Josephson junction of two-species Bose–Einstein condensates in a high-finesse optical cavity*

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The mean-field dynamics of undistinguishable two-species Bose Josephson junction coupled to a single mode high-finesse optical cavity is investigated. From the Hamiltonian, the phase portrait and the stationary points are given. It is shown that the role of the interspecies interaction equals the intraspecies interaction under suitable conditions. As the interspecies interaction increases, the trapped atoms will start tunneling between the two wells unnaturally for some special cases.

Keywords: Josephson effect, cavity, Bose–Einstein condensate
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1. Introduction

Recently, much attention to the interacting Bose–Einstein condensates (BECs) has been focused on the tunneling phenomena of BEC in a double-well potential,[1−5] which in this context is referred to as the Josephson effect occurring in the superconductor or superfluid. In the theoretical research, realisation of the Bose Josephson junction (BJJ) has already been investigated from earlier years.[6,7] And the BJJ consisting of two weakly coupled BEC was implemented in a macroscopic double-well potential in 2005 in the experiment.[4] In 2007, Gati and Oberthaler[8] studied the BJJ, focusing on the data obtained experimentally with the predictions of a mean-field approximation.

All of the above investigations are conducted in the presence of a single bosonic species, where the most important feature of its nonlinear dynamics comes from the interaction between the same atoms. As it is extended to two species BEC, the tunneling behaviour is expected to be far richer than that of the single component system due to the interspecies interaction between two species, which has attracted considerable theoretical and experimental attention.[9,10] Under the mean-field theory together with the two-mode approximation,[11] Xu *et al.*[12] has studied the fixed points and their stability of BJJ with mixture of components, and indicated that there existed two more fixed points than that in single component. Particles of different species tunneling as pairs through the potential barrier between the two wells in opposite directions were also investigated.[13] Such a correlated motion of tunneling atoms leads to the generation of quantum entanglement between two macroscopically coherent systems. Mazzarella *et al.*[14] has considered the atomic Josephson junction of two-species, and suggested the possibility to determine the inter-species s-wave scattering length experimentally by examining the oscillation frequency around the fixed points. By solving the coupled pendulum equation of binary BJJ numerically, the authors in Ref. [15] demonstrated a novel type of mode corresponding to symmetry-restoring phase which cannot appear in BJJ of single bosonic atoms. So, in comparison to the single-species case, the multispecies BECs can exhibit richer effects due to the existence of the interspecies interaction.

Even though many meaningful researches based on BJJ have been conducted, further study is also

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expected, for instance, on BJJ that couples to a cavity. In 2007, a great step was made as two groups succeeded independently in coupling a BEC to a single-cavity mode.[16,17] That is, a single-cavity mode dressed condensate has been achieved. All the atoms have the same motion mode and coupled identically to the cavity mode. This opens up a new regime in both the cavity quantum electrodynamics and ultracold atom physics. As shown by experiments, the strong interaction between the atoms and cavity would not destroy the condensates in the time scale of the experiments. In Ref. [18], in terms of the mean-field theory and two-mode approximation, the phase portrait shows that the strong condensate-field interplay alters the dynamics of the BJJ drastically.

Considering the above facts, it is very interesting to study the two-species BJJ coupled to a high-finesse cavity, then we would extend the investigation to the dynamics of binary BJJ,[8] which is coupled to the driven cavity mode. With the two species being two isotopes of an alkali metal which can both coupled to the same cavity mode, ^{85}Rb-^{87}Rb for example, we neglect the atomic spontaneous emission under the condition of large-detuning and low-excitation. In a one-species BJJ system, the parameters that can be varied to control the dynamics of the atoms are (i) detuning and loss rate, (ii) initial state.[18] For a two-species BJJ system, there are more controlling parameters, namely, intraspecies and interspecies interaction. Furthermore, the interspecies interaction can make a visible effect on the dynamics of the atoms and the intraphoton of the cavity.

This paper is organised as follows. In Section 2, we employ the mean-field approximation to map the quantum model into a classical Hamiltonian and derive the corresponding dynamical equations. In Section 3, we discuss the dynamics of a two-species Bose Josephson junction uncoupled and coupled to a cavity mode in the phase portrait. Our results are briefly summarised in Section 4.

2. Theoretical model

The Hamiltonian of the system consists of three parts

$$H = H_{\rm a} + H_{\rm f} + H_{\rm int}. \tag{1}$$

The normal method to solve this system is based on the two-mode approximation in which the evolution is confined by the left and right localised mode functions associated with the respective potential traps.[11,18–21] The two-mode model assumes two stationary wave functions in the individual traps, while neglecting modifications due to the atom–atom interaction.[11] So there are two kinds of wave functions. In the two-mode approximation the Hamiltonian $H_{\rm a}$ of the normal BJJ is, (in this paper we set $\hbar = 1$)

$$\begin{aligned} H_{\rm a} =& -\Omega(b_1^\dagger b_2 + b_2^\dagger b_1) - \Omega(c_1^\dagger c_2 + c_2^\dagger c_1) \\ &+ \frac{V}{2}(b_1^\dagger b_1^\dagger b_1 b_1 + b_2^\dagger b_2^\dagger b_2 b_2) \\ &+ \frac{V}{2}(c_1^\dagger c_1^\dagger c_1 c_1 + c_2^\dagger c_2^\dagger c_2 c_2) \\ &+ \frac{V'}{2} b_1^\dagger b_1 c_1^\dagger c_1 + \frac{V'}{2} b_2^\dagger b_2 c_2^\dagger c_2, \end{aligned} \tag{2}$$

where the subscripts 1 and 2 denote the localised modes in the left and right potential wells respectively. Since there are two modes available for each species, the model in fact consists of four-mode operators. We use $b_i^\dagger$ and $c_i^\dagger$ $(i = 1, 2)$ to denote the creation operators of the components b and c, respectively. Ω is the parameter of tunneling matrix element between the two modes, while V denotes the repulsive interaction intensity of two atoms in the same mode. V' is the repulsive interplay intensity between the different atoms. $H_{\rm f}$ is the single-mode field Hamiltonian,

$$H_{\rm f} = \omega_c a^\dagger a + \eta(t) {\rm e}^{-{\rm i}\omega_{\rm p} t} a^\dagger + \eta^*(t) {\rm e}^{{\rm i}\omega_{\rm p} t} a, \tag{3}$$

where ω_c and $\omega_{\rm p}$ are the frequencies of the cavity and the pump, $\eta(t)$ is the amplitude of the pump. Under the two-mode approximation, the interaction between the atoms and the cavity mode is[22]

$$H_{\rm int} = U_0 a^\dagger a (J_1 n_1 + J_2 n_2 + J_1' n_1' + J_2' n_2'), \tag{4}$$

where $U_0 = g_0^2/(\omega_{\rm c} - \omega_{\rm a})$, with g_0 being the atom–field coupling at the antinode. The parameters $J_1(J_2)$ and $J_1'(J_2')$ denote the overlap between the atomic mode and the cavity mode.[23] While $n_1(n_2)$, and $n_1'(n_2')$ describe the number of the atoms of the two species. The trapping potential is tilted provided that $J_1 \neq J_2$ and $J_1' \neq J_2'$ from the point of view of the atom ensemble. We would point out that in the Hamiltonian (4), we have dropped some terms $U_0 a^\dagger a J_{12}(b_1^\dagger b_2 + b_2^\dagger b_1)$ and $U_0 a^\dagger a J_{12}^{'}(c_1^\dagger c_2 + c_2^\dagger c_1)$, which are much smaller than the intrinsic tunneling coefficient;[18] and in the two-mode approximation, the overlap between different species atom modes can be also dropped.

According to the Heisenberg's equation, we have

$$\mathrm{i}\dot{b_1} = -\Omega b_2 + V b_1^\dagger b_1 b_1 + \frac{V'}{2} c_1^\dagger c_1 b_1 + J_1 U_0 a^\dagger a b_1, \tag{5a}$$

$$\mathrm{i}\dot{b_2} = -\Omega b_1 + V b_2^\dagger b_2 b_2 + \frac{V'}{2} c_2^\dagger c_2 b_2 + J_2 U_0 a^\dagger a b_2, \tag{5b}$$

$$\mathrm{i}\dot{c_1} = -\Omega c_2 + V c_1^\dagger c_1 c_1 + \frac{V'}{2} b_1^\dagger b_1 c_1 + J_1' U_0 a^\dagger a c_1, \tag{5c}$$

$$\mathrm{i}\dot{c_2} = -\Omega c_1 + V c_2^\dagger c_2 c_2 + \frac{V'}{2} b_2^\dagger b_2 c_2 + J_2' U_0 a^\dagger a c_2, \tag{5d}$$

$$\mathrm{i}\dot{a} = [\omega_c + U_0(J_1 n_1 + J_2 n_2 + J_1' n_1' + J_2' n_2')]a - \mathrm{i}\kappa a + \eta(t)\mathrm{e}^{-\mathrm{i}\omega_\mathrm{p} t}. \tag{5e}$$

We note that in Eq. (5e), there is a new term $-\mathrm{i}\kappa a$ that to describes the loss of the cavity, with κ being the loss rate of the cavity. In Eqs. (5a)–(5d), from the terms $J_i U_0 a^\dagger a b_i$ and $J_i^{'} U_0 a^\dagger a c_i$ $(i = 1, 2)$ we find that the changes of the number of atoms depend on the photon, in other words, the BJJ is titled with the photon distribution which depends on the number of atoms. So it is possible to modify the nature of the BJJ by adjusting the frequency of pump. In the mean-field approximation, we treat the operators b_1, b_2, c_1, c_2, and a as classical quantities, $b_1 = \sqrt{N_1}\mathrm{e}^{\mathrm{i}\theta_1}, b_2 = \sqrt{N_2}\mathrm{e}^{\mathrm{i}\theta_2}, c_1 = \sqrt{N_1'}\mathrm{e}^{\mathrm{i}\theta_1'}, c_2 = \sqrt{N_2'}\mathrm{e}^{\mathrm{i}\theta_2'}, a = \alpha$. Here $N_{1,2}$ and $N_{1,2}'$ are the particle number of the components b and c, in the two-side trap, respectively. In order to facilitate the discussion, we assume the number of particles are the same for the two species, i.e., $N_b = N_c = N$,[13] where $\theta_{1,2}$ and $\theta_{1,2}'$ are their phases, $\varphi = \Delta\theta$ is the phase difference.

In this system, the relaxation time of the cavity is of the order of $1/\kappa, \kappa \sim 2\pi \times 10^6$ Hz, but the vibration cycle of a BJJ is of the order of $1/\Omega, \Omega$ ranging from $2\pi \times 10$ to $2\pi \times 10^2$ Hz. This implies that the cavity field follows the motion of the condensates adiabatically and the phase diagram is stable. Thus, from Eq. (5e) we obtain

$$\langle a \rangle \equiv \alpha(t) = \frac{\eta(t)\mathrm{e}^{-\mathrm{i}\omega_\mathrm{p} t}}{\mathrm{i}\kappa + [\omega_\mathrm{p} - \omega_c - U_0(J_1 N_1 + J_2 N_2 + J_1^{'} N_1^{'} + J_2^{'} N_2^{'})]}, \tag{6}$$

and the photon number is

$$\langle a^\dagger a \rangle \equiv |\alpha(t) \times \alpha(t)| = |\alpha(t)|^2 = \frac{|\eta(t)|^2}{\kappa^2 + [\Delta - \delta U_0(N_1 - N_2/2) - \delta' U_0'(N_1' - N_2')/2]^2}, \tag{7}$$

where $\Delta \equiv \omega_\mathrm{p} - \omega_c - (J_1 + J_2)NU_0/2 - (J_1' + J_2')NU_0/2, \delta = J_1 - J_2, \delta^{'} = J_1' - J_2'$; δ and $\delta'(\delta = \delta')$ are the coupling differences between the atomic modes and the cavity mode, respectively.

By introducing two dimensionless parameters $z_b = (N_1 - N_2)/N$ and $z_c = (N_1' - N_2')/N$ the population of the two kinds of atoms between the two wells can be described. Thus Eq. (7) is expressed as

$$|\alpha(z_b, z_c, t)|^2 = \frac{A(t)^2}{(z_b + z_c - B)^2 + C^2}, \tag{8}$$

where the three parameters are defined as $A(t) = \eta(t)/[\delta U_0 N/2], B = \Delta/[\delta U_0 N/2]$, and $C = \kappa/[\delta U_0 N/2]$. Here the total particle number N is of the order of 10^4 and $A(t), B$, and C can be understood as the reduced pumping strength, reduced detuning, and reduced loss rate, respectively.[18]

Substituting Eq. (8) into Eqs. (6a)–(6d), defining the phase differences $\varphi_b = \theta_1 - \theta_2$, and $\varphi_c = \theta_1' - \theta_2'$, then we can rewrite the four equations expressed by the atom populations and phase differences as follows:

$$\dot{\phi_b} = \frac{z_b}{\sqrt{1 - z_b^2}} \cos\phi_b + r_b z_b + \frac{r_{bc} z_c}{2} + \frac{\delta U_0}{2\Omega}|\alpha|^2, \tag{9a}$$

$$\dot{z_b} = -\sqrt{1 - z_b^2}\sin\phi_b, \tag{9b}$$

$$\dot{\phi_c} = \frac{z_c}{\sqrt{1 - z_c^2}} \cos\phi_c + r_c z_c + \frac{r_{bc} z_b}{2} + \frac{\delta U_0}{2\Omega}|\alpha|^2, \tag{9c}$$

$$\dot{z_c} = -\sqrt{1 - z_c^2}\sin\phi_c, \tag{9d}$$

where the parameters r_b, r_c, and r_{bc} are proportional to the ratio of the interaction force to the tunneling element given in Ref. [19], and $r \geq r_{bc}$. The parameter $r_b = r_c = r = NV/(2\Omega)$ denotes the ratio of the intra-species atoms, and $r_{bc} = NV'/(2\Omega)$ denotes the ratio of the interspecies atoms. In Eqs. (9a) and (9c), there is one more term of r_{bc} than the one-species case in Ref. [18].

To study the relation between the parameters r_{bc} and r_b (or r_c), we define a Hamiltonian $H_n(z, \phi, t)(n = b, c)$ in which z and ϕ are two variables. Their relations are $\dot{z} = -\partial H_n/\partial\phi, \dot{\phi} = \partial H_n/\partial z$. So the forms

of the Hamiltonians are

$$H_b = -\sqrt{1-z_b^2}\cos\phi_b + \frac{rz_b^2}{2} + \frac{r_{bc}z_bz_c}{2} + \frac{\delta U_0}{2\Omega}F(z_b,z_c,t), \quad (10a)$$

$$H_c = -\sqrt{1-z_c^2}\cos\phi_c + \frac{rz_c^2}{2} + \frac{r_{bc}z_bz_c}{2} + \frac{\delta U_0}{2\Omega}F(z_b,z_c,t) \quad (10b)$$

with $F(z_{b,c},t) = A^2(t)/C\arctan((z_b+z_c-B)/C)$. In Eqs. (10), the first two terms of the two Hamiltonians are the Hamiltonian of a bare Bose Josephson junction.[6] The third term of each Hamiltonian indicates the interspecies interplay. The last terms of the two Hamiltonian are termed the cavity-field-induced tilt.[18] The introduction of the tilt indicates that the wells have been kept symmetric until just now they become asymmetric, because of the fluctuation of the atoms between the two traps. This term is similar to the potential an atom experiences when passing a cavity adiabatically.[24] In particular, we concentrate on that the pump strength is a constant, so that the Hamiltonian is conserved in time.

From another perspective, the number of stationary points is determined by $\partial H_n/\partial\phi = 0$ and $\partial H_n/\partial z = 0$, which reflects the dynamical nature of the quantum eigenstates. The equation $\partial H_n/\partial\phi = 0$ implies that $\phi_b = 0$ or π and $\phi_c = 0$ or π. In Figs. 1(a) and 1(b) we can see that there are some new points and motion modes after the BJJ coupling to the cavity at $\phi = 0, \pi$. So substituting the two values of ϕ into Eqs. (9a) and (9c), we obtain

$$F_1(z_b)|_{\phi_b=0} = \frac{z_b}{\sqrt{1-z_b^2}} + r_bz_b + \frac{r_{bc}z_c}{2} + \frac{\tilde{A}}{(z_b+z_c-B)^2+C^2} = 0, \quad (11a)$$

$$F_1(z_c)|_{\phi_c=0} = \frac{z_c}{\sqrt{1-z_c^2}} + r_cz_c + \frac{r_{bc}z_b}{2} + \frac{\tilde{A}}{(z_b+z_c-B)^2+C^2} = 0, \quad (11b)$$

$$F_2(z_b)|_{\phi_b=\pi} = -\frac{z_b}{\sqrt{1-z_b^2}} + r_bz_b + \frac{r_{bc}z_c}{2} + \frac{\tilde{A}}{(z_b+z_c-B)^2+C^2} = 0, \quad (11c)$$

$$F_2(z_c)|_{\phi_c=\pi} = -\frac{z_c}{\sqrt{1-z_c^2}} + r_cz_c + \frac{r_{bc}z_b}{2} + \frac{\tilde{A}}{(z_b+z_c-B)^2+C^2} = 0, \quad (11d)$$

where $\tilde{A} = \delta U_0A^2/(2\Omega)$. We only consider the case $z_b = z_c = z$, $\phi_b = \phi_c = \phi$, $F_1(z_b) = F_1(z_c) = F_1(z)$, $F_2(z_b) = F_2(z_c) = F_2(z)$ for simplicity. Let the two formulas equal to zero. Thus the solutions of the two equations are the stationary points which are shown in Fig. 2, under the condition of $\phi = 0, \pi$.

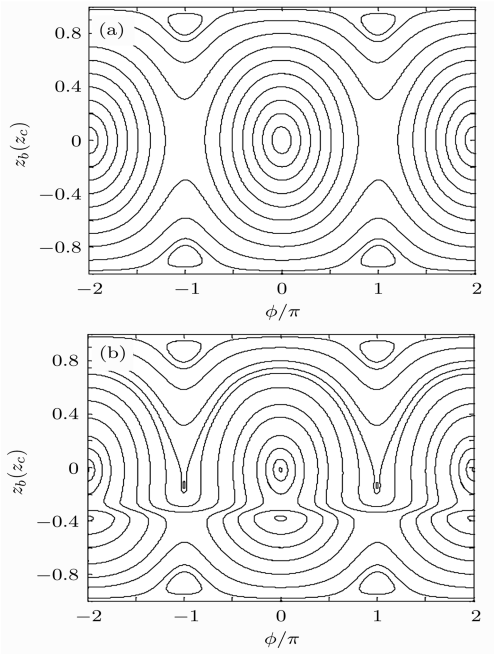


Fig. 1. Energy contours of a BJJ, (a) uncoupled and (b) coupled to a single-cavity mode. The parameters are $r = 2, r_{bc} = 0.01, \tilde{A} = 0.02, B = -0.65, C = 0.07$.[18]

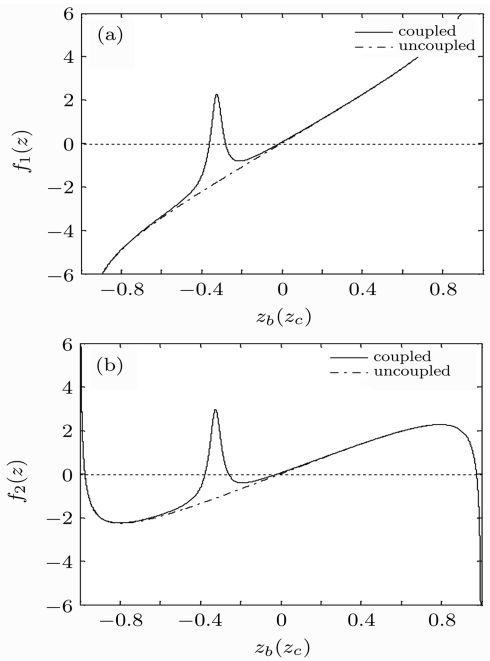


Fig. 2. The gradient of the energy along the lines, (a) $\varphi = 0$ and (b) $\varphi = \pi$. The parameters are: $\tilde{A} = 0.02, B = -0.65, C = 0.07$.

3. Bose Josephson junction in cavity

From Eqs. (9) and (11) we can plot the phase portrait and show the gradients of the energy along the line $\phi = 0$ and π in Figs. 1 and 2, respectively. There are two regimes, i.e. the Josephson regime[25] and self-trapping regime,[4,7,18,22] which make up the phase diagram in the uncoupled case. For the coupled case, the roots of Eqs. (11a) and (11c) have to be solved numerically. It is natural to expect that the last term will not only shift the position of the stationary points, but may also change the total number of them. So the phase portrait for the coupled case may be different from that for the uncoupled case, as shown in Fig. 1. As given in Figs. 2(a) and 2(b) for the set of parameters, $F_1(z)$ and $F_2(z)$ have two new solutions, respectively, when the BJJ couples to the cavity mode. In these figures we can see the new points along the lines $\varphi = 0$ and $\varphi = \pi$. By comparing Fig. 1(b) with Fig. 1(a), the coupled case has more complex behaviours than the uncoupled case. The coupled BJJ has two more types of zero-phase modes and two more types of π-phase modes than the uncoupled case. We attribute the appearance of new stationary points and new motional modes of the BJJ, to the nonlinearity of the cavity-field-induced tilt.[18]

As mentioned in Section 2, there are two types of interaction, i.e., interspecies and intraspicies interactions in our model. It is interesting to consider their effects on the dynamical behaviours. The role of intraspecies interaction without coupling to the cavity has been studied in detail. Figure 3 demonstrates the results for the BJJ coupled to the single-mode cavity. It can be found that with small intraspecies interaction, the Josephson oscillation will be observed, and with larger interaction, the self-trapping emerges. While the influences of the interspecies interaction on the BJJ dynamics is shown in Fig. 4. One may find that the self-regime increases with rising r_{bc}, and the Josephson regime reduces. It is illustrated that the role of the interspecies interaction equals the intraspecies interaction in ordinary cases. That is to say, for small interaction, Josephson oscillation will be achieved, while self-trapping appears when the interspecies interaction becomes large. This phenomenon can be understood from a simple fact, the bigger the r_{bc}, the more the atoms at the bottom of one of the wells, hence they will oscillate narrowly. In a word, the nonlinear self-trapping phenomenon of the BEC in a double-well potential can be modulated using both the intraspecies and interspecies interactions.

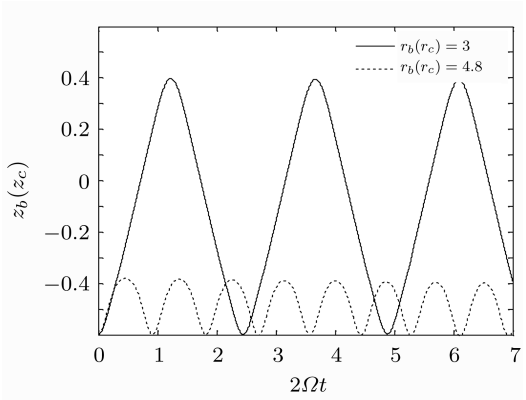


Fig. 3. For initial condition $Z_b(Z_c) = -0.6$, the population difference evolution with time for $r_{bc} = 2, r_b(r_c) = 3$ and $r_{bc} = 2, r_b(r_c) = 4.8$. $\tilde{A} = 0.02, B = -0.65, C = 0.07$.

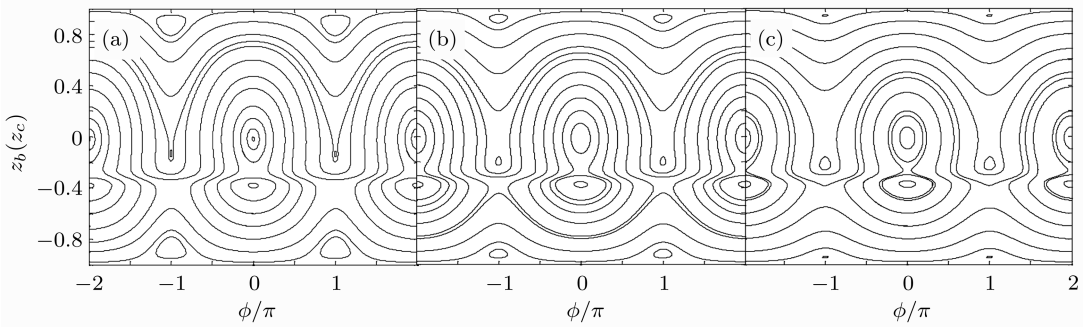


Fig. 4. Energy contours of a coupled BJJ. The interaction value are (a) $(r, r_{bc}) = (2, 0.02)$, (b) $(r, r_{bc}) = (2, 1)$ and (c) $(r, r_{bc}) = (2, 2)$ for $\tilde{A} = 0.02, B = -0.65, C = 0.07$.

We plot in Fig. 5 the time evolution of the atomic population imbalance and the number of intra cavity photons, using Eq. (8). Initially, we set the phase difference $\phi_b(0) = \phi_c(0) = 0$ and the population difference

$z_b(0) = z_c(0) = -0.5$, nonlinear parameter $r_{bc} = 1.88$ and 1.96. When $r_{bc} = 1.88$ during the whole time evolution, the atom number fluctuation displays a self-locked population imbalance, most atoms are trapped in one well. Normally, the atoms will be trapped in the well as before when enlarging r_{bc}. However, when we set the parameter $r_{bc} = 1.96$, the trapped atoms start tunneling between the two wells unnaturally. We also analysed the uncoupled case by using the numerical procedure. For the same conditions, the motion of the Bose Josephson junction is shown in Fig. 5(c). So this novel macroscopic quantum phenomenon results from the double effects of the cavity mode and the nonlinear interspecies interaction between the two-species BECs due to the mutual tunneling through the barrier. The influence of the cavity on the BJJ is shown in Figs. 6(a) and 6(b).

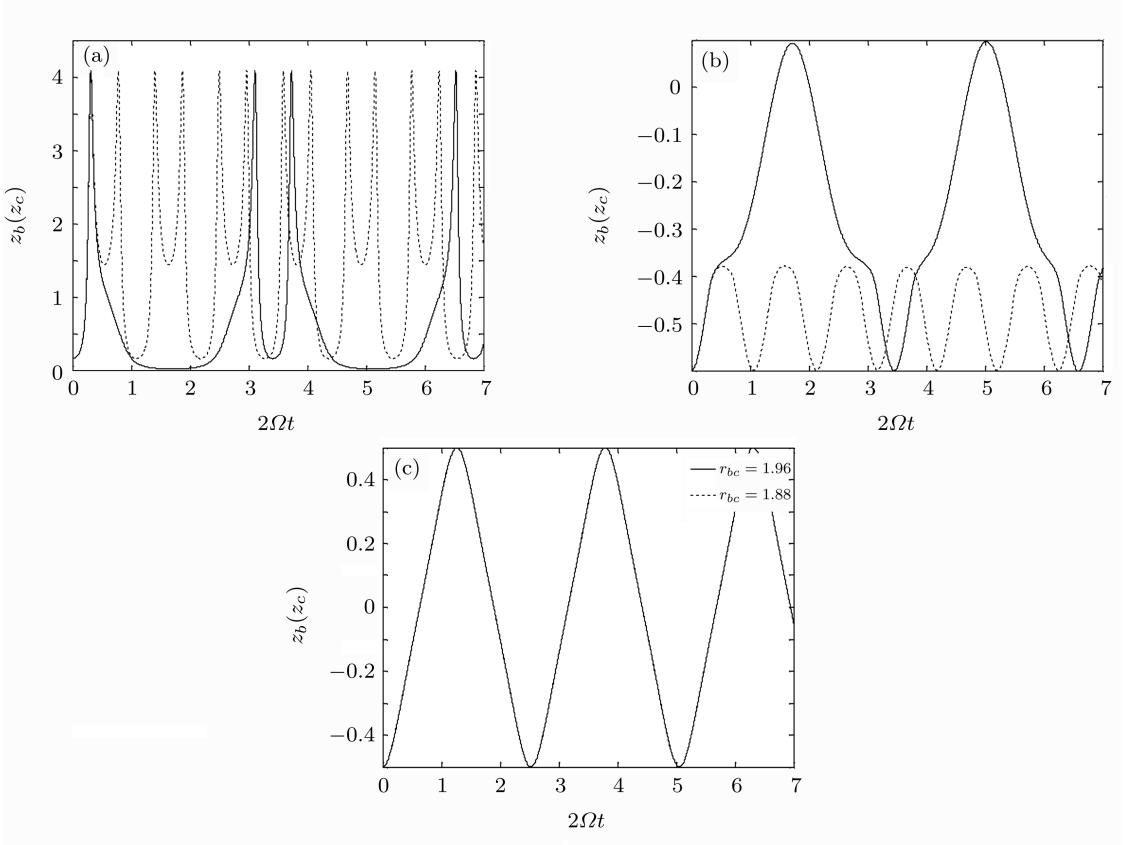


Fig. 5. EPS Motion of the BJJ for $\tilde{A} = 0.02, B = -0.65, C = 0.07$. (a) Atomic population imbalance $z_b(z_c)$ coupled to the cavity mode, and (b) intra cavity photon number $|\alpha|^2$ [in units of $2\Omega/(\delta U_0)$] versus reduced time $2\Omega t$. The initial values are $z_b(0) = z_c(0) = -0.5$ and $\phi_b(0) = \phi_c(0) = 0$. The conditions are $r = 2, r_{bc} = 1.88$ (dashed lines) and $r = 2, r_{bc} = 1.96$ (solid lines). (c) Atomic population imbalance $z_b(z_c)$ uncoupled to the cavity mode.

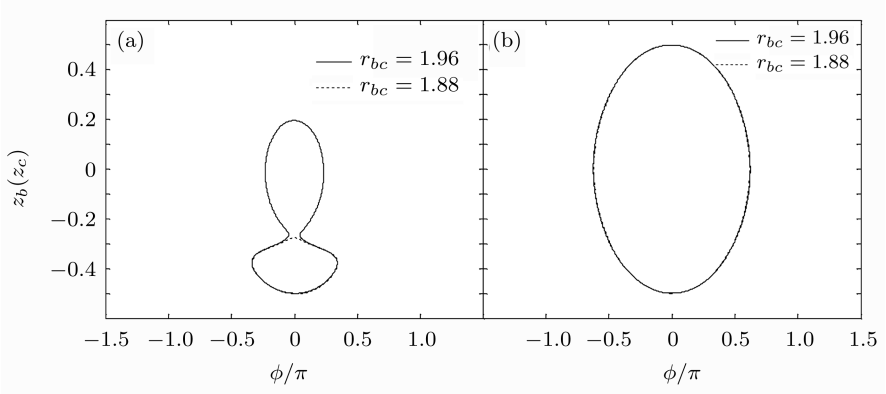


Fig. 6. Trajectories of the BJJ. (a) Coupled and (b) uncoupled to the cavity mode, under the two different conditions $r = 2, r_{bc} = 1.88$ (dashed lines) and $r = 2, r_{bc} = 1.96$ (solid lines), for $\tilde{A} = 0.02, B = -0.65, C = 0.07$.

4. Conclusion

In summary, we have derived the Hamiltonian for each part of the system under the mean-field approximation and obtained the atomic population evolution via the Heisenberg's equation. The Bose Josephson junction is investigated via the phase portrait, which is different from that of a bare Bose Josephson junction, due to the intra- and interspecies interactions and the atom-field interplay. By increasing the intraspecises interaction and interspecies interaction, the self-trapping can be achieved. Due to the more mutual interactions, at some special parameters regions, the trapped atom will turn to oscillate between the two wells.

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