Generalized projective synchronization via the state observer and its application in secure communication*

Wu Di(吴 迪) and Li Juan-Juan(李娟娟)†

School of Computer Science and Technology, Dalian University of Technology, Dalian 116023, *China*

(Received 21 June 2010; revised manuscript received 2 July 2010)

Based on the improved state observer and the pole placement technique, by adding a constant which extends the scope of use of the original system, a new design method of generalized projective synchronization is proposed. With this method, by changing the projective synchronization scale factor, one can achieve not only complete synchronization, but also anti-synchronization, as well as arbitrary percentage of projective synchronization, so that the system may attain arbitrary synchronization in a relatively short period of time, which makes this study more meaningful. By numerical simulation, and choosing appropriate scale factor, the results of repeated experiments verify that this method is highly effective and satisfactory. Finally, based on this method and the relevant feedback concept, a novel secure communication project is designed. Numerical simulation verifies that this secure communication project is very valid, and moreover, the experimental result has been greatly improved in decryption time.

Keywords: state observer, generalized projective synchronization, secure communication

PACC: 0545, 0555

1. Introduction

In 1990, Pecora and Corroll first proposed the "chaotic synchronization" concept,[1] and achieved the synchronization of two identical coupled chaotic systems driven by the same signal with circuitry in the laboratory. In recent years, adaptive synchronization,[2] generalized synchronization,[1] neural network synchronization[3] and T-S model-based fuzzy synchronization,[4] etc. are all widely used in a variety of chaotic systems. But the synchronization mentioned above usually need to break the controlled system into a number of stable subsystems, or calculate the Lyapunov function to determine the control parameter, and these requirements have limited the use of these synchronization algorithms. However, observer-based method of synchronization has no special requirements on the structure of the original system. It neither requires the solution of the Lyapunov function of error system nor needs the initial state of synchronized chaotic systems to be in the same domain of attraction. By making use of appropriate transformation of the control function, we can construct a series of driving-response systems.

Some recent progress in chaos synchronization has been made, and different types of chaotic synchronization in different chaotic systems have been achieved, for example, the complete synchronization,[1] phase synchronization,[5] lag synchronization,[6] generalized synchronization,[7−11] projective synchronization,[12,13] etc. Recently, a generalized projective synchronization approach was proposed.[14−18] This method associates projection synchronization with generalized synchronization. By changing the projective synchronization scale factor, it makes the response signal in an arbitrary ratio of the output of the driving chaotic system. When the scale factor is greater than zero or less than zero, respectively, it can access to in-phase and anti-phase synchronization of generalized projective synchronization. Obviously, complete synchronization and anti-synchronization are the special cases of generalized projective synchronization. In the chaotic secure communication, projective synchronization can extend binary numbers to Mary to achieve a faster transmission. Therefore, the investigation of the projective synchronization has important theoretical significance and application prospects.

In this paper, based on the improved state observer,[1] we shall study generalized projective synchronization of chaotic systems with a random perturbation. By adding a constant which extends the

*Project supported by the China Postdoctoral Science Foundation (Grant No. 20080431142).

†Corresponding author. E-mail: lijuanjuan1224@126.com

© 2010 Chinese Physical Society and IOP Publishing Ltd

scope of use of the original system, we present a new design method of the projective synchronization state observer under perturbation. With this method, if the scale factor $\partial = 1$, then complete synchronization occurs. However, if $\partial = -1$, then anti-synchronization arises. If ∂ takes other values, then arbitrary percentage of projective synchronization is obtained. Compared with the existing studies, it has a wide range of more practical applications. In numerical simulation, through the repeated experiments, selecting the most appropriate scale factor, using the new Lorenz system which makes the experiment more authoritative, the results show that this method is highly effective, and more importantly, satisfactory both in precision and in speed. Finally, based on this method and relevant feedback concept, a novel secure communication project is designed. We used the new Chu system to carry out simulation, which suggests that the proposed method can be adapted to the latest model. Numerical simulation verifies the validity of this secure communication. Moreover, the experimental result has great improvement in decryption time.

2. The design of generalized projective synchronization

Now, we consider a class of nonlinear feedback chaotic systems described as

$$\begin{cases} \dot{x} = Ax + Bu + Ew + D, \\ y = C^{\mathrm{T}}x, \\ u = f(x), \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^k$, $w \in R^m$ are the state variable, nonlinear feedback input and stochastic disturbance of the system, respectively. $y \in R^p$ is the output of the transmitting system. A, B, C, D and E are known constant matrices, while D is a constant term of the system.

Let rank $E = m$, rank $C^{\mathrm{T}} = p$, and rank $C^{\mathrm{T}}E = m$. Assume that $p \geq m$. At the receiver, when the nonlinear feedback input u can be measured, the state observer of the system equation (1) can be expressed as

$$\begin{cases} \dot{z} = Fz + Gu + Hy + ID, \\ \hat{x} = z - Jy, \end{cases} \tag{2}$$

where $z \in R^n$; $\hat{x} \in R^n$, F, G, H, J and I are unknown constant matrices.

The error dynamic system of projective synchronization can be expressed as

$$e = \hat{x} - \partial x = z - (JC^{\mathrm{T}} + \partial I_n)x.$$

Let $S = \partial I_n$, then S is a nonsingular diagonal matrix, where ∂ is a proportional factor. This item greatly expands the use of this method. By changing ∂, it can not only achieve complete synchronization, but also anti-synchronize, as well as arbitrary percentage of projective synchronization. Let

$$M = JC^{\mathrm{T}} + S. \tag{3}$$

Substituting Eq. (3) into the error dynamic system:

$$\begin{aligned} \dot{e} = {} & Fe + (G - MB)u + (FM + HC^{\mathrm{T}} - MA)x \\ & + (I - M)D - MEw. \end{aligned} \tag{4}$$

If system (4) satisfies

$$\begin{cases} FM + HC^{\mathrm{T}} - MA = 0, \\ G - MB = 0, \\ ME = (JC^{\mathrm{T}} + S)E = 0, \\ I - M = 0, \end{cases} \tag{5}$$

then equality (4) with some simple transformation gives

$$\dot{e} = Fe. \tag{6}$$

Obviously, if all the eigenvalues of F are less than zero, then $\lim_{t->\infty} \|e\| = 0$, $\hat{x}$ will be exponentially convergent to ∂x. Here we consider how to make F meet this requirement.

From $FM + HC^{\mathrm{T}} - MA = 0$, we can obtain

$$F = [MA - (FJ + H)C^{\mathrm{T}}]S^{-1}. \tag{7}$$

Let

$$N = FJ + H, \tag{8}$$

substituting Eq. (8) into Eq. (7) gives

$$F = MAS^{-1} - NC^{\mathrm{T}}S^{-1}.$$

From Eq. (8), we have

$$H = N - FJ. \tag{9}$$

Similarly, from Eq. (5), we further obtain

$$G = MB, \quad I = M. \tag{10}$$

Then the error dynamic system can be deduced as follows:

$$\dot{z} = (MAS^{-1} - NC^{\mathrm{T}}S^{-1})z + Gu + Hy + ID. \tag{11}$$

So that the design of the state observer under random perturbations becomes how to find a suitable J, satisfying $(JC^{\mathrm{T}}+S)E=0$, and a proper N meeting that all the eigenvalues of F, $F=MAS^{-1}-NC^{\mathrm{T}}S^{-1}$ are less than zero.

If we can find an N to make the eigenvalues of $MAS^{-1}-NC^{\mathrm{T}}S^{-1}$ be randomly configured, then the condition that all the eigenvalues of F are less than zero can necessarily be met. That is, if $(MAS^{-1}, C^{\mathrm{T}}S^{-1})$ is completely observable, then the eigenvalues of $MAS^{-1}-NC^{\mathrm{T}}S^{-1}$ can be randomly configured.

From $(JC^{\mathrm{T}}+S)E=0$, we can obtain: $JC^{\mathrm{T}}E=-SE$, and rank $C^{\mathrm{T}}E=m$. With the relevant knowledge of matrix, we have

$$J=-SE(C^{\mathrm{T}}E)^{+}+Y(I_p-(C^{\mathrm{T}}E)(C^{\mathrm{T}}E)^{+}), \quad (12)$$

where $(C^{\mathrm{T}}E)^{+}$ is $(C^{\mathrm{T}}E)$'s generalized inverse, Y is an arbitrary matrix of appropriate dimension. We know that $M=JC^{\mathrm{T}}+S$, then substituting Eq. (12) into it, we have

$$M=(S+YC^{\mathrm{T}})(I_n-E(C^{\mathrm{T}}E)^{+}C^{\mathrm{T}}). \quad (13)$$

Obviously, if $(S+YC^{\mathrm{T}})$ is nonsingular, then the matrix M has a maximal rank which equals $n-m$. Under this condition, the matrix $[C^{\mathrm{T}}S^{-1}, C^{\mathrm{T}}S^{-1}MAS^{-1}, \ldots, C^{\mathrm{T}}S^{-1}(MAS^{-1})^{n-1}]^{\mathrm{T}}$ has the maximal rank whose value is n. So $(MAS^{-1}, C^{\mathrm{T}}S^{-1})$ is completely observable, and the eigenvalues of $F=MAS^{-1}-NC^{\mathrm{T}}S^{-1}$ can be randomly configured.

And since rank $C=p$, for the convenience of its usage, let $Y=0$, then $(S+YC^{\mathrm{T}})$ is still nonsingular, which satisfies the conditions. In the meantime, $M=S(I_n-E(C^{\mathrm{T}}E)^{+}C^{\mathrm{T}})$. Namely, if $M=S(I_n-E(C^{\mathrm{T}}E)^{+}C^{\mathrm{T}})$, we will always find an appropriate N to satisfy that when $t\to\infty$, $\hat{x}$ will be exponentially convergent to ∂x, then systems (1) and (2) achieve projective synchronization.

3. The numerical simulation of generalized projective synchronization

Now, we select new Lü system which is a complex dynamics to perform this numerical simulation. Its chaotic oscillator is described as follows:

$$\begin{cases} \dot{x}_1=-\dfrac{ab}{a+b}x_1-x_2x_3+c, \\ \dot{x}_2=ax_2+x_1x_3, \\ \dot{x}_3=bx_3+x_1x_2, \end{cases} \quad (14)$$

where parameters a, b and c are constants. When $a=-10$, $b=-4$, $c=10$, the projections of chaos attractor are shown in Fig. 1.

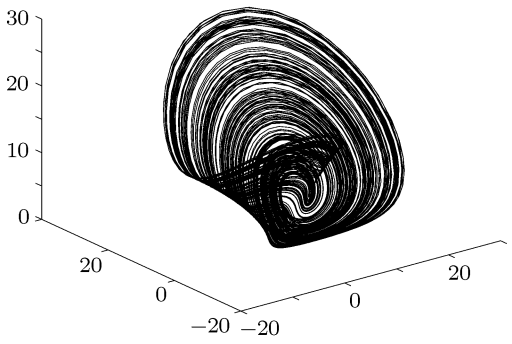


Fig. 1. Chaos attractor of new Lü system.

Transform the new Lü system (14) into the matrix with the form of Eq. (1)

$$\begin{cases} \begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{bmatrix}20/7 & 0 & 0\\ 0 & -10 & 0\\ 0 & 0 & -4\end{bmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{pmatrix}x_2x_3\\ x_1x_3\\ x_1x_2\end{pmatrix}+\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}W(t)+\begin{bmatrix}10\\ 0\\ 0\end{bmatrix}, \\ y=(\,1\ 0\ 0\,)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}. \end{cases} \quad (15)$$

Therefore

$$A=\begin{bmatrix}20/7 & 0 & 0\\ 0 & -10 & 0\\ 0 & 0 & -4\end{bmatrix}, \quad B=\begin{bmatrix}-1\ 0\ 0\\ 0\ 1\ 0\\ 0\ 0\ 1\end{bmatrix}, \quad u(t)=\begin{pmatrix}x_2x_3\\ x_1x_3\\ x_1x_2\end{pmatrix}, \quad C^{\mathrm{T}}=(\,1\ 0\ 0\,), \quad E=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}, \quad D=\begin{bmatrix}10\\ 0\\ 0\end{bmatrix}.$$

Here, we set the proportional factor $\partial = -0.8$. From Eqs. (7)–(10) and (12) we obtain

$$J = \begin{bmatrix} 0.8 \\ 0 \\ 0 \end{bmatrix}, \quad I = M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Let $F = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -4 \end{bmatrix}$, then $N = \begin{bmatrix} -8 \\ 0 \\ 0 \end{bmatrix}$.

Obviously, the eigenvalues of F are all less than 0. So when $t \to \infty$, $\hat{x}$ will be exponentially convergent to ∂x. Substituting F, G, H, J and I into Eq. (2), we can obtain the system equations of the receiver

$$\begin{cases} \dot{z} = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -4 \end{bmatrix} z + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{bmatrix} D, \\ \hat{x} = z - \begin{bmatrix} 0.8 \\ 0 \\ 0 \end{bmatrix} y. \end{cases} \tag{16}$$

Suppose the substantial interference with the signal is $w(t) = 0.5\sin(3t)$. We use the ODE45 method to solve Eqs. (15) and (16). The initial value of the driving system is $(x_1(0), x_2(0), x_3(0)) = (1, -5, 5)$, and the initial value of the response system is $(\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0)) = (1, 4, -2)$. We select $\partial = -0.8$, then the simulation results of generalized projection process between the driving system (15) and the response system (16) are shown in Figs. 2 and 3.

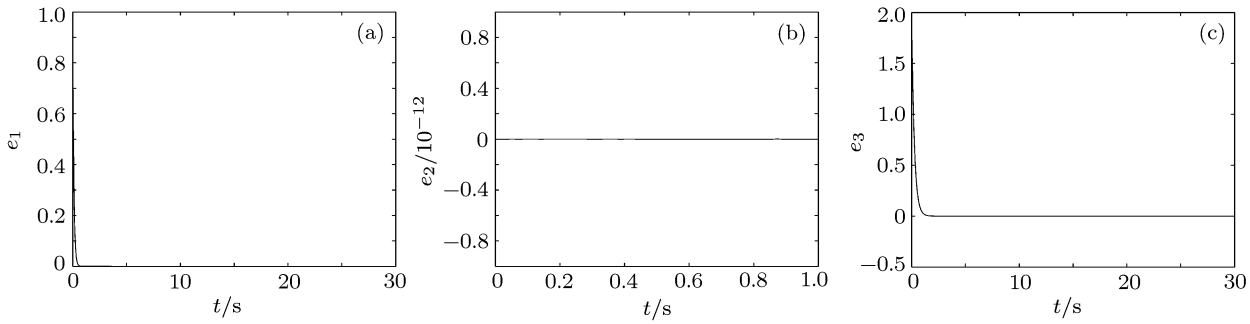


Fig. 2. The error curves of the projective synchronization: (a) e_1 curve with t, (b) e_2 curve with t, and (c) e_3 curve with t.

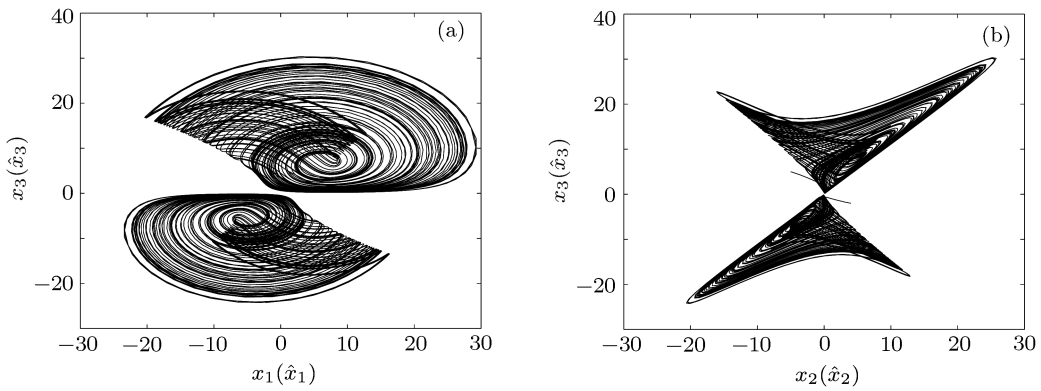


Fig. 3. The projection synchronization between the driving system and the response system: (a) $(x_1(\hat{x}_1), x_3(\hat{x}_3))$, (b) $(x_2(\hat{x}_2), x_3(\hat{x}_3))$.

From Fig. 2, we can see that when t is close to 0.5 s, 0 s and 1.5 s, the error systems $e_1(t)$, $e_2(t)$ and $e_3(t)$ become 0, which implies that the two chaotic systems in a short period of time have achieved generalized projective synchronization in accordance with the scale factor. Figure 3 shows the projective synchronization of chaotic attractor in the plane. Both are –0.8 times projection synchronized with the same phase. That is to say, even in the presence of large interfering signal, the projective synchronization can still be achieved within a short time using this method.

From the above experiment, we can see that the observer in our paper is valid. The precisions of synchronization results are all controlled in magnitude of 1×10^{-12}. What is more, as to the time, some error can become 0 even in less than 1 s, all other errors in less than 2 s. So the time and precision achieves a good combination. This is one of what our paper makes sence and at the same time lays good foundation for the following secure communication.

4. The application in secure communication

Chaotic signal is characterized by aperiodicity, continuous broadband spectrum, similar to the noise, unpredictability, which make it a natural concealment especially suitable for secure communications.

In this paper, we use the ideal of Yu and Lookman's improved programme.[19] By introducing the feedback of synthetic signal, we have designed a secure communication project using chaotic masking. The basic principle is shown in Fig. 4. The communication system consists of transmitter (i.e. driving system) and receiver (i.e. response system) components.

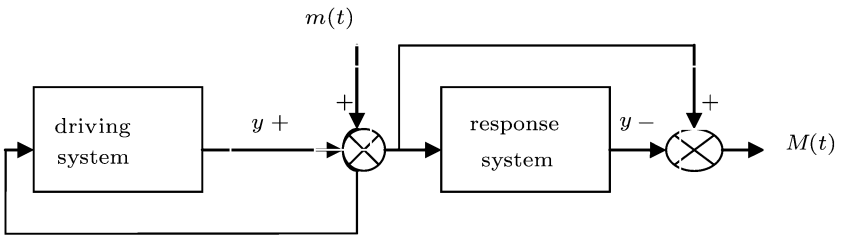


Fig. 4. The secret communication schematic diagram with chaotic masking.

In this paper, we see the superimposed information signal as the perturbation on the chaotic trajectory. Here, we consider the new Chu system and have designed a simple and effective secure communication project. The details are described below.

Now, let $m(t)$ be the information signal be sent to the receiver. The transmitting system is designed as

$$\begin{cases} \dot{x} = Ax + Bu + D + Em(t), \\ y = \partial(C^{\mathrm{T}}x + m(t)), \\ u = f(x), \end{cases} \tag{17}$$

where $x \in R^n$ and $u \in R^k$ are the state variable, and nonlinear feedback input, respectively, and $y \in R^p$ is the output of the transmitter system. A, B, C, and D are known constant matrices, while, E is an unknown constant matrix. D is the constant term of the system. ∂ is a proportional factor.

The receiving system is

$$\begin{cases} \dot{z} = Fz + Gu + Hy + KD, \\ \hat{x} = z - Jy, \end{cases} \tag{18}$$

where $z \in R^n$, $\hat{x} \in R^n$, F, G, H, K and J are unknown constant matrices.

From Eqs. (17) and (18), we know that this method puts the useful signal into the transmitting and receiving systems directly, so it is no longer out of the chaotic signal of communication device floating above the carrier, but directly modulated by the transmitting chaotic system, so the intensity of $m(t)$ can be given much larger. Additionally, $m(t)$ is directly modulated by dynamics, so the concealment is better and cannot easily be stolen. Meanwhile, because the chaotic carrier and the useful signal are integrated together with the same magnitude of strength, it is more

tolerant and robust to noise, and also greatly enhances the security of the communication.

The error of projective synchronization between the transmitting system and the state observer can be expressed as

$$e = \hat{x} - \partial x = z - \partial(JC^{\mathrm{T}} + I_n)x - \partial Jm(t).$$

Let $M = JC^{\mathrm{T}} + I_n$, then the error dynamic system is

$$\begin{aligned} \dot{e} = {} & Fe + (G - \partial MB)u + \partial(FM + HC^{\mathrm{T}} - MA)x \\ & + (K - \partial M)D + \partial(H + FJ - ME)m(t) \\ & + \partial Jm(t). \end{aligned} \tag{19}$$

If system (19) satisfies

$$\begin{cases} (FM + HC^{\mathrm{T}} - MA) = 0, \\ G - \partial MB = 0, \\ (H + FJ - ME) = 0, \\ K - \partial M = 0, \\ J = 0, \end{cases}$$

then equality (19) with some simple transformation gives

$$\dot{e} = Fe. \tag{20}$$

If all the eigenvalues of F are less than zero, then $\lim\limits_{t\to\infty} \|e\| = 0$, $\hat{x}$ will be exponentially convergent to ∂x. Then systems (17) and (18) achieve generalized projective synchronization. From the above analysis, we can obtain $F = A - EC^{\mathrm{T}}$, $H = E$, $G = \partial B$ and $K = \partial I_n$, using the pole placement technique, we make all the eigenvalues of F negative, then obtain E.

In 2007, Chu Yan-Dong *et al.* proposed a new chaotic system—the Chu system. Its dynamic equation is

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = x_1x_3 - x_2, \\ \dot{x}_3 = b - x_1x_2 - cx_3, \end{cases}$$

where $x = (x_1, x_2, x_3)^{\mathrm{T}} \in R^3$ is the state variable, a, b, $c \in R$ are parameters and $a > 0$, $b > 0$, $c > 0$. And from Ref. [20], when constant parameters $a = 5$, $b = 16$, $c = 1$, the system shows the chaotic state. Its attractor is shown in Fig. 5.

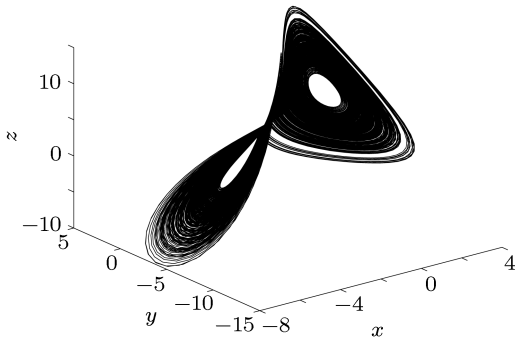


Fig. 5. The chaotic attractor of Chu system.

Select the new Chu system as the transmitting system, then

$$A = \begin{bmatrix} -5 & 5 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad u(t) = \begin{pmatrix} 0 \\ x_1x_3 \\ x_1x_2 \end{pmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ 16 \end{bmatrix}.$$

Set $C^{\mathrm{T}} = (\,1\ 1\ 0\,)$, $\partial = -0.8$, let $F = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then $E = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$. From these we obtain

$$J = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}, \quad H = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \quad K = \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{bmatrix}.$$

Substituting F, G, H, E, J and K into Eqs. (17) and (18), then the transmitting system is

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} -5 & 5 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{pmatrix} 0 \\ x_1x_3 \\ x_1x_2 \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ 16 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} m(t), \\ y = -0.8 * \left((1\ 1\ 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + m(t) \right). \end{cases} \tag{21}$$

The receiving system is

$$\begin{cases} \dot{z} = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} z + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \begin{pmatrix} 0 \\ x_1x_3 \\ x_1x_2 \end{pmatrix} + \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} y + \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 16 \end{bmatrix}, \\ \hat{x} = z - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} y. \end{cases} \tag{22}$$

From system (21), we know that the output signal of the transmitting system is $y = ax_1 + ax_2 + am(t)$. Driving the receiving system with y, after a period of time, the transmitting system and the receiving system will achieve generalized projective synchronization. Then the recovery signal in the receiver can be expressed as

$$\hat{M}(t) = \frac{1}{a}(y(t) - \hat{x}_1(t) - \hat{x}_2(t)) = \frac{1}{a}(ax_1 + ax_2 + am(t) - \hat{x}_1(t) - \hat{x}_2(t)) \cong m(t).$$

Here, we set the transmission signal as $m(t) = F\sin(wt)$ (single tones), $F = 0.01$, $w = 1.0$, and use ODE45 to do experiment. Figure 6 is the error curves of the projective synchronization, e_1 and e_2 respectively. It can be seen that the error signals get basically stable at zero in less than 1 s, achieving generalized projective synchronization effectively. In Fig. 7, we can see that in less than 1 s, the receiver can effectively restore the transmitted signal and realize the chaotic secure communication.

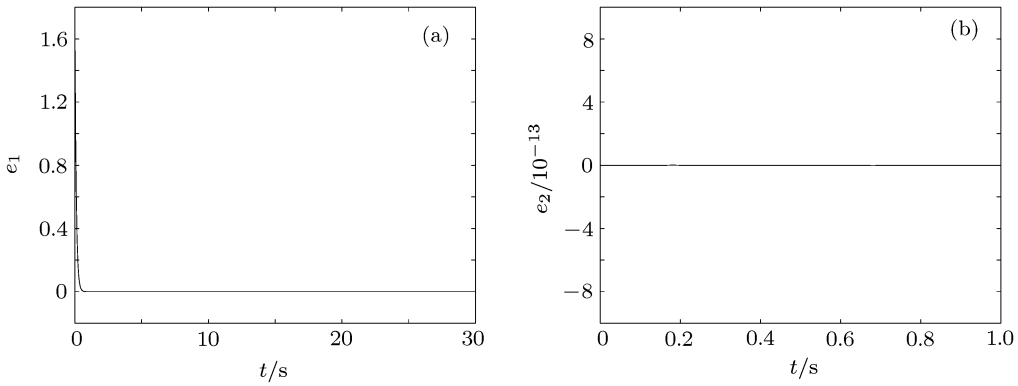


Fig. 6. The error curves of the projective synchronization: (a) e_1 curve with t and (b) e_2 curve with t.

From the above experiment, we can see that the designed secure communication in our paper has a great improvement in decryption time. Furthermore, the precisions of our system are all controlled at a magnitude of 1×10^{-12}. As we know, in secure communication, the decryption time has great influence on the communication quality. So the shorter the decryption time is, the better the communication quality is. At the same time, by adopting the feedback concept in our paper, the proposed method can send useful signal with arbitrary strength which is not limited by the strength of chaotic carrier. So it has greater practical value. Meanwhile, this secure communication project makes our previous designed generalized projective synchronization have important theoretical value and practical meaning.

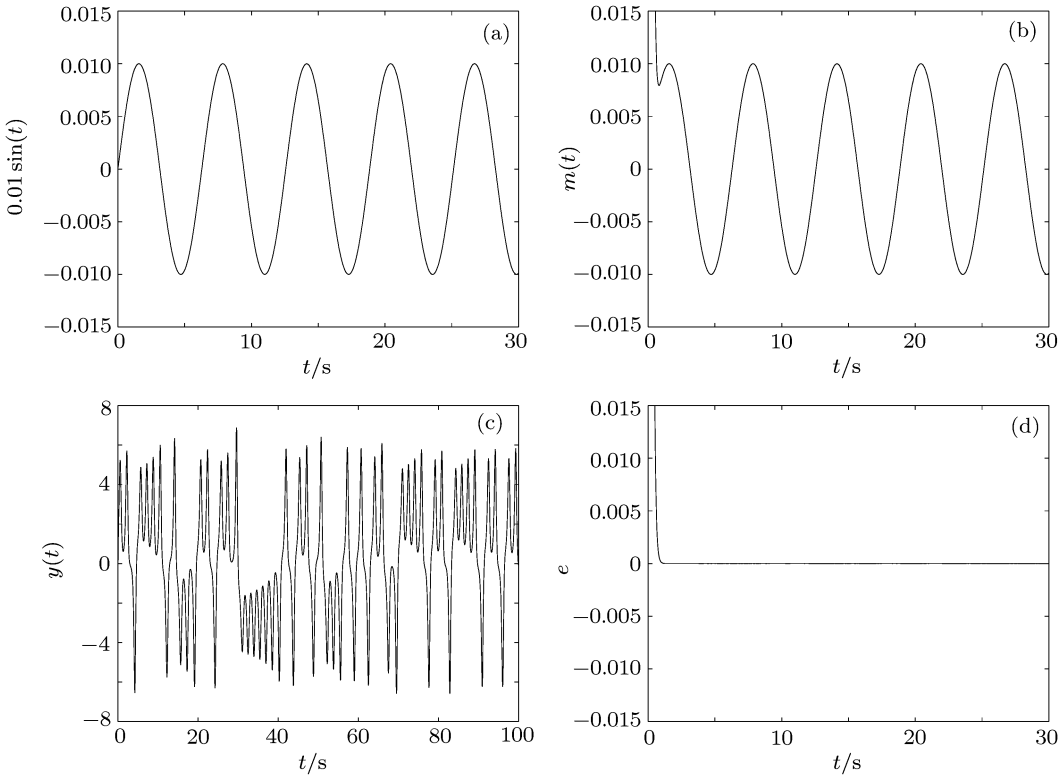


Fig. 7. (a) The waveform of useful signal $m(t)$, (b) the waveform of recovery signal $\hat{M}(t)$, (c) the waveform of the transmitting signal $y(t)$, and (d) the error curve between the recovery signal and the useful signal.

5. Conclusion

In this paper, we have studied the generalized projective synchronization of the autonomous chaotic system. Based on the improved state observer and the pole placement technique, we proposed an observer of generalized projective synchronization. The addition of a constant to the original system extends its scope of use. By the numerical simulation on the new Lorenz system, choosing suitable scale factor, we can achieve synchronization with certain precision in a very short time. This could meet the challenge of many existing methods. Finally, based on this method, a novel secure communication project is designed. We use the new Chu system for simulation and the proposed method can be adapted to the latest model. Numerical simulation verifies the effectiveness of the method. That is, we can mask and recover the useful signal effectively in a very short time. Therefore, our secure communication has certain significance in practical applications. As a result, our study is of significance both in theory and in application.

References

[1] Guan X P, Fan Z P, Chen C L and Hua C C 2002 *Chaotic Control, Synchronization and Its Application in Secure Communication* (Beijing: National Defence Industry Press) Chap. 9 (in Chinese)
[2] Wang X Y and Wu X J 2006 *Acta Phys. Sin.* **55** 606 (in Chinese)
[3] Ma W and Wang Z O 2003 *Commun. Theor. Phys.* **39** 385
[4] Wu Z Q 2003 *Journal of System Simulation* **15** 1314
[5] Rosenblum M G, Pikovsky A S and Kurths J 1996 *Phys. Rev. Lett.* **76** 180
[6] Taherionl S and Lai Y C 1999 *Phys. Rev.* E **59** 6247
[7] Kocarev L and Parlitz U 1996 *Phys. Rev. Lett.* **76** 1816
[8] Yang S S and Duan K 1998 *Chaos, Solitions and Fractals* **10** 1703
[9] Li G H 2004 *Acta Phys. Sin.* **53** 999 (in Chinese)
[10] Zhang W P, Tang G N and Luo X S 2005 *Acta Phys. Sin.* **54** 3497 (in Chinese)
[11] Wang X Y and Meng J 2008 *Acta Phys. Sin.* **57** 726 (in Chinese)
[12] Liu J, Chen S H and Lu J A 2003 *Acta Phys. Sin.* **52** 1595 (in Chinese)
[13] Wang X Y and Wang Y 2007 *Acta Phys. Sin.* **56** 2498 (in Chinese)
[14] Yan J P and Li C P 2005 *Chaos, Solitons and Fractals* **26** 1119
[15] Min F H and Wang Z Q 2007 *Acta Phys. Sin.* **56** 6239 (in Chinese)
[16] Li C P and Yan J P 2006 *Chaos, Solitons and Fractals* **30** 140
[17] Li G H 2007 *Chaos, Solitons and Fractals* **32** 1454
[18] Li G H 2006 *Chaos, Solitons and Fractals* **30** 77
[19] Yu P and Lookman T 1997 *Mini Symposium Cryptography*, Toronto: Canadian Applied Mathematics Society
[20] Chu Y D, Li X F and Zhang J G 2007 *Journal of Sichuan University* (Natural Science Edition) **44** 550 (in Chinese)