Cluster consensus of second-order multi-agent systems via pinning control*

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This paper investigates the cluster consensus problem for second-order multi-agent systems by applying the pinning control method to a small collection of the agents. Consensus is attained independently for different agent clusters according to the community structure generated by the group partition of the underlying graph and sufficient conditions for both cluster and general consensus are obtained by using results from algebraic graph theory and the LaSalle Invariance Principle. Finally, some simple simulations are presented to illustrate the technique.

Keywords: second-order multi-agent systems, cluster consensus, pinning control, LaSalle invariance principle

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1. Introduction

The coordinated and cooperative control of multiple autonomous vehicles is a problem which has been much researched in recent years because of its wide-ranging applications to many fields. This problem arises, for example, in the cooperative control of unmanned air vehicles, in formation control, in flocking problems, in distributed sensor networks, in the altitude alignment of satellite clusters, and in the control of communication networks congestion. Loosely speaking, a consensus is an agreement on a common value of a certain parameter which is reached by a number of agents (consider, for example, the common altitude of a fleet of spacecrafts, or the common flying direction of a flock of birds, or the average value of the results of a distributed computation).

Many studies have been conducted on consensus problems for first-order multi-agent systems[1−6] in the past decade after the foundation works of Olfati–Saber, Murray, Ren and Moreau.[7−10] For instance, Hong *et al.*[1] constructed a tracking control model for the consensus of a multi-agent system that has one active leader and a variable inter-connection topology and Hu *et al.*[3] investigated a collective coordination problem for multi-agent systems that are guided by multiple leaders. The results of Ref. [1] were then extended to general asymmetric adjacency matrices by Guo *et al.*[6] and, subsequently, attention switched to second-order consensus problems in which the time-derivatives of the information state motions are considered.[11−14] The results of Ref. [3], for example, were then extended by Hu *et al.*[12] to second-order consensus problems with coupling time delays between the agents, Zhang *et al.*[13] also obtained some new results for systems that are subjected to external disturbances and switching topologies. The results of Hu *et al.*,[12] in turn, were further extended to flocking problems by Su and Wang[14] by considering a virtual leader.

Despite the above literature on the ultimate consensus state of the whole multi-agent system, little is yet known about networks with community structures (i.e. sub-groups of nodes with high inter-linking densities). Community structures are common in biological and social networks and cluster consensus, in particular, are often seen in flocking processes because of the inherently communal structure of the flocks. Cluster consensus is the phenomenon in which the system's agents are segregated into classes, called *clusters*, in such a way that every class attains a consensus state which is distinct from those attained by other classes and it is a phenomenon also appearing in physics and engineering applications in the form of cluster formations,[15,16] cluster-based protocols for wireless sensor networks[17,18] and cluster aggregation

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problems.[19] Second-order consensus is therefore an important problem which must be investigated.

In this paper, we extend our previous results for first-order consensus problems[20] to second-order systems by applying the general synchronization idea,[21,22] and the classical pinning control methods.[23,24] Pinning control is a particularly practical technique because of the small number of injected controllers which are needed for consensus, we shall show in this paper that this technique can also be applied to achieve cluster consensus provided that the pinned agents are chosen in accordance with the topological structure of the underlying graphs. Specifically, for any given group partition of the underlying graph, we shall show that cluster consensus is guaranteed when the pinnings are applied to all those nodes with direct connections between sub-groups and those which are the root vertices of either isolated sub-groups or sub-groups containing connected components. These results will be rigorously proved and stated in the form of sufficient conditions for general and cluster consensus of multi-agent systems.

This paper is organized as follows. In Section 2, we formulate the model for second-order multi-agent systems on graphs and write down the definitions, some well-known results and assumptions which will be needed for the rest of the paper. The pinning control strategies and the main results are contained in Section 3 and some simulation results are given in Section 4. Section 5 concludes the paper.

The notation used in this paper is standard. The R^n is the n-dimensional Euclidean space. $R^{m\times n}$ denotes the set of $m \times n$ real matrix, $\|\cdot\|$ denotes Euclidean norm, diag$\{\cdots\}$ denotes the block diagonal matrix.

2. Model description and preliminaries

Algebraic graph theory is a natural framework for analysing consensus problems[1,20] because of its discrete and inter-connected nature. Let $G = (V, \varepsilon, A)$ be an undirected graph of order N with nodes $V = \{1, 2, \ldots, N\}$, arcs $\varepsilon \subseteq V \times V$ (with (i, j) denoting an arc of G from node i to node j) and adjacency matrix $A = (a_{ij}) \in R^{N\times N}$ where $a_{ii} = 0$, $a_{ij} = a_{ji} \geq 0$ for $i \neq j$ and $a_{ij} > 0$ if and only if node i receives information from node j. A degree matrix of G is a diagonal matrix $D = \text{diag}\{d_1, d_2, \ldots, d_n\} \in R^{N\times N}$, where $d_i = \sum_{j\in N_i} a_{ij}$ for $i = 1, 2, \ldots, N$ and the Laplacian of the undirected graph G is then defined as $L = D - A \in R^{N\times N}$. A symmetric Laplacian of order N is m-reducible if it is not $(m+1)$-reducible for $1 \leq m < N$ and every such matrix has the form

$$L = P \begin{pmatrix} L_{11} & 0 & \cdots & 0 \\ 0 & L_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L_{mm} \end{pmatrix} P^{\mathrm{T}}, \qquad (1)$$

for some permutation matrix P and irreducible symmetric Laplacian matrices L_{ii} for $i = 1, \ldots, m$. Clearly, the graph G is connected if and only if the Laplacian matrix L associated with G is irreducible. A spanning tree is a subgraph of G with N nodes and $N - 1$ edges such that there is a path from one vertex (called the root vertex) to every other vertex. In general, the root vertex is not unique.

Some well-known results will be used later. The following well-known result establishes a direct relationship between the graph connectivity and its Laplacian. It can be found in Ref. [6].

If the Laplacian matrix $L = (l_{ij}) \in R^{N\times N}$ corresponding to the undirected graph G is irreducible, then (i) L has an eigenvalue 0 of multiplicity 1 with corresponding eigenvector $[1, 1, \ldots, 1]^{\mathrm{T}}$ and all the nonzero eigenvalues of L are positive; (ii) all the eigenvalues of $L + B$ are positive, where $B = \text{diag}\{b_1, b_2, \ldots, b_N\}$, $\sum_{i=1}^{N} b_i > 0$ and $b_i \geq 0$ for $i = 1, 2, \ldots, N$.

In this paper, we consider a system made up of N agents (indexed by $1, 2, \cdots, N$) with interrelationships which are described by a graph G with N vertices (one agent at each vertex). Obviously, different community structures can arise by taking different group partitions of this graph and so, for simplicity, we make the following assumptions for our model.[21] Let $N_k = \{l_{k-1} + 1, \ldots, l_k\}$, $k \in \{1, 2, \ldots, p\}$ be the index set of all the agents in the k-th group with $l_0 = 1, l_p = N$ and $l_{k-1} < l_k$, $l_k \in N$ and let E_k be the index set of all the nodes in the k-th group which have direct connections with other groups. Let V_{ik} be the set of nodes outside the k-th group which have direct connections with node i in the k-th group and R_k be the root nodes of the k-th group. It is then clear that $E_k \subseteq N_k$, $R_k \subseteq N_k$, $\cup_{k=1}^{p} N_k = \{1, 2, \ldots, N\}$ and $\cup_{k=1}^{p} (E_k \cup R_k)$ is a subset of some sub-group of nodes (because of the community structure). Our aim is to

control the nodes in $\cup_{k=1}^{p}(E_k \cup R_k)$ so that the agents of group N_k will all converge to one single state and the convergent states of groups N_k and $N_{\tilde{k}}$ are distinct for all $k \neq \tilde{k}$ (i.e. cluster consensus is produced).

Let x_i be the state of agent i for $i = 1, \ldots, N$ and assume that the agents are moving in an n-dimensional Euclidean space with a motion described by two integrators

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = \sum_{j=1}^{N} a_{ij}\left[(x_j - x_i) + (v_j - v_i)\right] + u_i, \end{cases} \quad x_i, v_i, u_i \in R^n, i \in N_k, k = 1, \ldots, p, \tag{2}$$

where x_i and v_i are the position and velocity vectors of agent i respectively and u_i is the control input which can be taken to be the acceleration regulator of agent i.

3. Main results

In this section, we consider the cluster consensus problem for the dynamical system (2) when it is under a certain control protocol. Letting $G = \cup_{k=1}^{p} G_k$ be a group partition of the underlying graph G, we shall first consider the case when all the G_k's are connected and then extend it to the case where G_k has m_k connected components for $k = 1, 2, \ldots, p$.

Define the error variables $ex_i = x_i - sx_k$ and $ev_i = v_i - sv_k$ for $k = 1, 2, \ldots, p$ and $i \in N_k$, where sx_k and sv_k can be equilibrium points, periodic orbits or chaotic orbits in phase space such that $s\dot{x}_k = sv_k$ and $s\dot{v}_k = 0$. The N agents are said to achieve cluster consensus if $\lim_{t\to\infty} \|ex_i\| \leq C$ and $\lim_{t\to\infty} \|ev_i\| = 0$ for all $i = 1, 2, \ldots, N$ and some real constant C. Take the control protocol

$$u_i = -b_i[(x_i - sx_k) + (v_i - sv_k)] - d_i \sum_{j \in V_{ik}} a_{ij}[(x_j - x_i) + (v_j - v_i)], \tag{3}$$

where

$$\begin{cases} b_i > 0, \ i \in R_k, \\ b_i = 0, \ i \in N_k \backslash R_k, \end{cases} \quad \begin{cases} d_i = 1, \ i \in E_k, \\ d_i = 0, \ i \in N_k \backslash E_k, \end{cases} \quad k = 1, 2, \ldots, p.$$

Then the corresponding error system with respect to Eq. (2) is

$$\begin{cases} e\dot{x}_i = ev_i, \\ e\dot{v}_i = \sum_{j \in N_k} a_{ij}\left[(ex_j - ex_i) + (ev_j - ev_i)\right] - b_i\left(ex_i + ev_i\right) \end{cases} \tag{4}$$

for $i \in N_k$. Now construct an energy function

$$V\left(ex_i, ev_i\right) = \frac{1}{2} \sum_{j \in N_k} \left(\sum_{j \in N_k} a_{ij}\left(ex_i - ex_j\right)^2 + b_i ex_i^{\mathrm{T}} ex_i + ev_i^{\mathrm{T}} ev_i \right),$$

and denote the initial energy by V_0. It is clear that $V \geq 0$.

Taking the energy V as a Lyapunov function, we have

$$\begin{aligned} \frac{\mathrm{d}V(t)}{\mathrm{d}t} &= \frac{1}{2} \sum_{i \in N_k} \Big\{ 2 \sum_{j \in N_k} ev_i^{\mathrm{T}} a_{ij}\left(ex_i - ex_j\right) + 2 b_i ev_i^{\mathrm{T}} ex_i \\ &\quad + 2 ev_i^{\mathrm{T}} \Big[\sum_{j \in N_k} a_{ij}\left[(ex_j - ex_i) + (ev_j - ev_i)\right] - b_i\left(ex_i + ev_i\right) \Big] \Big\} \\ &= \sum_{i \in N_k} \Big\{ \sum_{j \in N_k} ev_i^{\mathrm{T}} a_{ij}\left(ev_j - ev_i\right) - b_i ev_i^{\mathrm{T}} ev_i \Big\} \\ &= \sum_{i,j \in N_k} ev_i^{\mathrm{T}} a_{ij} ev_j - \sum_{i,j \in N_k} ev_i^{\mathrm{T}} a_{ij} ev_i - \sum_{i \in N_k} b_i ev_i^{\mathrm{T}} ev_i \\ &= \left(ev^k\right)^{\mathrm{T}} \left(A_k - D_k - B_k\right) ev^k = -\left(ev^k\right)^{\mathrm{T}} \left(L_k + B_k\right) ev^k = -\left(ev^k\right)^{\mathrm{T}} \left(L_k^0 + L_k^1 + B_k\right) ev^k, \end{aligned}$$

where $ev^k = [v_{l_{k-1}+1} - sx_k, \ldots, v_{l_k} - sx_k]^{\mathrm{T}}$, A_k, D_k and L_k^0 are respectively the velocity error, the adjacency matrix, the diagonal matrix and the Laplacian of the k-th group, $B_k = \mathrm{diag}\{b_{l_{k-1}+1}, \ldots, b_{l_k}\}$, $L_k = D_k - A_k$ and $L_k^1 = L_k - L_k^0$ (which is the residual part). Since G_k is connected, L_k^0 is semi-positive definite for $k = 1, 2, \ldots, p$ (by the result in Section 2). Also, $L_k^1 \neq 0$ if the k-th group G_k is not isolated and $B_k \neq 0$ if it is. Hence it is easy to claim that $L_k^0 + L_k^1 + B_k$ is positive-definite and so

$$\dot{V} = -\sum_{k=1}^{p} \left(ev^k\right)^{\mathrm{T}} \left(L_k^0 + L_k^1 + B_k\right) ev^k \leq 0,$$

which means that the energy function $V(ex_i, ev_i)$ is decreasing. Thus $b_i ex_i^{\mathrm{T}} ex_i \leq 2V_0$ for each $i \in N_k$ and $k = 1, 2, \ldots, p$ (provided that the initial energy V_0 is finite).

On the other hand, for those agents with $b_i \neq 0$, we have $\lim\limits_{t\to\infty} \|ex_i\| \leq \sqrt{2V_0/b_i}$, and for those agents with $b_i = 0$, there always exists a path of finite length between the root nodes and other nodes in the same group, then $\lim\limits_{t\to\infty} \|ex_i\| \leq \max\limits_{\forall j, b_j \neq 0} \sqrt{2V_0/b_j} + h_i$ (for some positive constant h_i). Hence

$$\lim_{t\to\infty} \|ex_i\| \leq C = \max_i(\sqrt{2V_0/b_i} + h_i)$$

for each $i \in N_k$ and $k = 1, 2, \ldots, p$.

Finally, since

$$S = \left\{ \left[(ev^1)^{\mathrm{T}}, (ev^2)^{\mathrm{T}}, \ldots, (ev^p)^{\mathrm{T}}\right]^{\mathrm{T}} \in R^{2Nn} : V \leq V_0 \right\}$$

is a compact invariant set (because V is positive-definite and $\dot{V} \leq 0$), all agent trajectories which start from S will converge to the largest invariant set

$$S_0 = \left\{ \left[(ev^1)^{\mathrm{T}}, (ev^2)^{\mathrm{T}}, \ldots, (ev^p)^{\mathrm{T}}\right]^{\mathrm{T}} \in R^{2Nn} : \dot{V} = 0 \right\},$$

in accordance with the LaSalle invariance principle. Therefore the positive definiteness of the matrix $L_k^0 + L_k^1 + B_k$ implies that $\dot{V} = 0$, i.e., $ev^1 \equiv ev^2 \equiv \cdots \equiv ev^p = 0$, which also means that $v_{l_{k-1}+1} \equiv v_{l_{k-1}+2} \equiv \cdots \equiv v_{l_k} \equiv sv_k$ for all $k = 1, 2, \ldots, p$. Hence $\lim\limits_{t\to\infty} \|ev_i\| = 0$ for all $i \in N_k$ and $k = 1, 2, \ldots, p$. Then we have the following result.

Consider a system of N mobile agents, each with dynamic equation (2). Suppose that the initial energy V_0 is finite and the graph G_k is connected for $k = 1, 2, \ldots, p$, then the N agents can achieve cluster consensus.

Now suppose that the k-th group G_k is not connected but has m_k connected components. Then it follows from the definition of the linear control protocol u_i and the previous result in Section 2 that $L_k^0 + L_k^1 + B_k$ is positive-definite.

In view of the discussion of the result above, it suffices to claim that $L_k^0 + L_k^1 + B_k$ is positive-definite. Now, because there are m_k connected components in the k-th group G_k, the Laplacian L_k^0 corresponding to the graph of G_k has the form $L_k^0 = \mathrm{diag}\{L_{11}, L_{22}, \ldots, L_{m_k m_k}\}$, where L_{ii} is an irreducible symmetric block matrix for each $i = 1, 2, \ldots, m_k$. Since B_k can be rewritten in the form $B_k = \mathrm{diag}\{B_k^1, B_k^2, \ldots, B_k^{m_k}\}$, where $B_k^i \neq 0$ for each $i = 1, 2, \ldots, m_k$ (by definition of b_i in B_k), it is easy to verify that $L_{ii} + B_k^i$ is positive-definite for all $i = 1, 2, \ldots, m_k$. Hence, both $L_k^0 + B_k$ and $L_k^0 + L_k^1 + B_k$ are positive-definite and it can be shown (by proceeding in the same way as in the discussion of the result above) that cluster consensus can be attained by the N agents. Therefore, the following result holds.

Consider a system of N mobile agents, each with dynamic equation (2). Suppose that the initial energy V_0 is finite and the k-th graph G_k has m_k connected components for $k = 1, 2, \ldots, p$, then the N agents can achieve cluster consensus.

In particular, if we consider the graph G as one group (i.e., $p = 1$), then the cluster consensus problem becomes the general consensus problem. From the previous results, we conclude that if the initial energy V_0 is finite and the graph G has m connected components (or G is connected), then the N agents (each with dynamic equation (2)) can achieve consensus.

4. Numerical simulation

In this section, we demonstrate the efficiency of the pinning scheme introduced in this paper by considering a tree-shaped graph with 10 nodes. Community structures are common in tree-shaped graphs and this one (shown in Fig. 1) is group-partitioned into two communities N_1 and N_2 with N_2 containing two connected components. The adjacency and Laplacian matrices are

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$L = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix},$$

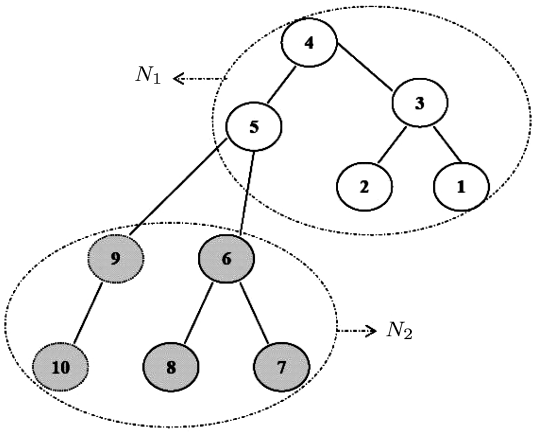


Fig. 1. The coupling topology G of 10 agents with a tree-shaped graph that is group-partitioned into two communities N_1 and N_2. The white circles represent members of community N_1 and the gray circles represent those of N_2. Community N_2 has two connected components, namely, the nodes $\{6, 7, 8\}$ and $\{9, 10\}$.

and if we partition G into two graphs G_1 and G_2 with nodes $N_1 = \{1, 2, 3, 4, 5\}$ and $N_2 = \{6, 7, 8, 9, 10\}$, then the corresponding matrices are

$$L_1 = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 2 \end{pmatrix},$$

and

$$L_2 = \begin{pmatrix} 3 & -1 & -1 & & \\ -1 & 1 & 0 & & 0 \\ -1 & 0 & 1 & & \\ & & & 2 & -1 \\ & 0 & & -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & -1 & & \\ -1 & 1 & 0 & & 0 \\ -1 & 0 & 1 & & \\ & & & 1 & -1 \\ & 0 & & -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & & \\ & & & 1 & 0 \\ & 0 & & 0 & 0 \end{pmatrix}. \quad (5)$$

Take $B = \mathrm{diag}\{B_1, B_2\}$, where $B_1 = \mathrm{diag}\{0, 0, 0, 0, 3\}$ and $B_2 = \mathrm{diag}\{2, 0, 0, 2, 0\}$. The time-evolution of the positions and velocities of the two groups are then shown in Figs. 2 and 3 where cluster consensus has clearly been attained by pinning only the agents 5, 6 and 9. The evolution of the position and velocity

errors of the two groups are shown in Figs. 4 and 5.

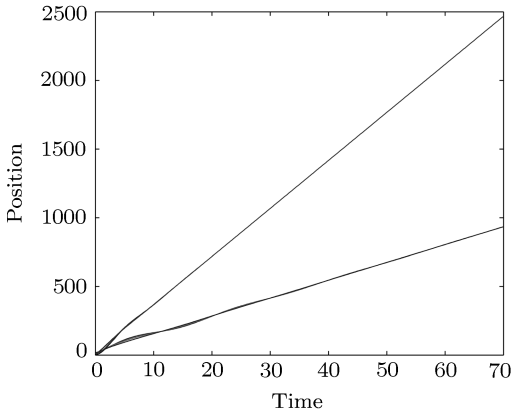


Fig. 2. Time evolution of the positions of the two groups N_1 and N_2.

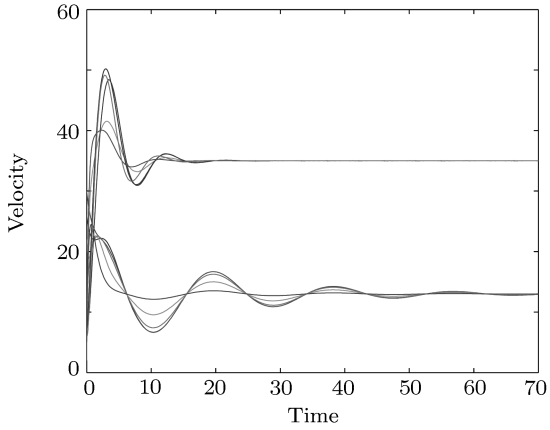


Fig. 3. Time evolution of the velocities of the two groups N_1 and N_2.

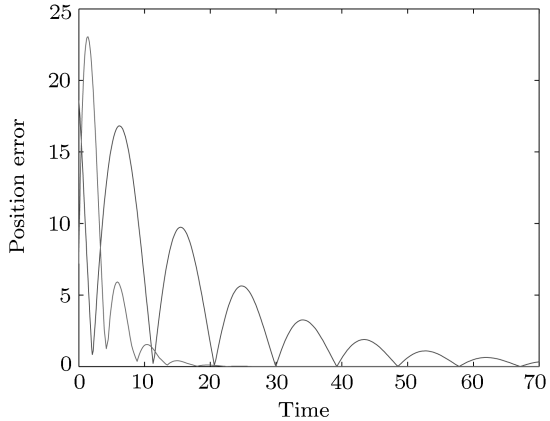


Fig. 4. Time evolution of the position errors of the two groups N_1 and N_2.

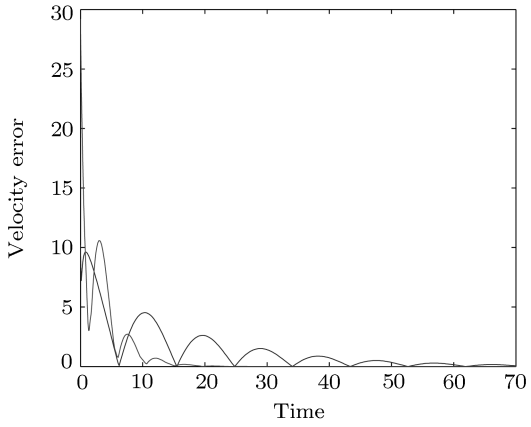


Fig. 5. Time evolution of the velocity errors of the two groups N_1 and N_2.

Note that the pinning scheme discussed in this paper is not unique in the sense that the root vertices of groups with connected components may, for example, be pinned in different ways. Consider the graph of Fig. 1 where the group N_2 is made up of two connected components $\{6,7,8\}$ and $\{9,10\}$ with root vertices $\{6,9\}$ and $\{8,10\}$. The nodes $\{6,9\}$ are pinned instead of $\{8,10\}$ because they provide direct connections between N_1 and N_2 which, in practical terms, could translate into a lowering of the control costs. The same choice is possible, in general, for the sets E_k and R_k of an N-agent system.

Cluster consensus of second-order multi-agent systems is an important phenomenon in the study of the coordinated movements of agents in a network because it has wide-ranging potential applications to commercial and military technologies such as the designing of space-based interferometers and combat reconnaissance systems. Future investigations, however, should be focused on the cluster consensus problem of second-order systems with directed underlying graphs because it is an area as yet poorly understood.

5. Conclusion

In this paper, we investigated the cluster consensus problem for second-order multi-agent systems with community structure and obtained sufficient conditions for both cluster and general consensus. We introduced a pinning control scheme which is based on the topological structure of the communities of the underlying graphs and illustrated the results by simulating a 10-agent system with a tree-shaped graph.

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