Adaptive synchronization of uncertain chaotic systems via switching mechanism*

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This paper deals with the problem of synchronization for a class of uncertain chaotic systems. The uncertainties under consideration are assumed to be Lipschitz-like nonlinearity in tracking error, with unknown growth rate. A logic-based switching mechanism is presented for tracking a smooth orbit that can be a limit cycle or a chaotic orbit of another system. Based on the Lyapunov approach, the adaptation law is determined to tune the controller gain vector online according to the possible nonlinearities. To demonstrate the efficiency of the proposed scheme, the well-known chaotic system namely Chua's circuit is considered as an illustrative example.

Keywords: chaos synchronization, adaptive synchronization, switching mechanism

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1. Introduction

Since the discovery of chaos synchronization,[1] there has been tremendous interest in studying the synchronization of chaotic systems, see Refs. [2] and [3] and the references therein for a survey of the recent development. As chaotic signals could be used to transmit information in a secure and robust manner, chaos synchronization has been intensively studied in communications research[4−20] and other research fields[21−28] (name just a few).

Recently, specialists of nonlinear control theory turned their attention to the study of chaos synchronization and its potential applications in communications. After the pioneering work on controlling chaos introduced by Ott *et al.*,[10] there have been many other attempts to control chaotic systems. Generally, we can classify the existing methods into two main streams: parameter perturbations of an accessible system parameter, see Ref. [11] and the references therein, and introduction of an additive control law to the original chaotic system.[12−14] Our paper falls within the second stream. In Ref. [12] Yang *et al.* designed a sliding mode feedback control in order to control Lorenz chaos in the presence of parameter uncertainties. Tian *et al.*[15] imposed an inequality constraint to obtain a bounded control that stabilizes the unstable equilibrium of Lorenz chaos. Hwang *et al.* proposed a linear[13] and a nonlinear[16] controller to construct a stable equilibrium manifold so that any point on it will be trackable. These controllers are only suitable to control chaotic systems to fixed points or to limit cycles.

To achieve tracking of a reference signal for chaotic systems, Solak *et al.*[17] proposed a linearizing controller, therein an observer has been used to estimate the unmeasured states. An adaptive synchronization and control method have been developed in Refs. [8] and [18]. In the former, John and Amritkar showed that feedback synchronization can be achieved by using only a few state variable feedback. In the latter, Bernardo proposed an adaptive estimation of the nonlinearity bound to avoid very high gain controller. In Refs. [19] and [20] the authors considered chaotic systems with some unknown parameters, and designed adaptive controllers which lead to parameter estimation and state variable synchronization.

This paper addresses chaos control through an adoption of an adaptive observer-based approach. We propose an adaptive controller for nonlinear systems which can be chaotic or nonchaotic, but systems with chaotic behaviour are addressed herein. Moreover, our method concerns chaotic systems which are transformable into Brunovsky form (e.g. Lur'e system, Duffing equations, Chua's circuit, ...). Our idea con-

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sists in tuning the gain vector of an adaptive controller during control procedure. We wish to update this gain vector with a proper adaptation law such that the proposed control can track a reference signal for chaotic systems in the presence of strong uncertainties in the unmeasurable states. This kind of uncertainties has been presented by Lei and Lin[29] for research on nonlinear systems, which contains an unknown growth rate for the normal Lipschitz-like uncertainties. The unknown growth rate in uncertainties can be handled by a proposed switching mechanism. Eventually, using the Lyapunov approach, we prove that the desired trajectory, such as a limit cycle or a chaotic orbit, can be tracked by the output signal. It is shown that if the desired trajectory is a well chosen signal of another chaotic system, synchronization can be achieved.

This paper is outlined as follows. In Section 2 we give a brief preliminary on input–output linearizing control for trajectory tracking and give the problem statement. Then in Section 3 we present our main result: adaptation law and controller construction. Next in Section 4 we illustrate the results presented by using a Chua's circuit. Finally, Section 5 includes some perspectives and concluding remarks.

2. Problem formulation and assumptions

Let us consider the nonlinear continuous-time dynamic system

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R$ is the controlling signal and $y \in R$ is the output of the system. Moreover let us assume that system (1) has relative degree $\rho = n$ (see Ref. [30]). Therefore we can define a nonlinear transformation $z = \phi(x)$ as

$$z_1 = h(x), \quad z_{i+1} = L_f^i h(x), \quad i = 2, \ldots, n-1. \tag{2}$$

Using z as the new state variable, system (1) becomes

$$\begin{cases} \dot{z}_i = z_{i+1}, \quad i = 1, \ldots, n-1, \\ \dot{z}_n = F(z) + G(z)u, \\ y = z_1, \end{cases} \tag{3}$$

where $F(z) = L_f^n h(\Phi^{-1}(z))$ and $G(z) = L_g L_f^{n-1} h(\Phi^{-1}(z))$. We want to design an adaptive controller which forces the output y to track a smooth (infinitely differentiable) reference trajectory y_r. To accomplish this aim we define a tracking error $e = Y_r - z$ where $Y_r = (y_r, \dot{y}_r, \ldots, y_r^{(n-1)})^{\mathrm{T}}$, here the superscript T denotes the transpose of the vector.

In the existing literatures, one can choose an input–output linearizing controller

$$u = u_{io} = \frac{1}{G(z)}(-F(z) + y_r^{(n)} + K^{\mathrm{T}} e), \tag{4}$$

with $K = (k_1, k_2, \ldots, k_n)$, then the closed loop system can be written as $\dot{e} = A_c e$

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_{n-1} \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ -k_1 & -k_2 & \ldots & \ldots & -k_n \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \\ e_n \end{bmatrix}. \tag{5}$$

Let K be a Hurwitz vector, that is all the roots of the polynomial

$$p(s) = s^n + k_n s^{n-1} + \cdots + k_3 s^2 + k_2 s + k_1 \tag{6}$$

have negative real parts, then the error is stable at the origin, i.e.

$$\lim_{t\to\infty} e(t) = 0 \Longleftrightarrow \lim_{t\to\infty} z(t) = Y_r, \tag{7}$$

and in particular

$$\lim_{t\to\infty} y(t) = y_r. \tag{8}$$

Although, in general, nonlinear controllers show better performance than linear controllers in controlling nonlinear systems which is rather conceivable. One of the main drawbacks of the conventional input–output linearizing controller is the realization of the nonlinear part in addition to a necessary exact knowledge of the system model (i.e. $f(x)$ and $g(x)$). When the knowledge is not exact, e.g., there is an uncertainty in the unmeasurable states, this controller probably will lose its effectiveness. In this paper, we will consider the following error systems with a continuous mapping which represents a family of time-varying parameters or disturbances bounded by an unknown constant δ,

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_{n-1} \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \ldots & \ldots & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \\ e_n \end{bmatrix}$$

$$+\begin{bmatrix}0\\0\\\vdots\\0\\u_{\rm aio}\end{bmatrix}+\begin{bmatrix}\psi_1(e)\\\psi_2(e)\\\vdots\\\psi_{n-1}(e)\\\psi_n(e)\end{bmatrix}, \qquad (9)$$

where the functions $\psi_i(e), i=1,\ldots,n$, are locally Lipschitz in e, and they represent the system uncertainties and need not to be precisely known.

Throughout this paper, the following condition is assumed.

Assumption 1 There exists an unknown constant δ such that

$$|\psi_i(e)| \le \delta\|e\|, \quad \forall e \in R^n, \quad i=1,\ldots,n. \qquad (10)$$

In most existing literatures, the Lipschitz constant δ is often required to be known for the control design purpose. In fact, it is often difficult to obtain a precise δ in some practical systems, hence the Lipschitz constant is often selected to be larger, which will induce the control gain to be higher, and the obtained results would be conservative. In this paper, we will assume the Lipschitz constant δ unknown, and an adaptive method will be proposed. Under Assumption 1, we will design $u_{\rm aio} = \frac{1}{G(z)}(-F(z) + y_r^{(n)} + K^{\rm T}(\theta)e)$ with an on-line updating parameter θ associated with the controller, to achieve the objective given by Eq. (7). The structure of $u_{\rm aio}$ will be given in detail in the following section.

3. Adaptive synchronization via switching mechanism

We first propose a dynamic output feedback controller $u_{\rm aio}$ and show that it globally exponentially stabilizes systems (1) and (2) with a properly tuned parameter θ in the following.

Controller with on-line tuning parameter

$$u = u_{\rm aio} = \frac{1}{G(z)}(-F(z) + y_r^{(n)} + K^{\rm T}(\theta)e) \qquad (11)$$

with $K(\theta) = (k_1/\theta^n, k_2/\theta^{n-1}, \ldots, k_n/\theta)$ and $\theta > 0$. Then system (9) can be written as

$$\begin{bmatrix}\dot e_1\\\dot e_2\\\vdots\\\dot e_{n-1}\\\dot e_n\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & \ldots & 0\\0 & 0 & 1 & \ldots & 0\\\vdots & \vdots & \ddots & \ddots & \vdots\\\vdots & \vdots & \vdots & \ddots & 1\\-k_1/\theta^n & -k_2/\theta^{n-1} & \ldots & \ldots & -k_1/\theta\end{bmatrix}\begin{bmatrix}e_1\\e_2\\\vdots\\e_{n-1}\\e_n\end{bmatrix}+\begin{bmatrix}\psi_1(e)\\\psi_2(e)\\\vdots\\\psi_{n-1}(e)\\\psi_n(e)\end{bmatrix}, \qquad (12)$$

which has a compact form $\dot e = A_c(\theta)e + \Psi(e)$ and $\Psi = [\psi_1(e), \ldots, \psi_n(e)]^{\rm T}$.

Lemma 1 Suppose that Assumption 1 holds. Select $K = [k_1, \ldots, k_n]$ such that matrix A_c is Hurwitz. Then, there exists a constant $\theta^* > 0$ such that for $0 < \theta < \theta^*$, system (1) is globally exponentially stable by Eqs. (3) and (4).

Proof Consider the error dynamics $\dot e = A_c(\theta)e + \Psi$. Since A_c is Hurwitzian, we have a Lyapunov equation $A_c^{\rm T}P + PA_c = -I$. Denote $E_\theta = {\rm diag}[1, \theta, \theta^2, \ldots, \theta^{n-1}]$. Then, we have a relation $\theta A_c(\theta) = -E_\theta^{-1}A_cE_\theta$, hence we have a new Lyapunov equation $A_c^{\rm T}(\theta)P(\theta) + P(\theta)A_c(\theta) = \theta^{-1}E_\theta^2$, where $P(\theta) = E_\theta P E_\theta$ and a Lyapunov function $V(e) = {\rm e}^{\rm T}P(\theta)e$. Now, along the trajectory of Eq. (12), the time derivative of $V(e)$ is

$$\begin{aligned}\dot V(e) &= -\theta^{-1}\|E_\theta e\|^2 + 2{\rm e}^{\rm T}E_\theta P E_\theta \Psi(e)\\ &\le -\theta^{-1}\|E_\theta e\|^2 + 2\|E_\theta e\|\|P\|\|E_\theta\Psi(e)\|.\end{aligned}$$

Noting that $\|E_\theta\Psi(e)\| = \sqrt{\psi_1^2 + \ldots + \theta^{n-1}\psi_n^2} \le |\psi_1| + \ldots + |\theta^{n-1}\psi_n| = \|E_\theta\Psi(e)\|_1$, one can show that

$$\dot V(e) \le -\theta^{-1}\|E_\theta e\|^2 + 2\|P\|\|E_\theta e\|\|E_\theta\Psi(e)\|_1.$$

Using Assumption 1, we have $\|E_\theta(e)\Psi(e)\|_1 \le \sqrt{n}\delta\|E_\theta e\|$. Thus, we have

$$\dot V(e) \le -\theta^{-1}\|E_\theta e\|^2 + 2\sqrt{n}\delta\|P\|\|E_\theta e\|^2,$$

which is a finite constant independent of θ and δ. From the above equation, $\dot V(e)$ becomes negative-definite if $\theta^{-1} - 2\sqrt{n}\delta\|P\| > 0$, which leads to $0 < \theta < \theta^* := 1/2\sqrt{n}\delta\|P\|$. This ends the proof of Lemma 1.

To implement controllers (11) and (12), we need to choose θ which requires a priori knowledge on δ. Moreover, when the system nonlinearities change, we need to tune the gain-related parameter θ accordingly if necessary. In the following, we introduce an adaptive switching mechanism which can tune this parameter on-line depending on system nonlinearities.

On-line tuning mechanism: We will choose for j a sequence $H = \{\theta_j : j = 0, 1, 2, \ldots, \infty\}$, we start with $\theta = \theta_0$, switch to θ_1, then to θ_2 and so on. The switching time is determined by a switching logic, which is defined below.

Initialization

i) Preselect three positive numbers $\kappa_0 > 1$, $\mu > 0$, and $a > 1$.

ii) Set initial time $t_0 > 0$, $j = 0$ and preselect $\theta_0 = \kappa(t_0) = T_0 = \kappa_0$, $t_1, t_2, \ldots$, are the first, second, ... switching time respectively.

Turning mechanism

$$\dot{\kappa}(t) = \mu|e_1| \quad \text{with} \quad \kappa(t_0) = \kappa_0, \quad \mu > 0,$$

Switching logic

Step 1 Obtain $T_{j+1} = aT_j$, $\theta_j = \kappa(t_j)$.

Step 2 If there exists a time $t > t_j$ such that $\kappa(t) > T_j$, then set $t_{j+1} = t$, $\theta_{j+1} = 1/\sqrt{n}\|P\|T_{j+1}$ and $j = j + 1$, (Switching), go to Step 1; else $\theta_j = 1/\sqrt{n}\|P\|T_j$, (No switching), go to Step 2.

We briefly explain the idea of the tuning mechanism with the switching logic: first, $\theta(t)$ is a monotonically nondecreasing function of time by taking integral value of $|e_1|$. Obviously, $\theta(t)$ stops increasing only when $e_1 \equiv 0$. Also, from the switching logic, it is obvious that $T(t)$ is a piecewise constant function of time and $T(t)$ increases stepwise with some time-interval. At some point of switching time, $T(t)$ will become sufficiently large such that during each time interval, Lemma 1 can be applied for each switched subsystem. With this, the increment of $\theta(t)$ may become smaller and eventually there would be the last switching point. After the last switching occurs, the proposed controllers (11) and (12) become the same one as static case of controllers (4) and (5). Overall, the proposed scheme leads to a switched nonlinear system and its Lyapunov stability can be proved by the following theorem.

Theorem 1 Suppose that Assumption 1 holds. Select $K = [k_1, \ldots, k_n]$ such that each matrix A_c is Hurwitzian. Then, the closed-loop system with controllers (11) and (12) is globally asymptotically stable. Also, $\theta(t) \to \theta_{ss} < \infty$ as $t \to \infty$.

Proof First, note that the uncertainty $\Psi(e)$ satisfies the linear growth condition. Thus, there is no finite escape phenomenon with the proposed controllers (11) and (12). Consider a case that $\theta(t)$ increases with the time. Then, there exists a switching time t_j , i.e., a time at which $\theta_j = 1/\sqrt{n}\|P\|T_j$ such that T_j is large enough that the conclusion of Lemma 1 holds for each interval $\Delta t_k, k \geq j$. The time-interval Δt_k is defined as $t \in [t_k, t_{k+1}]$.

Now, we only consider the case of $t \geq t_j$. Note that for each time-interval $\Delta t_k, k \geq j$, $T(t)$ remains as a constant value T_k. Thus, as a whole, the closed-loop system can be viewed as a switched system $\dot{e} = A_c(\theta_j)e + \Psi, k = j, j+1, j+2, \ldots, l$ and for each time-interval Δt_k, each subsystem $\dot{e} = A_c(\theta)e + \Psi$ is engaged. In the following, we need to show that l is indeed a finite number. For each subsystem $\dot{e} = A_c(\theta)e + \Psi$, utilizing the proof of Lemma 1, we can set a Lyapunov function $V_c(e)$ where $V_c(e) = \mathrm{e}^{\mathrm{T}}P(\theta)e$, and T_k is a sufficiently large constant. Then, we have

$$\lambda_{\min}(P)\|E_{\theta_k}e\|^2 \leq V_k(e) \leq \lambda_{\max}(P)\|E_{\theta_k}e\|^2. \quad (13)$$

For a sufficiently small θ_k, we have

$$\dot{V}_k(e) \leq -\left(\frac{1}{\theta_k} - \sqrt{n}\delta\|P\|\right)\|E_{\theta_k}e\|^2. \quad (14)$$

From inequalities (13) and (14), it is easy to obtain

$$\|E_{\theta_k}e\| \leq \sqrt{\frac{M}{m}}\|E_{\theta_k}e(t_k)\|\mathrm{e}^{-(2\lambda_{\max}(P)(\theta_k{}^{-1} - \sqrt{n}\|P\|\delta))t},$$

which can be rewritten as

$$\|E_{\theta_k}e\| \leq \rho_k\mathrm{e}^{-\pi_k t}, \quad \rho_k > 0, \pi_k > 0. \quad (15)$$

Here, ρ_k is a nonincreasing sequence with respect to index k. It is obvious that ρ_k increases as θ_k decreases, when θ_k is sufficiently small. Now, for each fixed θ_k, using inequality (15), we obtain

$$|e_1(t)| \leq \sqrt{n}\rho_k\mathrm{e}^{-\pi_k t}. \quad (16)$$

Using inequality (16) and from $\dot{\kappa}(t) = \mu|e_1(t)|$, we have $\dot{\kappa}(t) = \mu\sqrt{n}\rho_k\mathrm{e}^{-\pi_k t}$ for $t \in \Delta t_k$ with $\kappa(t_k) = \theta_k$. Since $\kappa(t)$ is a monotonically nondecreasing function, the increment of $\kappa(t)$ for $t \in \Delta t_k$ is

$$\begin{aligned}\Delta\kappa(\Delta t_k) :&= \int_{t_k}^{t_{k+1}} \dot{\kappa}(\tau)\mathrm{d}\tau \\ &= \frac{\mu\sqrt{n}\rho_k}{\pi_k}(\mathrm{e}^{-\pi_k t_k} - \mathrm{e}^{-\pi t_{k+1}}).\end{aligned}$$

On the other hand, the increment of sequence T_k is

$$\Delta T_k := T_{k+1} - T_k = a^k(a-1)T_0, \ a > 1.$$

According to the switching logic, it is obvious that there exists a finite time $t_l, l \geq j$ such that t_l would be the last switching time. Therefore, the switched system consists of only a finite number of subsystems. Thus, the closed-loop switched system is globally asymptotically stable. Finally, utilizing Barbalat's Lemma, we conclude that $\theta(t)$ converges to

$\theta_{ss} < \infty$ as $t \to \infty$ by the proposed scheme. This ends the proof of Theorem 1.

Remark 1 Our proposed scheme does not require a priori knowledge on δ. The advantage of our result is twofold: (i) many system parameters $\psi_1, \ldots, \psi_n$ are not needed to be measured or estimated. For the standard adaptive control technique,[31] if the uncertainties are given in form of $\vartheta\Phi$, where ϑ stands for an unknown constant vector and Φ for known vector of functions, n unknown constants are needed to be estimated. In this paper, the uncertainties in Assumption 1 can also consider this kind of uncertainties, but the proposed adaptive controller just handle one unknown parameter δ. (ii) Our controller is adaptive to parameter variations such that once the controller is designed, the same controller may give good performance to various different systems without redesigning control parameters. For the uncertainties $\vartheta\Phi$, if the vector of functions Φ changes, then the standard adaptive controller[31] should be changed also. Whereas, the proposed controller does not need to accommodate the structure.

Remark 2 By the derivation of Theorem 1, from $\|E_{\theta_k} e\| \leq \rho_k \mathrm{e}^{-\pi_k t}$, $\rho_k > 0, \pi_k > 0$ and $\pi_k = 2\lambda_{\max}(P)\sqrt{n}\|P\|(a^k(a-1)T_0 - \delta)$, we can see that large value of a will result in quick convergence in error. From $\Delta\kappa(\Delta t_k) = \frac{\mu\sqrt{n}\rho_k}{\pi_k}(\mathrm{e}^{-\pi_k t_k} - \mathrm{e}^{-\pi t_{k+1}})$, large value of μ tenders quick increase of the integral of $\kappa(t)$, and leads to early switching time. So the parameters μ and a affects the convergence rates of the system trajectories as indicated in the proof.

Remark 3 From the structure of the switching mechanism, a pure integrator is utilized in the tuning mechanism, so there may be a case that a small measurement error can drive $\theta(t) \to \infty$ as $t \to \infty$, which results in infinite number of switching points so that $K(\theta(t))$ becomes unbounded as $t \to \infty$. In this case, the following modified rule may deal with this robustness issue against measurement noise in the corresponding switching mechanism.

Step 3 If there exists a time $t > t_j$ such that $\kappa(t) > T_j$ and $|e_1| > \epsilon$, then set $t_{j+1} = t$, $\theta_{j+1} = \frac{1}{\sqrt{n}\|P\|T_{j+1}}$ and $j = j + 1$, (Switching) where $\epsilon > 0$ is a pre-specified constant which sets the tolerance of measurement error. Under this modified rule, normal necessary switching actions still occur and once the system is stabilized, any measurement error smaller than ϵ will not cause further switching.

4. Example

In this section, we will use the Chua's circuit, which is a typical chaotic system and has been thoroughly studied.[13,14] A voltage source V_L (controller) is added in series with the inductor. The circuit model in dimensionless form is as follows:

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = \alpha x_2 - \alpha x_1 - \alpha \mathscr{N}(x_1), \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = x_1 - x_2 + x_3, \\ \dfrac{\mathrm{d}x_3}{\mathrm{d}t} = \beta x_2 + u, \end{cases} \tag{17}$$

where $\mathscr{N}(x_1) = bx_1 + 0.5(a-b)(|x_1+1| - |x_1-1|)$. The state variables x_1 and x_2 represent the voltages across the two capacitors, x_3 is the current through the inductor and $u = \beta V_L$. Typical value of the system parameter (α, β, a, b) are chosen to be $(10, 18.6, -6, 161/44, -3/4)$ so that equation (17) behaves chaotically. We have chosen x_1 as the output signal, thus the relative degree of Eq. (17) is $\rho = 3$. Next, we apply state transformation, it can be seen that $G(z) = \alpha$. An adaptive controller was designed to force the output x_1 to track another signal x_{1r} of an identical autonomous ($u = 0$) Chua's circuit.

The whole system was numerically simulated using a fourth order Runge–Kutta algorithm of MATLAB6 with the following initial conditions and parameters:

$$P = \begin{bmatrix} 3.8 & 2.4 & 0.1 \\ 2.4 & 4 & 0.3 \\ 0.1 & 0.3 & 0.2 \end{bmatrix},$$

$(x_1(0), x_2(0), x_3(0)) = (1, 0, 0)$,

$(x_{1r}(0), x_{2r}(0), x_{3r}(0)) = (2, 1, 1)$,

$(k_1, k_2, k_3) = (8, 12, 6)$, $\delta = 1$, $\theta(0) = 0.5$, $\mu = 0.5$,

$\psi_1(e) = e_1$, $\psi_2(e) = \sin(e_1) + e_2$, $\psi_3(e) = |e_3|$.

The controller has been stated at $t = 0$, figures 1 and 2 show that the trajectories diverge from each other due to the sensitivity to the initial condition and the uncertainties. However, as soon as the controller is applied, not only tracking of the reference signal, but also synchronization of all the state variables are achieved. The signals $T(t)$ and θ used in the switching mecha-

nism are shown in the first sub-figure of Fig. 4.

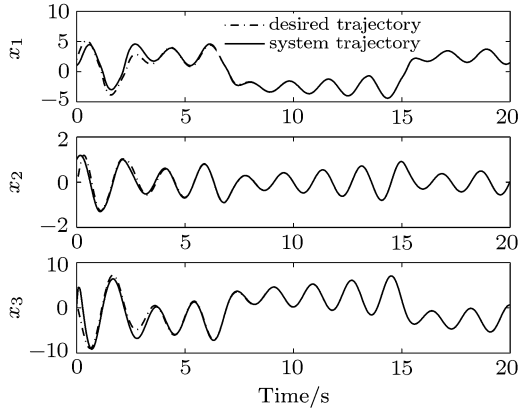


Fig. 1. Synchronization of Chua's circuit.

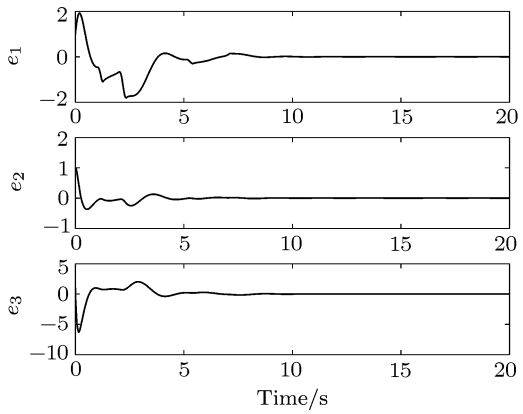


Fig. 2. Tracking error fade to zero.

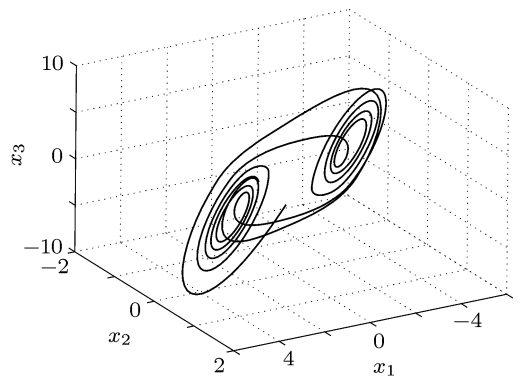


Fig. 3. System behaviour of the desired Chua's circuit.

It can be seen that the gained related parameter converges to a constant value after a finite number of switching, which is used in the controller finally. The values of θ_j in H are: $H = \{\theta_0, \theta_1, \ldots, \theta_{14}\} = \{0.5, 0.6, 0.72, 0.864, 1.0368, 1.2442, 1.493, 1.7916, 2.1499, 2.5799, 3.0959, 3.715, 4.4581, 5.3497, 6.4196\}$, which are generated by the on-line tuning mechanism. The input signal is shown in the second sub-figure of Fig. 4.

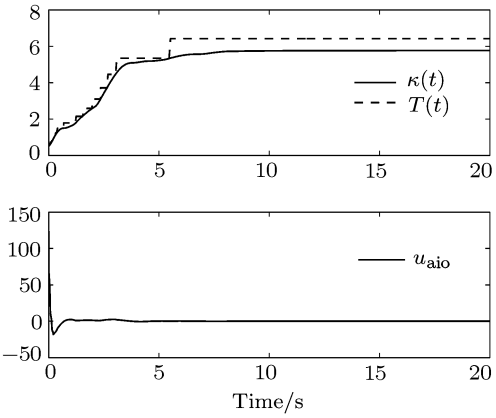


Fig. 4. The signals in the switching mechanism and control input.

5. Conclusion

In this paper we have proposed a logic-based switching adaptive controller approach to control chaotic systems. The constructed controller may be used for chaos control as well as chaotic system synchronization. The main advantage of this construction is the characteristic of the controller which can on-line tune its parameter according to the varying uncertainties automatically. Besides, if the chaotic system is in Brunovsky form, the construction of the controller is independent of the drift function of the system model. This advantage is of crucial importance in practice, since an exact model cannot always be obtained. A simulation example of the electronic realization of the Chua's circuit is given to show the effectiveness of the proposed adaptive synchronization.

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