Continuum modeling for two-lane traffic flow with consideration of the traffic interruption probability*

Tian Chuan(田 川)† and Sun Di-Hua(孙棣华)

College of Automation, Chongqing University, Chongqing 400030, *China*

(Received 15 March 2010; revised manuscript received 21 June 2010)

Considering the effects that the probability of traffic interruption and the friction between two lanes have on the car-following behaviour, this paper establishes a new two-lane microscopic car-following model. Based on this microscopic model, a new macroscopic model was deduced by the relevance relation of microscopic and macroscopic scale parameters for the two-lane traffic flow. Terms related to lane change are added into the continuity equations and velocity dynamic equations to investigate the lane change rate. Numerical results verify that the proposed model can be efficiently used to reflect the effect of the probability of traffic interruption on the shock, rarefaction wave and lane change behaviour on two-lane freeways. The model has also been applied in reproducing some complex traffic phenomena caused by traffic accident interruption.

Keywords: two lanes, traffic interruption probability, traffic flow dynamics model, numerical simulation

PACC: 0520, 5110, 0547

1. Introduction

In the last decades, traffic flow research has attracted much attention of physicists and engineers. A number of traffic models, including microscopic car-following models,[1−4] cellular automaton (CA) models,[5−7] macroscopic continuum models,[8−10] and macroscopic gas-kinetic-based models,[11] have been proposed to describe the physical mechanisms of various complex traffic phenomena. However, the models mentioned above do not involve the effects of traffic interruption probability on traffic flow, and thus they cannot be used to capture the complex phenomena resulted by various traffic interruptions.

In fact, some traffic interruptions (e.g. accident) always occur with some probabilities and produce complex phenomena. Very recently, considering the effects of the traffic interruption probability with dynamic equation,

$$\frac{\partial v}{\partial t}+(v-c(1-p))\frac{\partial v}{\partial x}=\frac{v_{\rm e}-v}{T}+\frac{1}{\tau}p(-v), \qquad (1)$$

Tang *et al.*[12] proposed a macroscopic continuum model. But this model focuses on the traffic flow on single lane road. Nevertheless, in real traffic, the road usually has more than one lane. Therefore, it is necessary and important to extend the well-developed single-lane traffic models to multi-lane traffic systems. In this paper, we concentrate on the research of traffic flow on two-lane freeways.

At present, many works have been done to describe the nature of traffic flow on two-lane freeways. Daganzo[13] developed a continuum theory of traffic dynamics for freeways with two lanes allocated for two sorts of vehicles (lane 1 for faster vehicles and lane 2 for slower vehicles). The theory defines the conservation equation as

$$\frac{\partial P}{\partial t}+\frac{\partial Q}{\partial x}=S, \qquad (2)$$

in which $P=(\rho_1,\rho_2)^{\rm T}$, $Q=[q_1(\rho_1,\rho_2),q_2(\rho_1,\rho_2)]^{\rm T}$, $S=(s_1(x,t),s_2(x,t))^{\rm T}$, where ρ_i, q_i and s_i represent the traffic density, the flow rate and the generation rate on lane i ($i=1,2$), respectively. Later, Tang and Huang[14,15] presented a continuum model for two-lane freeways, in which Jiang's momentum equation[16] was introduced into Daganzo's modeling framework[13] of multi-lane traffic flow. Hereafter, other different two-lane traffic models[17−22] were successively presented to reproduce complex traffic phenomena. However, to our knowledge, the effects that the probability of traffic interruptions on two-lane freeways have not been explored so far.

In this paper, we extended the traffic flow model proposed by Tang *et al.*[12] to describe the asymmetric traffic system of two-lane freeways. In our model, the

*Project supported by the National High Tech Research and Development Program of China (Grant No. 511-0910-1031).

†Corresponding author. E-mail: tianchuan_168@163.com

© 2010 Chinese Physics Society and IOP Publishing Ltd

factors including the probability of traffic interruption, the friction between two lanes and the lane change behaviours are considered in the same time. Then, the effects of the probability of traffic interruption on the shock wave, rarefaction wave and lane change rate in two-lane freeways are investigated by simulation.

The remainder of the present paper is organized as follows. In Section 2, we establish a new continuum traffic flow model starting from a car-following theory modified to suit the two-lane traffic flow. In Section 3, we numerically investigate the effects of the probability of traffic interruption on shock waves and rarefaction waves. In Section 4, we give a case study in which the presented model is applied to describe complex phenomena resulted by traffic accident interruption. In Section 5 we draw some conclusions from the present study.

2. Model

The classical car-following model used to describe the motion of vehicles following each other on a single lane without overtaking is put forward by Gazis *et al.*[23] In their model, the distance between successive vehicles can be arbitrarily close, which is unrealistic. Bando *et al.*[24] proposed optimal velocity model (OVM) which considers that the optimal velocity depends on the distance from the leading vehicle. Then OVM is improved by Helbing and Tilch[25] and developed into a generalized force model. Taking into account the effects of both the distance and relative speed of two successive vehicles, Jiang *et al.*[26] proposed a full velocity difference model recently. Theoretically, Jiang's model is more realistic and accurate than the previous ones. However, all these models just considered single lane traffic flow in which overtaking is not allowed.

In the case of two-lane traffic, the frictions between the vehicles on two lanes must be considered. As shown in Fig. 1, the driving behaviour of vehicle

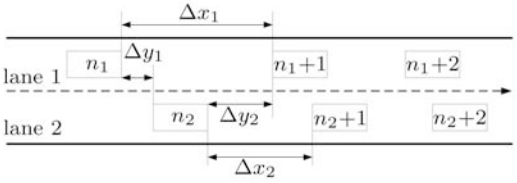


Fig. 1. The sketch of the two-lane traffic flow on a freeway.

n_1 is influenced by the distance and the relative speed of two successive vehicles on the same lane, as well as the location of vehicles n_2 on the lateral lane. Furthermore, the motion of each vehicle may be interrupted with some probability in real traffic.

In view of the above reasons, based on the car-following theory, we modified Tang's model[12] to describe the motion of vehicles on two-lane freeways with dynamic equation as follows:

$$\begin{aligned}\frac{\mathrm{d}u_{n_i}(t)}{\mathrm{d}t} =& k_i[u_i(\Delta x_i, \Delta y_i) - u_{n_i}(t)] \\ &+ \lambda_{i,1} p_{i,n_i+1}(-u_{n_i}) \\ &+ \lambda_{i,2}(1 - p_{i,n_i+1})\Delta u_{n_i}, \quad i = 1, 2,\end{aligned} \tag{3}$$

where $\Delta x_i = x_{n_i+1} - x_{n_i}$, $\Delta u_{n_i} = u_{n_i+1} - u_{n_i}$, and x_{n_i+1} (x_{n_i}) denotes the position of the leading (following) cars on lane i, u_{n_i+1} (u_{n_i}) represents the speed of the leading (following) cars on lane i. The Δy_1 is the distance between the car n_1 and the car n_2. The Δy_2 represents the distance between the car n_2 and the car $n_1 + 1$. The k_i, $\lambda_{i,1}$ and $\lambda_{i,2}$ are the reaction coefficients on lane i, and p_{i,n_i+1} is the probability that the leading vehicle on lane i is interrupted. For simplicity, p_{i,n_i+1} is set to be a same constant for two lanes. Once the leading vehicle is completely interrupted, its speed immediately becomes zero, thus the relative velocity between the preceding car n_{i+1}-th and following one n_i-th on lane i becomes $(-u_{n_i})$. The idea of the extended model is that the acceleration of the n_i-th vehicle on lane i is determined by the factors including the speed $u_{n_i}(t)$, the headway Δx_i, Δy_i, the relative speed Δu_{n_i} and the probability p_{i,n_i+1}.

To investigate the effects of the probability of traffic interruption on the dynamic properties of traffic flow, we should transform the micro variables in Eq. (3) into the macro ones by using the method of Ref. [27], i.e.,

$$\begin{cases} u_{n_i}(t) \to v_i(x,t),\ u_{n_i+1}(t) \to v_i(x + \Delta_i, t), \\ u_i(\Delta x_i, \Delta y_i) \to v_{ie}(\rho_1, \rho_2), \\ k_i \to \dfrac{1}{T_i},\ \lambda_{i,1} \to \dfrac{1}{\tau_{i,1}},\ \lambda_{i,2} \to \dfrac{1}{\tau_{i,2}}, \\ p_{i,n_i+1} \to p_i(x,t). \end{cases} \tag{4}$$

The parameter Δ_i in the $(x + \Delta_i)$ is the distance between the leading and following vehicles on lane i. The details of the inner relationship between macro and micro variables can be found in Refs. [12] and [15]. Then, equation (3) can be rewritten as follows:

$$\begin{aligned}\frac{\mathrm{d}v_i(x,t)}{\mathrm{d}t} =& \frac{v_{ie}(\rho_1, \rho_2) - v_i(x,t)}{T_i} + \frac{1}{\tau_{i,1}} p(-v_i) \\ &+ \frac{1}{\tau_{i,2}}(1 - p)(v_i(x + \Delta_i, t) - v_i(x,t)),\end{aligned} \tag{5}$$

where v_{ie}, v_i, and ρ_i denote the equilibrium speed, the speed, and the density on lane i ($i = 1, 2$), respectively. The T_i denotes the reactive time required by drivers for adjusting their acceleration based on equilibrium speed, while $\tau_{i,1}$ and $\tau_{i,2}$ respectively represent that reactive time based on the speed difference $(-v_i)$ caused by traffic interruption and the speed difference $v_i(x + \Delta_i, t) - v_i(x, t)$ without traffic interruption on lane i. In general, $k_i > \lambda_{i,2}$, $\tau_{i,1} < \tau_{i,2}$ hold, we then have $\tau_{i,1} < \tau_{i,2} < T_i$ on lane i.

Expanding the right-hand side of Eq. (5), and neglecting high-order terms, we obtain

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = \frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = \frac{v_{ie} - v_i}{T_i} + \frac{1}{\tau_{i,1}}p(-v_i) + \frac{\Delta_i}{\tau_{i,2}}(1-p)\frac{\partial v_i}{\partial x}, \quad i = 1, 2. \tag{6}$$

Combining with the conservation equation (2) of traffic flow, we find that the dynamics model for two-lane traffic consists of the following equations:

$$\frac{\partial \rho_i}{\partial t} + v_i\frac{\partial \rho_i}{\partial x} + \rho_i\frac{\partial v_i}{\partial x} = s_i(x, t), \quad i = 1, 2, \tag{7}$$

$$\frac{\partial v_i}{\partial t} + (v_i - c_{i,0}(1-p))\frac{\partial v_i}{\partial x} = \frac{v_{ie} - v_i}{T_i} + \frac{1}{\tau_{i,1}}p(-v_i), \quad i = 1, 2, \tag{8}$$

$$s_1(x, t) + s_2(x, t) = 0, \tag{9}$$

where $c_{i,0} = \Delta_i/\tau_{i,2} \geq 0$ is the propagating speed of small perturbation on lane i ($i = 1, 2$). For simplicity, $c_{i,0}$ is assumed to be a constant in this study.

In fact, on two-lane freeways, lane change behaviour will occur. To model this effect according to Tang's method,[28] a source term is added into the continuity equation to describe the flow rate changing from the other lane, and a sink term is added into the continuity equation to describe the flow rate changing to the other lane, thus, with consideration of the lane change, the final form of the traffic model involving interruption for two-lane freeways is obtained as follows:

$$\frac{\partial \rho_i}{\partial t} + v_i\frac{\partial \rho_i}{\partial x} + \rho_i\frac{\partial v_i}{\partial x} = s_{ji} - s_{ij}, \tag{10}$$

$$\frac{\partial v_i}{\partial t} + (v_i - c_{i,0}(1-p))\frac{\partial v_i}{\partial x} = \frac{v_{ie} - v_i}{T_i} + \frac{1}{\tau_{i,1}}p(-v_i) + r_1 s_{ij} - r_2 s_{ji}. \tag{11}$$

Here $i = 1, 2$ denotes one lane and $j = 1, 2$ ($j \neq i$) denotes the other lane. The term s_{ij} represents lane change rate from lane i to lane j, r_1 and r_2 are two constant parameters. Note that our model given by Eqs. (10) and (11) for two lanes is formally the same as the traffic flow model proposed by Tang *et al.*[12] (when $s = 0$ on single lane). However, attention should be paid to such a difference that all velocities in Eqs. (10) and (11) are functions of ρ_1 and ρ_2.

According to Liu's idea[29] and Tang's method,[28] we suppose the analogous terms of lane change rate with the form as follows:

$$s_{12} = \gamma a Q_{\mathrm{e}1}\rho_1(1 - (\rho_2/\rho_{2\mathrm{jam}})^{\sigma}), \tag{12}$$

$$s_{21} = \gamma a(1 + b(Q_{\mathrm{e}1} - Q_{e2}))Q_{\mathrm{e}2}\rho_2(1 - (\rho_1/\rho_{1\mathrm{jam}})^{\phi}), \tag{13}$$

where $Q_{\mathrm{e}i}$ denotes the equilibrium flow ($Q_{\mathrm{e}i} = \rho_i u_{\mathrm{e}i}(\rho_i)$), σ, ϕ are parameters which reflect the intensity of the lane change effect from one lane to another, γ is a constant used to describe the rate of the driver's lane change behaviour. The values of these three parameters depend on the condition of road and is limited by $0 \leq \sigma, \phi, \gamma \leq 1$. The special case $Q_{\mathrm{e}1} = Q_{\mathrm{e}2}$ corresponds to symmetric lane change rules, while $Q_{\mathrm{e}1} \neq Q_{\mathrm{e}2}$ corresponds to asymmetric lane change rules. The a and b are assumed to be constant parameters, $\rho_{i\mathrm{jam}}$ denotes the jam density of lane i.

In real traffic, the lane change rate should be zero under certain conditions. For example, when either $\rho_2 = \rho_{2\mathrm{jam}}$ (lane 2 is in complete jam) or $\rho_1 = 0$ (there is no vehicle on lane 1), then the lane change rate from lane 1 to lane 2 becomes zero. Furthermore, this rate obviously depends on the factors including the equilibrium speed-density relationship as well as the density of present lane and the lateral lane. These properties are satisfied in Eqs. (12) and (13).

The system consists of Eqs. (10) and (11) can be rewritten into a vector form as follows:

$$\frac{\partial V}{\partial t} + [A]\frac{\partial V}{\partial x} = E, \tag{14}$$

where

$$V = \begin{bmatrix} \rho_1 \\ u_1 \\ \rho_2 \\ u_2 \end{bmatrix},$$

$$E = \begin{bmatrix} s_{12} - s_{21} \\ \frac{(v_{1\mathrm{e}} - v_1)}{T_1} + \frac{1}{\tau_{1,1}}p(-v_1) + \gamma_1 s_{12} - \gamma_2 s_{21} \\ s_{21} - s_{12} \\ \frac{(v_{2\mathrm{e}} - v_2)}{T_2} + \frac{1}{\tau_{2,1}}p(-v_1) + \gamma_1 s_{21} - \gamma_2 s_{12} \end{bmatrix},$$

$$[A] = \begin{bmatrix} v_1 & \rho_1 & 0 & 0 \\ 0 & v_1 - c_{1,0}(1-p) & 0 & 0 \\ 0 & 0 & v_2 & \rho_2 \\ 0 & 0 & 0 & v_2 - c_{2,0}(1-p) \end{bmatrix}.$$

Next, matrices were employed to analyse characteristic speeds of the system (14). The eigenvalues λ of the $[A]$ matrix can be obtained by the eigenvalue analysis,

$$\det([A] - \lambda[I]) = 0, \tag{15}$$

where $[I]$ is an identity matrix. From Eq. (15), we obtain

$$\lambda_{11} = v_1, \quad \lambda_{12} = v_1 - c_{1,0}(1-p),$$

$$\lambda_{21} = v_2, \quad \lambda_{22} = v_2 - c_{2,0}(1-p).$$

For the condition mentioned above that $c_{i,0} \geq 0$ $(i = 1, 2)$, it is found that the characteristic speeds of each subsystem are not larger than their macroscopic traffic flow speeds, which means that the rear disturbance on each lane cannot propagate forward, but we should not take it for granted that the driving behaviour of the leading vehicle will not be affected by the rear disturbance in a system composed of two lanes because lane changing is allowed in these two-lane traffic systems.

3. Numerical simulation

In this section, the finite difference method was employed to illustrate that our model can be used to reproduce complex traffic phenomena. Let index j and index k represent the time interval and the road section, respectively. Then the difference equations corresponding to Eqs. (10) and (11) are as follows:

$$\rho_{ik}^{j+1} = \rho_{ik}^{j} + \frac{\Delta t}{\Delta x}\rho_{ik}^{j}\left(v_{ik}^{j} - v_{i(k+1)}^{j}\right) + \frac{\Delta t}{\Delta x}v_{ik}^{j}\left(\rho_{i(k-1)}^{j} - \rho_{ik}^{j}\right) + \Delta t s_{mik}^{j} - \Delta t s_{imk}^{j}, \quad m, i = 1, 2, m \neq i. \tag{16}$$

If the traffic is heavy, $v_{ik}^{j} < (1-p)c_{i0}$,

$$\begin{aligned} v_{ik}^{j+1} =& v_{ik}^{j} + \frac{\Delta t}{\Delta x}\left[(1-p)c_{i0} - v_{ik}^{j}\right]\left(v_{i(k+1)}^{j} - v_{ik}^{j}\right) + \frac{\Delta t}{T_i}\left(v_{ie} - v_{ik}^{j}\right) + \frac{1}{\tau_{i,1}}p(-v_{ik}^{j}) \\ &+ \gamma_1 \Delta t s_{imk}^{j} - \gamma_2 \Delta t s_{mik}^{j}, \quad m, i = 1, 2, i \neq m. \end{aligned} \tag{17}$$

Otherwise

$$\begin{aligned} v_{ik}^{j+1} =& v_{ik}^{j} + \frac{\Delta t}{\Delta x}\left[(1-p)c_{i0} - v_{ik}^{j}\right]\left(v_{ik}^{j} - v_{i(k-1)}^{j}\right) + \frac{\Delta t}{T_i}\left(v_{ie} - v_{ik}^{j}\right) + \frac{1}{\tau_{i,1}}p(-v_{ik}^{j}) \\ &+ \gamma_1 \Delta t s_{imk}^{j} - \gamma_2 \Delta t s_{mik}^{j}, \quad m, i = 1, 2, i \neq m. \end{aligned} \tag{18}$$

To investigate congestion and dissipation of the traffic flow, we adopted two Riemann initial conditions as follows:[1,15]

$$\text{(i)} \quad \begin{cases} \rho_{1\mathrm{u}}^{1} = 0.03, \\ \rho_{2\mathrm{u}}^{1} = 0.04, \end{cases} \quad \begin{cases} \rho_{1\mathrm{d}}^{1} = 0.12, \\ \rho_{2\mathrm{d}}^{1} = 0.18, \end{cases} \tag{19a}$$

$$\text{(ii)} \quad \begin{cases} \rho_{1\mathrm{u}}^{2} = 0.12, \\ \rho_{2\mathrm{u}}^{2} = 0.18, \end{cases} \quad \begin{cases} \rho_{1\mathrm{d}}^{2} = 0.03, \\ \rho_{2\mathrm{d}}^{2} = 0.04, \end{cases} \tag{19b}$$

where $\rho_{1\mathrm{u}}^{1}$, $\rho_{2\mathrm{u}}^{1}$, $\rho_{1\mathrm{d}}^{1}$, $\rho_{2\mathrm{d}}^{1}$ and $\rho_{1\mathrm{u}}^{2}$, $\rho_{2\mathrm{u}}^{2}$, $\rho_{1\mathrm{d}}^{2}$, $\rho_{2\mathrm{d}}^{2}$ are, respectively, the upstream and downstream densities of two lanes. Equation (19a) corresponds to the appearance of shock waves when free-flow traffic meets stopped vehicles, while equation (19b) corresponds to the rarefaction wave as a queue dissolves. Two initial conditions are

$$v_{1\mathrm{u}}^{1,2} = v_{1\mathrm{e}}(\rho_{1\mathrm{u}}^{1,2}, \rho_{2\mathrm{u}}^{1,2}), \quad v_{1\mathrm{d}}^{1,2} = v_{1\mathrm{e}}(\rho_{1\mathrm{d}}^{1,2}, \rho_{2\mathrm{d}}^{1,2}), \tag{20a}$$

$$v_{2\mathrm{u}}^{1,2} = v_{2\mathrm{e}}(\rho_{1\mathrm{u}}^{1,2}, \rho_{2\mathrm{u}}^{1,2}), \quad v_{2\mathrm{d}}^{1,2} = v_{2\mathrm{e}}(\rho_{1\mathrm{d}}^{1,2}, \rho_{2\mathrm{d}}^{1,2}). \tag{20b}$$

Here, the free boundary condition, i.e., $\partial\rho_i/\partial x = 0$, $\partial v_i/\partial x = 0$ are adopted. The equilibrium speed-density relationships are as follows:

$$v_{1\mathrm{e}} = v_{1\mathrm{f}}\left[1 - \exp\left(1 - \exp\left(\frac{c_{1\mathrm{jam}}}{v_{1\mathrm{f}}}\left(\frac{\rho_{1\mathrm{jam}} + 0.1\rho_{2\mathrm{jam}}}{\rho_1 + 0.1\rho_2} - 1\right)\right)\right)\right], \tag{21a}$$

$$v_{2e} = v_{2f}\left[1 - \exp\left(1 - \exp\left(\frac{c_{2jam}}{v_{2f}}\left(\frac{0.1\rho_{1jam} + \rho_{2jam}}{0.1\rho_1 + \rho_2} - 1\right)\right)\right)\right], \tag{21b}$$

where ρ_{1jam}, ρ_{2jam} denote the jam densities, and v_{1f}, v_{2f} are free-flow velocity on lane 1 and lane 2, respectively. The c_{1jam} and c_{2jam} are the propagation velocity of a disturbance under the jam density ρ_{1jam}, ρ_{2jam}, respectively. In the simulation, set the length of freeway to be $x = 20$ km long, divide it into 100 cells, and take the time interval as $\Delta t = 1$ s. According to real observation and the parameter identification process, the values of the parameters are chosen as follows:[8,10,28]

$\rho_{1j} = 0.15$ veh/m, $v_{1f} = 40$ m/s, $c_{10} = c_{1jam} = 15$ m/s, $T_1 = 15$ s, $\tau_{1,1} = 9$ s, $p = 0.2$, $\sigma = \phi = 1$; $\rho_{2j} = 0.2$ veh/m, $v_{2f} = 30$ m/s, $c_{20} = c_{2jam} = 11$ m/s, $T_2 = 10$ s, $\tau_{2,1} = 8$ s, $a = 0.01$, $b = 5$, $\gamma_1 = 150$ m^2/s, $\gamma_2 = 50$ m^2/s, $\gamma = 1$.

Note that the jam density of lane 2 is higher than that of lane 1. The computational results under the two Riemann initial conditions (19a) and (19b) are shown in Figs. 2–5 respectively.

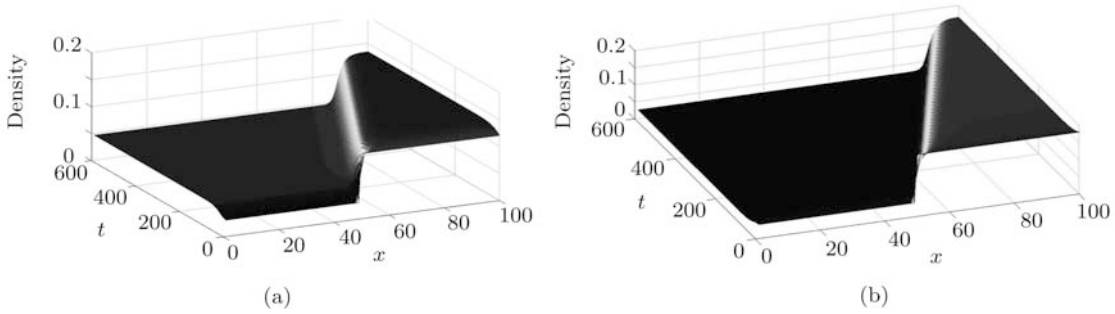


Fig. 2. Shock waves under the Riemann initial conditions of Eq. (19a): (a) and (b) illustrate evolutions of density of lane 1 and lane 2 respectively.

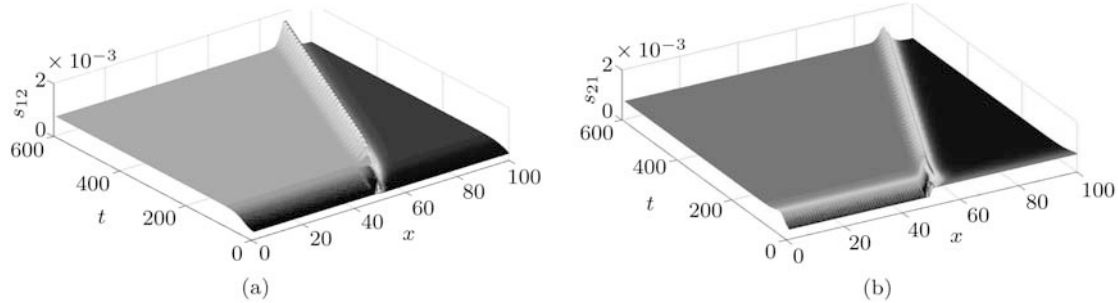


Fig. 3. Lane change rate under the Riemann initial conditions of Eq. (19a): (a) s_{12} denotes the lane change rate from lane 1 to lane 2, and (b) s_{21} is the lane change rate from lane 2 to lane 1.

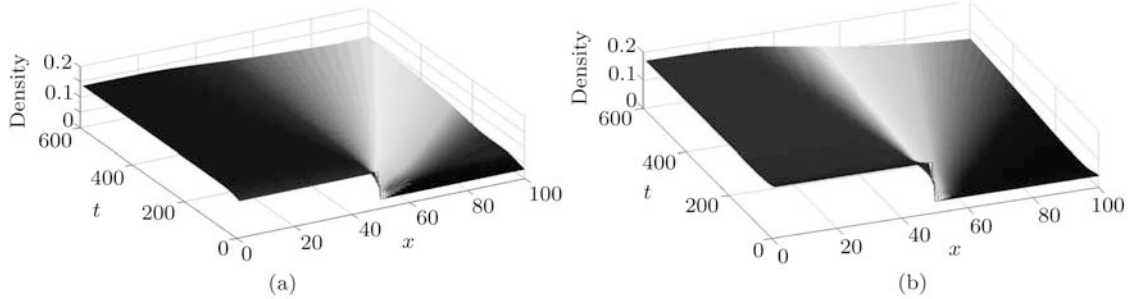


Fig. 4. Rarefaction waves under the Riemann initial conditions of Eq. (19b): (a) and (b) are the evolution of density of lane 1 and lane 2 respectively.

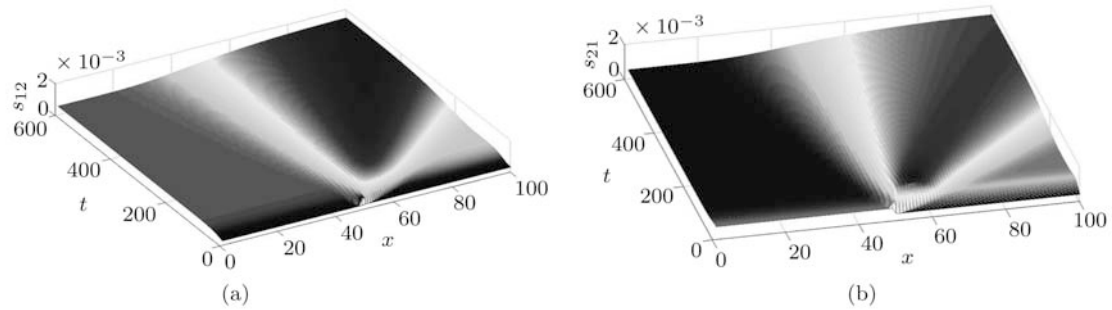


Fig. 5. Lane change rate under the Riemann initial conditions of Eq. (19b): (a) s_{12} denotes the lane change rate from lane 1 to lane 2, and (b) s_{21} is the lane change rate from lane 2 to lane 1.

Figures 2 and 4 show that different Riemann initial conditions lead to different fronts between the congested and free-flow traffic, while the lane change rate corresponding to those two initial conditions is illustrated in Figs. 3 and 5 respectively. The wave shape of two lanes (see Figs. 2 and 4) are similar to that reported in Refs. [8], [10], [12], and [16], which indicates that our new two-lane model is reasonable. Figure 2 shows the evolution of the backward-moving shock wave under the condition (19a) on lanes 1 and 2, respectively. This means that the traffic state becomes more congested, which can often be observed in rush hours. Figure 4 shows the trend that the rarefaction wave front evolves under the condition (19b). This illustrates the dissolution process of a queue, which is in agreement with our common experiences in actual traffic.

4. A case study

In this section, we explore the effects of one traffic accident interruption on the waves of two-lane traffic under the following two cases.

Case 1 the accident appears at $x = 4$ km (the upstream of the midpoint) on lane 1 and lasts for 5 min;

Case 2 the accident appears at $x = 16$ km (the downstream of the midpoint) on lane 1 and lasts for 5 min.

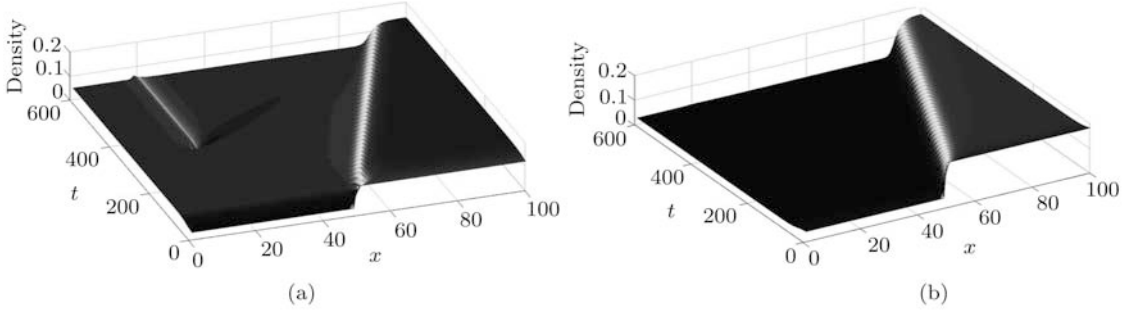


Fig. 6. Evolution of shock wave under Riemann initial conditions of Eq. (19a) for Case 1: (a) and (b) are the evolutions of density corresponding to lane 1 and lane 2 respectively.

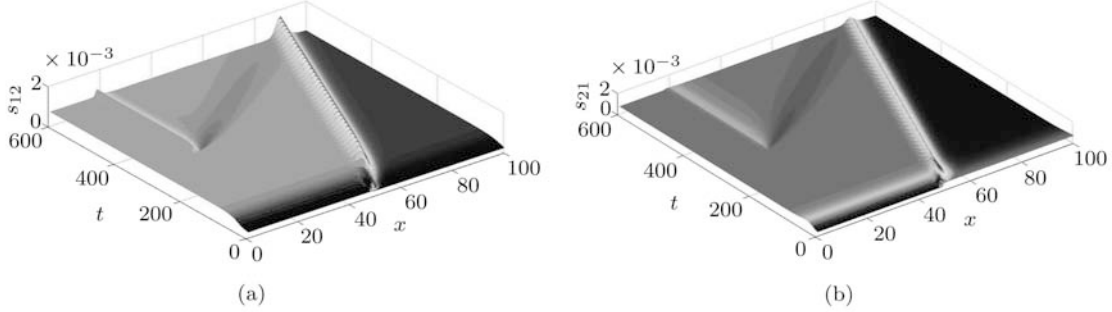


Fig. 7. Lane change rate under the Riemann initial conditions of Eq. (19a) for Case 1: (a) s_{12} denotes the lane change rate from lane 1 to lane 2, and (b) s_{21} is the lane change rate from lane 2 to lane 1.

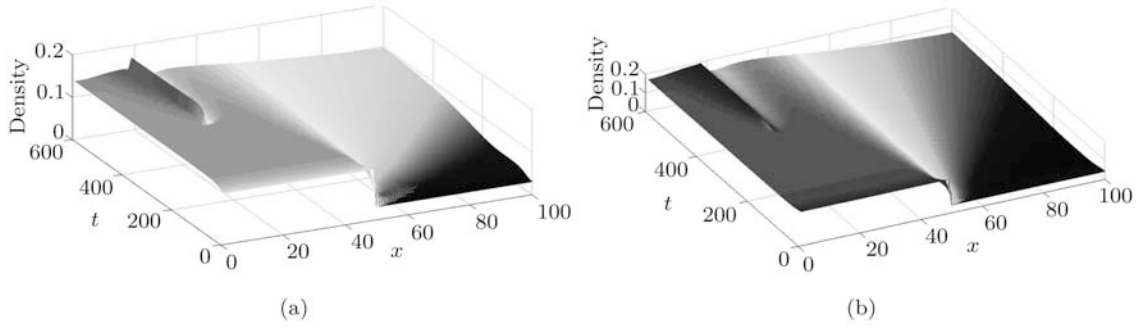


Fig. 8. Evolution of rarefaction wave under Riemann initial conditions of Eq. (19b) for Case 1: (a) and (b) are the evolution of density on lane 1 and lane 2 respectively.

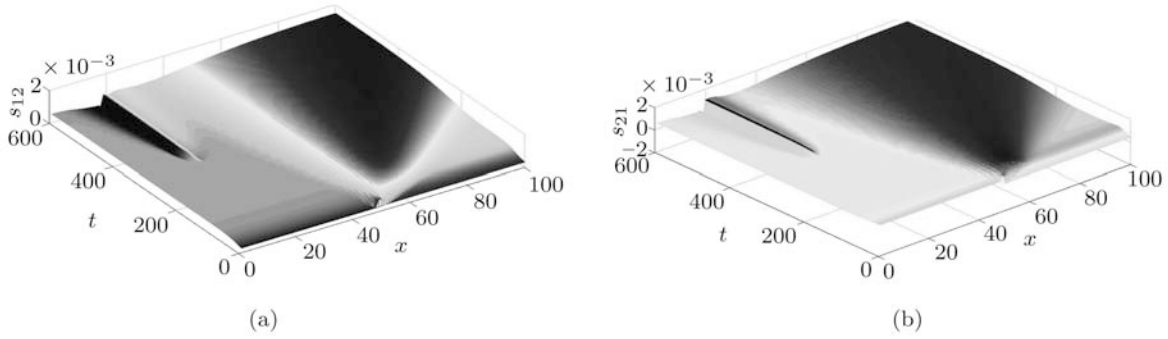


Fig. 9. Lane change rate under the Riemann initial conditions of Eq. (19b) for Case 1: (a) s_{12} is the lane change rate from lane 1 to lane 2, and (b) s_{21} is the lane change rate from lane 2 to lane 1.

The density evolution and lane change rate in Case 1 under the two Riemann initial conditions (19a) and (19b) are shown in Figs. 6–9.

In Figs. 6–9, the following observations can be obtained.

(i) When an accident appears at the upstream of the midpoint on the lane 1, the rarefaction wave and lane change rate under Riemann initial conditions of Eq. (19b) are influenced on each lane (see Figs. 8 and 9), since the clustering vehicles at the upstream cannot be dissipated quickly.

(ii) When an accident appears at the upstream of the midpoint on the lane 1 under the conditions of Eq. (19a), we can observe that the shock wave is influenced little and traffic cluster appears at the upstream of the accident site on lane 1 (see Fig. 6(a)). However, the shock wave on lane 2 is not affected (see Fig. 6(b)). The reason of the phenomena is that the lane change rate is low between lane 1 and lane 2 under this situation (see Fig. 7).

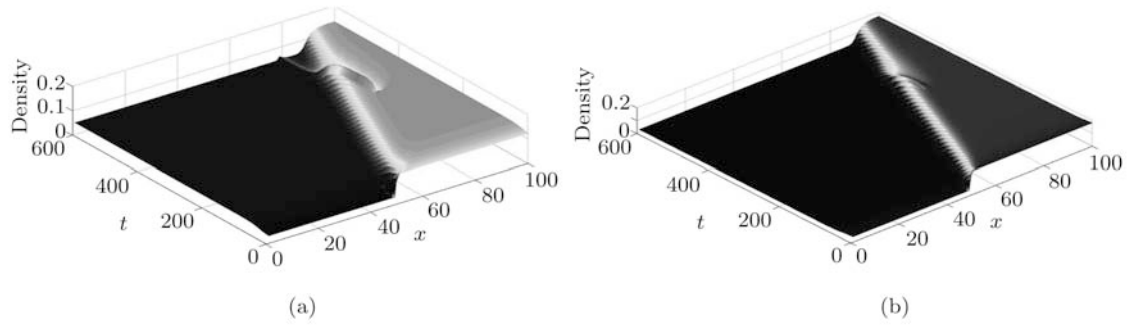


Fig. 10. Evolution of shock wave under Riemann initial conditions of Eq. (19a) for Case 2: (a) and (b) are evolutions of density of lane 1 and lane 2 respectively.

The density evolution and lane change rate for Case 2 under the two Riemann initial conditions are given in Figs. 10–13.

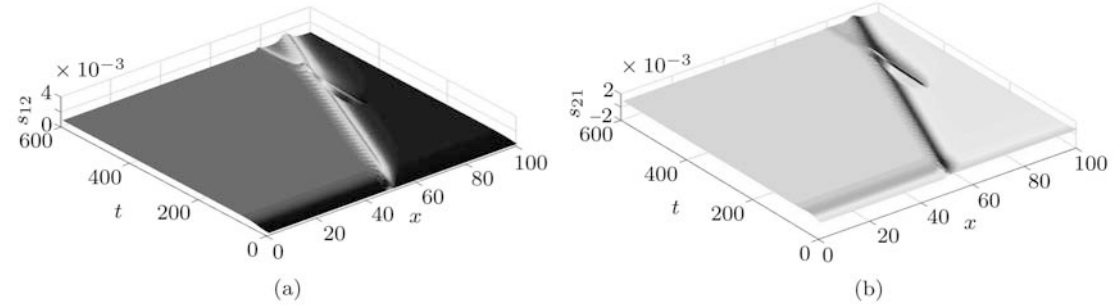


Fig. 11. Lane change rate under the Riemann initial conditions of Eq. (19a) for Case 2: (a) s_{12} is the lane change rate from lane 1 to lane 2, and (b) s_{21} is the lane change rate from lane 2 to lane 1.

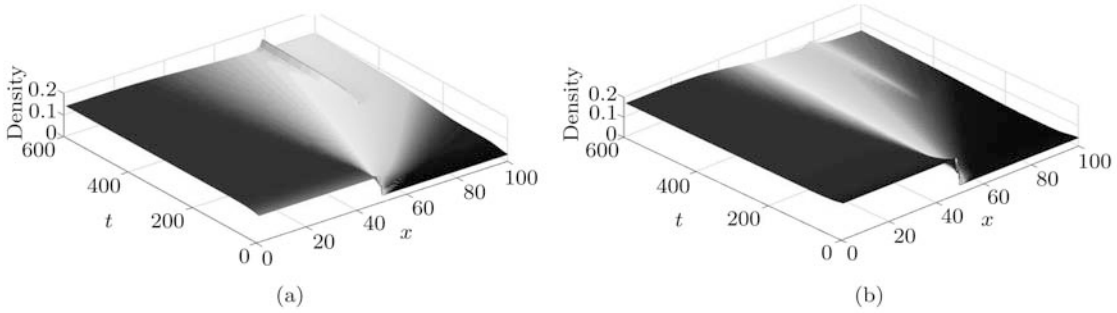


Fig. 12. Evolution of rarefaction wave under Riemann initial conditions of Eq. (19b) for Case 2: (a) and (b) are evolutions of density of lane 1 and lane 2 respectively.

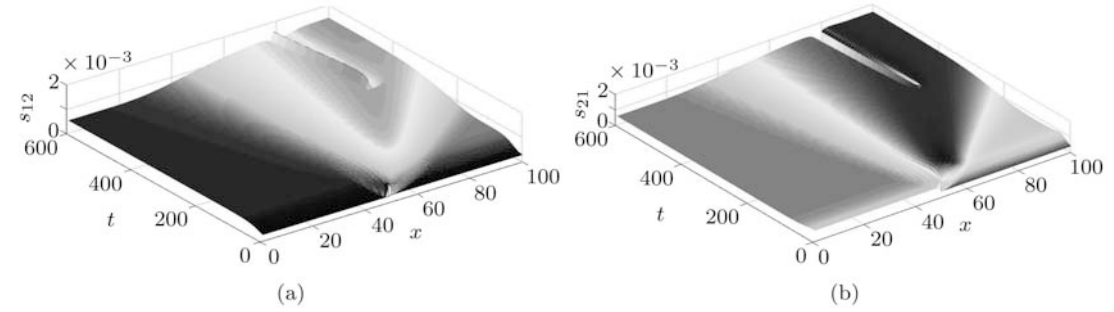


Fig. 13. Lane change rate under the Riemann initial conditions of Eq. (19b) for Case 2: (a) s_{12} is the lane change rate from lane 1 to lane 2, and (b) s_{21} is the lane change rate from lane 2 to lane 1.

From Figs. 10–13, the following points can be obtained.

(I) When an accident takes place at the downstream of the midpoint on lane 1 under condition (19a), the shock waves and corresponding lane change rate on both lanes are affected by the accident. The backward propagating cluster occurs at the upstream of the accident site on each lane (see Figs. 10 and 11). The cause of this phenomenon is that the density of downstream is higher than that of the upstream, and the occurrence of traffic accident on lane 1 leads to the formation of bottleneck, and then the vehicles of upstream are forced to decelerate, thus traffic jam emerges and propagates backward.

(II) When the accident occurs on lane 1, light cluster phenomenon in the rarefaction wave can be observed on lane 1 (Fig. 12(a)), while it has no effect on the rarefaction wave on lane 2 (Fig. 12(b)), the

cause of this phenomenon is that the density of downstream of the midpoint on both lanes is rather low, once traffic accident takes place on lane 1, the drivers can change lane easily, and thus the cluster can be dissipated quickly.

Combining with the outcomes mentioned in Case 1 and Case 2, we can conclude that the above results are in good agreement with the real traffic, so the new model can be employed to describe complex phenomenon resulted by traffic accident interruption for two-lane traffic system.

5. Conclusion

There are a few traffic flow models to study the effects of the probability of the traffic interruption on traffic flow for two-lane freeway. In this paper, we presented a new macromodel in which the effects of interruption probability and friction for two lanes are considered. Furthermore, the lane change behaviour is incorporated into the extended model by introducing a source term and a sink term into both the continuity equation and the dynamic equations. By means of this method, the lane-changing rate has been investigated.

We carried out numerical simulation of the shock waves, rarefaction waves and lane change rate caused by probability of traffic interruption, and verified that the new model can reasonably describe complex traffic phenomena caused by traffic interruptions in the two-lane freeway traffic system. Therefore, our proposed model can provide an intuitively acceptable explanation of some common two-lane traffic behaviour caused by traffic interruptions.

References

[1] Peng G H, Sun D H and He H P 2008 *Acta Phys. Sin.* **57** 7541 (in Chinese)

[2] Xue Y, Dong L Y, Yuan Y W and Dai S Q 2002 *Acta Phys. Sin.* **51** 492 (in Chinese)

[3] Mo Y L, He H D, Xue Y, Shi W and Lu W Z 2008 *Chin. Phys.* B **17** 4446

[4] Chen X, Gao Z Y, Zhao X M and Jia B 2007 *Acta Phys. Sin.* **56** 2024 (in Chinese)

[5] Li X L, Kuang H, Song T, Dai S Q and Li Z P 2008 *Chin. Phys.* B **17** 2366

[6] Ge H X, Zhu H B and Dai S Q 2005 *Acta Phys. Sin.* **54** 4621 (in Chinese)

[7] Zheng Z Z and Wang A L 2009 *Chin. Phys.* B **18** 489

[8] Peng G H, Sun D H and He H P 2009 *Chin. Phys.* B **18** 468

[9] Ge H X, Dai S Q and Dong L Y 2008 *Chin. Phys.* B **17** 23

[10] Sun D H and Peng G H 2009 *Chin. Phys.* B **18** 3724

[11] Treiber M, Hennecke A and Helbing D 1999 *Phys. Rev.* E **59** 239

[12] Tang T Q, Huang H J and Xu G 2008 *Physica* A **387** 6845

[13] Daganzo C F 1997 *Transpn. Res.* B **31** 83

[14] Tang T Q and Huang H J 2004 *Chin. Sci. Bull.* **49** 1937

[15] Tang T Q and Huang H J 2005 *J. Beijing University of Aero. Astro.* **31** 1121 (in Chinese)

[16] Jiang R, Wu Q S and Zhu Z J 2002 *Transpn. Res.* B **36** 405

[17] Peng L, Tan H L, Wu D Y, Liu M R and Kong L J 2004 *Acta Phys. Sin.* **53** 2899 (in Chinese)

[18] Zheng R S, Tan H L, Kong L J and Liu M R 2005 *Acta Phys. Sin.* **54** 4614 (in Chinese)

[19] Tang T Q, Huang H J and Xue Y 2006 *Acta Phys. Sin.* **55** 4027 (in Chinese)

[20] Wu K F, Kong L J and Liu M R 2006 *Acta Phys. Sin.* **55** 6275 (in Chinese)

[21] Chen X, Gao Z Y, Jia B and Zhao X M 2007 *Acta Phys. Sin.* **56** 2024 (in Chinese)

[22] Teng Y F, Gao Z Y, Jia B and Li F 2008 *Acta Phys. Sin.* **57** 1365 (in Chinese)

[23] Gazis D C, Herman R and Rothery R W 1961 *Oper. Res.* **9** 545

[24] Bando M, Hasebe K, Nakyaama A, Shibata A and Sugiyama Y 1995 *Phys. Rev.* E **51** 1035

[25] Helbing D and Tilch B 1998 *Phys. Rev.* E **58** 133

[26] Jiang R, Wu Q S and Zhu Z J 2001 *Phys. Rev.* E **64** 017101

[27] Liu G Q, Lyrintiz A S and Michalopoulos P G 1998 *Transport. Res. Rec.* **1644** 37

[28] Tang C F, Jiang R and Wu Q S 2007 *Chin. Phys.* **16** 1570

[29] Liu G Q 1996 *Appl. Math. Model* **20** 459