Up to now, there have been only a few papers investigating FPS method. Furthermore, in these designs of FPS scheme, there is always a term associated with the derivative of the scaling function. So the scaling function is always asked to be differentiable, bounded and non-vanished. In this paper we investigate FPS in partially linear drive–response chaotic systems by only using conventional linear feedback coupling technique, moreover it does not need to calculate the derivative of the scaling function, and the scaling function is even not to be bounded and non-vanished.

The organization of this paper is as follows. In Section 2, the definition of FPS is given. In Section 3, the general scheme of FPS is presented. In Section 4, numerical examples are given in order to verify the effectiveness of the proposed scheme. The conclusion is finally drawn in Section 5.

2. The definition of FPS

The drive–response systems are described as follows:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \\ \dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{H}(\boldsymbol{x}, \boldsymbol{y}, t), \end{cases} \tag{1}$$

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†Corresponding author. E-mail: ronia62@yahoo.com

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where $\boldsymbol{x}$, $\boldsymbol{y} \in \boldsymbol{R}^n$ are the state vectors, $\boldsymbol{f}$, $\boldsymbol{g}: \boldsymbol{R}^n \to \boldsymbol{R}^n$ are two vector functions and $\boldsymbol{H}: \{\boldsymbol{R^n}, \boldsymbol{R^n}, \boldsymbol{R}\} \to \boldsymbol{R^n}$ is a controller.

Definition 1 (FPS) The drive–response systems (1) are said to realize FPS if there exists a continuous scaling function $\alpha(t)$ such that

$$\lim_{t\to\infty} \| \boldsymbol{y} - \alpha(t)\boldsymbol{x} \| = 0. \tag{2}$$

Remark 1 Concerning FPS definitions in most previous papers, we define the FPS as that $\alpha(t)$ is not only continuous, but is differentiable, bounded, and $\alpha(t) \neq 0$ for all t. However in this definition it is not necessary for $\alpha(t)$ to be differentiable, and $\alpha(t)$ is even does not need to be bounded and $\alpha(t) \neq 0$ for all t.

3. The scheme of FPS

A partially linear system can be described as follows:

$$\begin{cases} \dot{\boldsymbol{u}} = \boldsymbol{M}(z) \cdot \boldsymbol{u}, \\ \dot{z} = f(\boldsymbol{u}, z), \end{cases} \tag{3}$$

in which the state vector is broken into two parts ($\boldsymbol{u}$, z). The matrix $\boldsymbol{M}(z)$ is only dependent on the variable z.

For a given scaling function $\alpha(t)$, the drive–response chaotic systems can be formed when the two identical systems (3) are linked through the variable z and unidirectionally linearly coupled in the form as

$$\begin{cases} \dot{\boldsymbol{u}} = \boldsymbol{M}(z) \cdot \boldsymbol{u}, \\ \dot{z} = f(\boldsymbol{u}, z), \\ \dot{\boldsymbol{v}} = \boldsymbol{M}(z) \cdot \boldsymbol{v} - c(\boldsymbol{v} - \alpha(t)\boldsymbol{u}), \end{cases} \tag{4}$$

where $\boldsymbol{u} = (u_1, u_2, \ldots, u_n) \in \boldsymbol{R}^n$, $\boldsymbol{v} = (v_1, v_2, \ldots, v_n) \in \boldsymbol{R}^n$, ($\boldsymbol{u}$, z) and $\boldsymbol{v}$ are the state vectors of the drive and response systems, respectively, $\boldsymbol{M}(z) = (m_{ij}(z))_{n\times n} \in \boldsymbol{R}^{n\times n}$, c is the coupling strength constant. The FPS would be achieved in the drive–response systems (4), if $\lim_{t\to\infty} \| \boldsymbol{v} - \alpha(t)\boldsymbol{u} \| = 0$.

Theorem 1 Let the matrix $\boldsymbol{M}(z)$ be bounded with $\| \boldsymbol{M}(z) \| \leq L$. If the coupling strength c satisfies $(n+1)L - c < 0$, then drive–response systems (4) can realize FPS with desired synchronization scaling function $\alpha(t)$.

Proof Let $\boldsymbol{e}_i = \boldsymbol{u}v_i - u_i\boldsymbol{v}$, $i = 1, 2, \ldots, n$. If every e_i tends to 0 in evolution, FPS between the drive system and response system in Eq. (4) would be realized.

The time derivative of $\boldsymbol{e}_i$ is

$$\dot{\boldsymbol{e}}_i = \dot{\boldsymbol{u}}v_i + \boldsymbol{u}\dot{v}_i - \dot{u}_i\boldsymbol{v} - u_i\dot{\boldsymbol{v}}. \tag{5}$$

Substituting Eq. (4) into the above equation, we have

$$\begin{aligned} \dot{\boldsymbol{e}}_i =& \boldsymbol{M}(z)\boldsymbol{u}v_i + \boldsymbol{u}\sum_{k=1}^{n} m_{ik}(z)v_k \\ & - c\boldsymbol{u}(v_i - \alpha(t)u_i) - \sum_{k=1}^{n} m_{ik}u_k\boldsymbol{v} \\ & - u_i\boldsymbol{M}(z)\boldsymbol{v} + cu_i(\boldsymbol{v} - \alpha(t)\boldsymbol{u}) \\ =& \boldsymbol{M}(z)\boldsymbol{e}_i + \sum_{k=1}^{n} m_{ik}(z)\boldsymbol{e}_k - c\boldsymbol{e}_i. \end{aligned} \tag{6}$$

Denote $\boldsymbol{e} = (\boldsymbol{e}_1^{\mathrm{T}}, \boldsymbol{e}_2^{\mathrm{T}}, \ldots, \boldsymbol{e}_n^{\mathrm{T}})^{\mathrm{T}} \in \boldsymbol{R}^{n^2}$, construct dynamical Lyapunov function

$$V = \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} = \frac{1}{2}\sum_{i=1}^{n} \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i. \tag{7}$$

The time derivative of V is

$$\begin{aligned} \dot{V} =& \boldsymbol{e}^{\mathrm{T}}\dot{\boldsymbol{e}} = \sum_{i=1}^{n} \boldsymbol{e}_i^{\mathrm{T}}\dot{\boldsymbol{e}}_i \\ =& \sum_{i=1}^{n} \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{M}(z)\boldsymbol{e}_i + \sum_{i=1}^{n}\sum_{k=1}^{n} m_{ik}(z)\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_k - c\sum_{i=1}^{n} \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i \\ \leq& L\sum_{i=1}^{n} \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i + nL\sum_{i=1}^{n} \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i - c\sum_{i=1}^{n} \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i \\ \leq& [(n+1)L - c]\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}. \end{aligned} \tag{8}$$

Note that the condition $(n+1)L - c < 0$ holds, according to the Lyapunov stability theory, the error vector $\boldsymbol{e}$ exponentially asymptotically tends to zero. This completes the proof. □

Remark 2 Note that the given scaling function has no effect on the time derivative $\dot{V}$. Therefore one can change the scaling function arbitrarily without adjusting the coupling strength, which also shows the robustness of the scheme.

Remark 3 Du *et al.* in their paper[22] also investigated FPS in partially linear chaotic systems. As compared with the control law derived in Ref. [22], the scheme of FPS in this paper is more simple and easier to realize in practice since it only uses the conventional linear feedback coupling technique.

4. Numerical illustrative examples

We use a seven-dimensional chaotic system[22] as an example, to illustrate the effectiveness of the scheme proposed in Section 3. The drive–response systems are described by

$$\begin{cases} \dot{\boldsymbol{u}} = \boldsymbol{M}(z) \cdot \boldsymbol{u}, \\ \dot{z} = 3u_1 u_3 - 14z, \\ \dot{\boldsymbol{v}} = \boldsymbol{M}(z) \cdot \boldsymbol{v} - c(v - \alpha(t)\boldsymbol{u}), \end{cases} \tag{9}$$

where

$$\boldsymbol{M}(z) = \begin{pmatrix} -14 & 8 & -6 & 1 & 2 & 3 \\ -8 & 12 & -(8+3z) & 2 & 3 & 4 \\ 6 & 8+3z & -22 & 3 & 4 & 5 \\ -1 & -2 & -3 & -13 & 5 & 6 \\ -2 & -3 & -4 & -5 & -14 & 8+7z \\ -3 & -4 & -5 & -6 & -(8+7z) & -15 \end{pmatrix}.$$

In the numerical simulations, the coupling strength is always selected as $c = 1.5$. The initial condition is $\{u_1, \ldots, u_6, z, v_1, \ldots, v_6\} = \{0.1, \ldots, 0.6, 0.7, 0.8, \ldots, 0.13\}$.

First, we choose the scaling function $\alpha(t) = 0.5 + 0.1\sin(2\pi t/30)$, note that $\alpha(t)$ is a continuously differentiable and bounded function, and $\alpha(t) \neq 0$ for all t. Figure 1 displays the time evolutions of the scaling functions $\alpha_i(t) = v_i/u_i\,(i = 1, \ldots, 6)$. The projection of the FPS attractors in u–v plane is shown in Fig. 2. The components of the FPS error vector $\boldsymbol{v} - \alpha(t)\boldsymbol{u}$ are shown in Fig. 7(a).

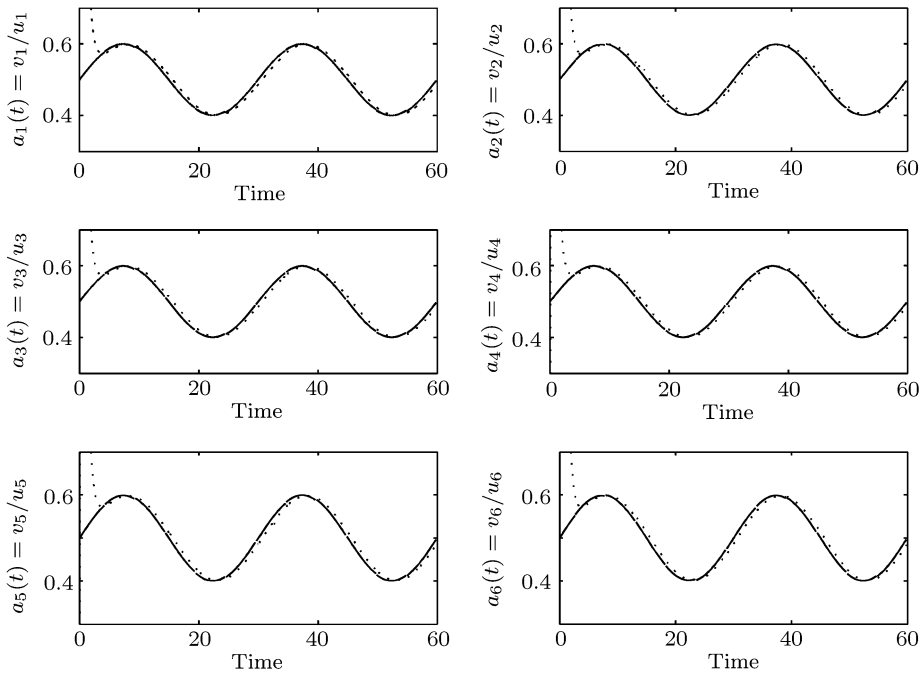


Fig. 1. The time evolution of the scaling function, where the desired scaling function $\alpha(t) = 0.5 + 0.1\sin(2\pi t/30)$ (solid line).

Then, when the scaling function is chosen as $\alpha(t) = 0.2\sin(2\pi t/30)$ which does not satisfy $\alpha(t) \neq 0$ for all t. The time evolutions of the scaling functions $\alpha_i(t) = v_i/u_i\,(i = 1, \ldots, 6)$ are displayed in Fig. 3. The corresponding components of the FPS error vector $\boldsymbol{v} - \alpha(t)\boldsymbol{u}$ are shown in Fig. 7(b).

Furthermore, the scaling function is chosen as $\alpha(t) = |\sin(2\pi t/30)|$, while $\dot{\alpha}(t)$ is not existing for all t. Now figure 4 displays the time evolutions of the corresponding scaling functions $\alpha_i(t) = v_i/u_i\,(i = 1, \ldots, 6)$. The corresponding components of the FPS error vector $\boldsymbol{v} - \alpha(t)\boldsymbol{u}$ are shown in Fig. 7(c).

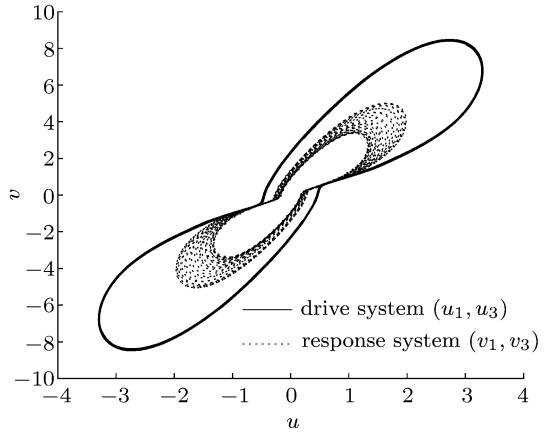


Fig. 2. The projections of the FPS attractors in u–v plane with the desired scaling function $\alpha(t) = 0.5 + 0.1\sin(2\pi\, t/30)$.

Finally we select the scaling function $\alpha(t) = \sqrt[3]{t}$ which is unbounded. Figure 5 displays the time evolutions of the scaling functions $\alpha_i(t) = v_i/u_i\,(i = 1, \ldots, 6)$. The projection of the FPS attractors in u–v plane is shown in Fig. 6. The corresponding components of the FPS error vector $\boldsymbol{v} - \alpha(t)\boldsymbol{u}$ are shown in Fig. 7(d).

Remark 4 They are shown from these figures that various types of the scaling function $\alpha(t)$ have little effect on the simulation results, the FPS can be realized in all cases with the same coupling strength value. Our simulation results also show that the larger the coupling strength value is, the better the effectiveness of the FPS is. And along the idea of Ref. [22], FPS can also be used in secure communication in theory by using the new FPS scheme in this paper.

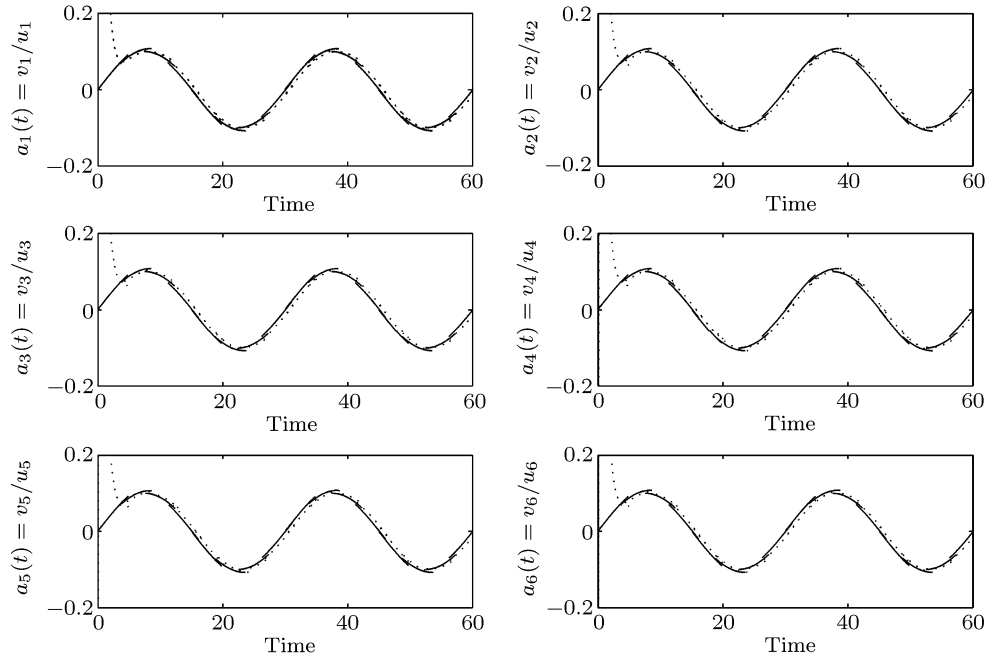
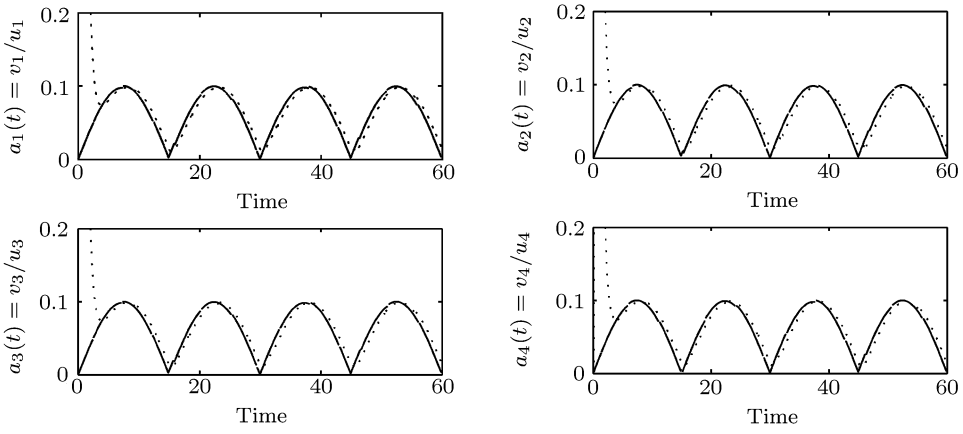


Fig. 3. The time evolution of the scaling function, where the desired scaling function $\alpha(t) = 0.1\sin(2\pi\, t/30)$ (solid line).



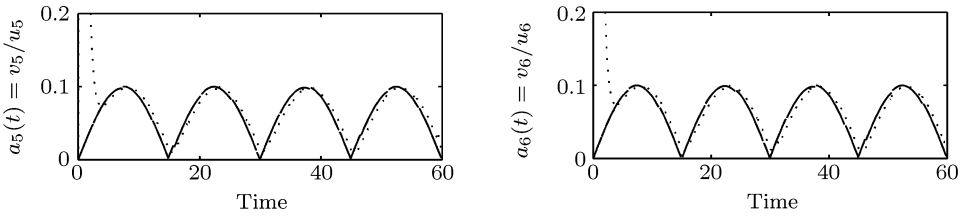


Fig. 4. The time evolution of the scaling function, where the desired scaling function $\alpha(t) = |0.1\sin(2\pi\, t/30)|$ (solid line).

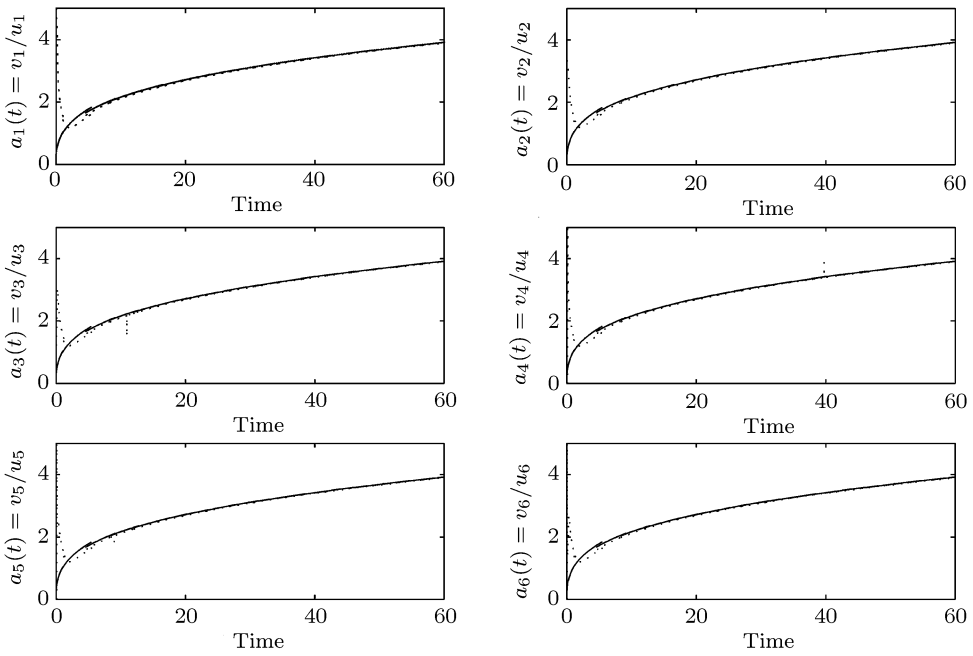


Fig. 5. The time evolution of the scaling function, where the desired scaling function $\alpha(t) = \sqrt[3]{t}$ (solid line).

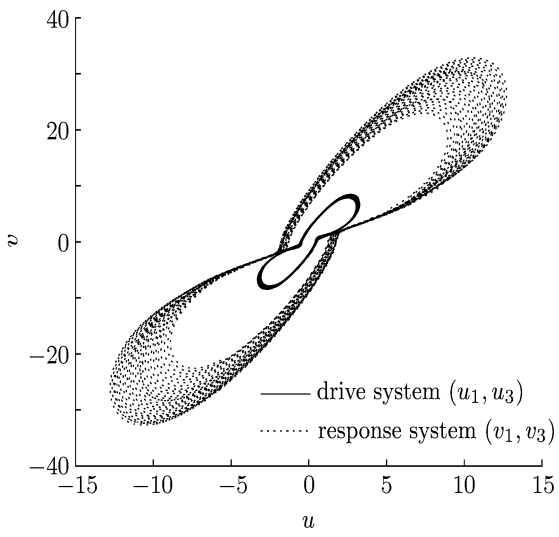


Fig. 6. The projection of the FPS attractors in u–v plane with the desired scaling function $\alpha(t) = \sqrt[3]{t}$.

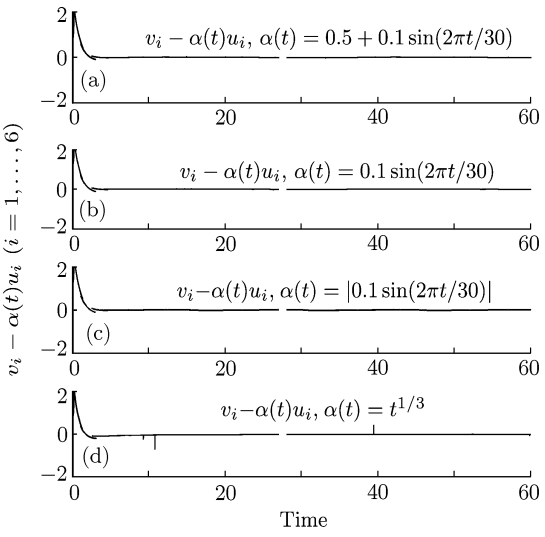


Fig. 7. The function projection synchronization errors.

5. Conclusions

In this paper, we gave the definition of FPS with less conservative demand for the desired scaling function and a general scheme for the FPS in partially linear drive–response chaotic systems by using a conventional linear feedback coupling technique. Numerical simulations showed the effectiveness of the scheme. Further work regarding this topic could be considered to realize FPS on networks or non-partially-linear system by using a simple feedback coupling technique.

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