Investigation of electric fields inside and outside a magnetised cold plasma sphere*

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(Received 9 October 2009; revised manuscript received 15 April 2010)

Based on the scales transformation of electromagnetic theory, the analytical expressions of electric fields inside and outside a magnetised cold plasma sphere are presented by reforming the spherical electromagnetic parameter. The obtained results are in good agreement with that in the literature. The angle between the direction of inside field and that of outside field is derived. In S wave band, calculations for the effects induced by parameters of the inner field are established. Simulations show that the angle between incident field and the outside magnetic field influences the inner field remarkably. The inner field will increase as the electron density increases. The inner field varies with frequency nonlinearly. There is an angle between the inner field and the incident field, it changes nonlinearly with the frequency.

Keywords: magnetised cold plasma, scales transformation, electric field

PACC: 5235H, 4110H, 9580D

1. Introduction

The investigations of electromagnetic (EM) scattering features from a spherical target and their applications have become increasingly important in recent years, particularly in the areas of remote-sensing and detection.[1−3] The electric fields inside and outside a single isotropic dielectric sphere have been researched.[4−7] The scattering features of an isotropic dielectric sphere and a conducting sphere, which are irradiated by an EM wave propagating in the z-direction and polarised in the x-direction, have been studied.[8] The scattering properties of spherical targets that are irradiated by a wave from an arbitrary direction was further studied in Ref. [9]. In Refs. [10]–[13], the EM scattering features for an isotropic dielectric ellipsoidal target and the power to seize radiation for a coated sphere in the Gaussian beam were separately investigated by the well-known Mie theory. In a word, the subjects of electromagnetic scattering of an isotropic dielectric sphere and their applications have been discussed in detail. However, topics of scattering features for the magnetised cold plasma have not been fully understood in theory. The main reason for this is the lack of analytical expression of electric field inside the magnetised cold plasma when it is irradiated by a wave from an arbitrary direction. If the inside electric field is known, the absorption cross section and the scattering cross section of the sphere will be obtained accurately.[14] In the present paper, the expression of the electric field inside a magnetised cold plasma sphere is presented in detail by reforming the anisotropic electromagnetic parameters into an isotropic one. The angle between the field inside the magnetised plasma sphere and that outside the sphere is calculated. The influence induced by the fundamental parameters such as electric density, outside magnetic field and the azimuth angle on the inner field is simulated. The method used has the features of briefness in computation and distinctness in physical significance.

2. Electric fields inside and outside a magnetised cold plasma sphere

2.1. Foundation of the differential equation

We assume that a magnetised cold plasma sphere has a radius of R_0 and its centre is located at the

*Project supported by the National Natural Science Foundation of China (Grant No. 60971079), the Natural Science Foundation of Shaanxi Province of China (Grant No. 2009JM8020) and the Natural Science Foundation of Shaanxi Educational Office of China (Grant No. 09JK800).

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origin of the primary coordinate system Σ. The outside magnetic field B_0 is in the z-axis. The dielectric constant tensor of this plasma sphere[8] is given as

$$\boldsymbol{\varepsilon} = \varepsilon_0 \begin{bmatrix} \varepsilon & -\mathrm{j}\varepsilon_\mathrm{p} & 0 \\ \mathrm{j}\varepsilon_\mathrm{p} & \varepsilon & 0 \\ 0 & 0 & \varepsilon_1 \end{bmatrix}, \tag{1}$$

where

$$\varepsilon_\mathrm{p} = \frac{\dfrac{ne^3 B_0}{m^2\omega^3\varepsilon_0}}{1-\dfrac{e^2B_0^2}{m^2\omega^2}}, \quad \varepsilon_1 = 1-\frac{ne^2}{m\omega^2\varepsilon_0},$$
$$\varepsilon = 1-\frac{\dfrac{ne^2}{m\omega^2\varepsilon_0}}{1-\dfrac{e^2B_0^2}{m^2\omega^2}},$$

with n being the electron density and ω the frequency of the incident wave. When the frequency is very low, the condition $\lambda >> R_0$ is satisfied, so in approximation the magnetised cold plasma sphere can be considered to be located in an electrostatic field.[4,6] The plasma has no electric charge in the whole, according to the formulae $\nabla\cdot\boldsymbol{D}=0, \boldsymbol{E}=-\nabla u$; and considering that the second derivative of potential u is not related to the order for x and y, $\partial^2u/\partial x\partial y = \partial^2u/\partial y\partial x$, so the derivative equation of u inside the plasma is obtained as in the primary coordinate system, we name it Σ system,

$$\varepsilon\frac{\partial^2u}{\partial x^2}+\varepsilon\frac{\partial^2u}{\partial y^2}+\varepsilon_1\frac{\partial^2u}{\partial z^2}=0. \tag{2}$$

Now, a scale coordinate system Σ' is introduced as a new coordinate system. The coordinates of this system are indicated with x', y' and z'. The relation of coordinates between the two systems is written as

$$x'=\frac{x}{\sqrt{\varepsilon}}, \quad y'=\frac{y}{\sqrt{\varepsilon}}, \quad z'=\frac{z}{\sqrt{\varepsilon_1}}.$$

The differential equation of the potential in the scaled coordinate system is derived by substituting the above expressions into Eq. (2) and using the condition $u = u'$[15−18] at any spatial point. It is expressed as

$$\frac{\partial^2u'}{\partial x'^2}+\frac{\partial^2u'}{\partial y'^2}+\frac{\partial^2u'}{\partial z'^2}=0. \tag{3}$$

The condition $u=u'$ is understandable, since the potential is defined as the work done by the electric field to move a unit charge from one point to the reference point, namely W/q, both the numerator and the denominator are scaled invariants. Equation (3) shows that a magnetised cold plasma sphere in the primary coordinate system is transformed into an isotropic sphere in the scaled coordinate system. This manipulation can greatly simplify the calculation of electric fields and electromagnetic scattering problems.

2.2. Expressions of electric fields outside and inside a magnetised cold plasma sphere

Now we find that Eq. (3) is entirely the same as that in Refs. [18] and [19], so the same technique, method of separation of variables, may be used to acquire the solution of Eq. (3). The detailed derivation of solution of Eq. (3) is not presented here, but final solution of Eq. (3) may be obtained as follows:

$$u(R,\theta,\varphi) = \frac{3A}{2+\varepsilon_1}R\cos\theta+\frac{3\left(2B+B\varepsilon+\mathrm{j}D\varepsilon_\mathrm{p}\right)}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}R\cos\varphi\sin\theta+\frac{3\left(2D+D\varepsilon-\mathrm{j}B\varepsilon_\mathrm{p}\right)}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}R\sin\varphi\sin\theta, \tag{4}$$

$$\begin{aligned} u_1(R,\theta,\varphi) = {} & AR\cos\theta+BR\sin\theta\cos\varphi+DR\sin\theta\sin\varphi+\frac{AR_0^3}{R^2}\frac{1-\varepsilon_1}{2+\varepsilon_1}\cos\theta \\ & -\frac{R_0^3\left(B\varepsilon^2+B\varepsilon-2B-B\varepsilon_\mathrm{p}^2-3\mathrm{j}D\varepsilon_\mathrm{p}\right)}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}\sin\theta\cos\varphi \\ & -\frac{R_0^3\left(D\varepsilon^2+D\varepsilon-2D-D\varepsilon_\mathrm{p}^2+3\mathrm{j}B\varepsilon_\mathrm{p}\right)}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}\sin\theta\sin\varphi, \end{aligned} \tag{5}$$

where

$$A=-E_0\cos\theta_0, \quad B=-E_0\sin\theta_0\cos\varphi_0, \quad D=-E_0\sin\theta_0\sin\varphi_0.$$

Expressions (4) and (5) are the electric potentials for the inside and outside magnetised cold plasma sphere, respectively.

2.3. Discussion

From the above expressions it follows that, when $\theta_0 = \phi_0 = 0$, the incident electric field E_0 is in the z-direction; $B = D = 0$, and $A = -E_0$. If we suppose that $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon/\varepsilon_0, \varepsilon_\mathrm{p} = 0$, the problem of the electric field in a magnetised cold plasma medium under study becomes a subject in an isotropic medium. Expressions (4) and (5) are transformed, respectively, into the following expressions

$$u(R,\theta,\varphi) = \frac{-3E_0\varepsilon_0}{2\varepsilon_0+\varepsilon} R\cos\theta,$$

$$u_1(R,\theta,\varphi) = -E_0 R\cos\theta + \frac{-E_0R_0^3}{R^2}\frac{\varepsilon_0-\varepsilon}{2\varepsilon_0+\varepsilon}\cos\theta.$$

It can be seen that the results are consistent with those in the literature, thereby indirectly testifying the correctness of the obtained results. Let α and β be the angles between E_0 and the x-axis and between E_0 and the y-axis, respectively, then it will be easily proved that $\cos\alpha = \sin\theta\cos\varphi, \cos\beta = \sin\theta\sin\varphi$. So the second term and the third terms in expression (4) are the potentials produced by the polarised electrical dipole moment respectively in x- and y-directions. In the scattering of small particles, for example, Raleigh scattering, the electric field inside the target is of great importance. So we discuss the distribution of the inside electric field in the following. By making a gradient of the potential in expression (4) and utilising the transformation between a vector in the spherical coordinate system and that in the rectangular coordinate system, the electric field is obtained as follows:

$$\boldsymbol{E} = -\frac{3(2B+B\varepsilon+\mathrm{j}D\varepsilon_\mathrm{p})}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}\hat{x} - \frac{3(2D+D\varepsilon-\mathrm{j}B\varepsilon_\mathrm{p})}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}\hat{y} - \frac{3A}{2+\varepsilon_1}\hat{z}. \quad (6)$$

Expression (6) demonstrates that the electric field inside a magnetised cold plasma sphere is a uniform field which is a complex function of the incident azimuthal angle, outside magnetic field, the electric density, and the frequency etc. Since ε_p is generally not zero, so we conclude from the first term in expression (6) that the inside field in x-direction is in advance in phase as compared with the incident wave in x-direction. This phase angle is related to the incident wave in y-direction. The second term in expression (6) shows that the inside field in y-direction lags in phase as compared with the incident wave in y-direction. This phase angle is also related to the incident wave in x-direction. The inner field inside the magnetised cold plasma generally turns off the incident field by an angle δ.

This cosine function for the angle δ is easily derived by taking the scalar product of these two vectors as

$$\cos\delta = \frac{\dfrac{3D(2D+D\varepsilon-\mathrm{j}B\varepsilon_\mathrm{p})}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4} + \dfrac{3B(2B+B\varepsilon+\mathrm{j}D\varepsilon_\mathrm{p})}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4} + \dfrac{3A^2}{2+\varepsilon_1}}{\left[\left(\dfrac{3(2D+D\varepsilon-\mathrm{j}B\varepsilon_\mathrm{p})}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}\right)^2 + \left(\dfrac{3(2B+B\varepsilon+\mathrm{j}D\varepsilon_\mathrm{p})}{\varepsilon^2+4\varepsilon-\varepsilon_\mathrm{p}^2+4}\right)^2 + \left(\dfrac{3A}{2+\varepsilon_1}\right)^2\right]^{1/2}}. \quad (7)$$

The displacement vector $\boldsymbol{D}$ is easily obtained from expressions (6) and (1), which is not presented here in detail. Part of simulation results is given as follows.

According to the Ref. [18], the electric density in the simulation is chosen to be $n = 4.5\times10^{17}$ m^{-3} and other parameters are shown in the figures. It is concluded from Fig. 1 that the inner field will increase when the outside magnetic field increases. The angle θ_0 between the outside electric field and the outside magnetic field has a great effect on the field inside the plasma, and further simulations show that angle φ_0 has not affected on the inner field. This is because the anisotropic property in the plasma is caused by the outside magnetised field, and the components of outside electric field are related to the angles θ_0 and φ_0, but this outside electric field has a good symmetry with the outside magnetic field, so the inner field does not vary with angle φ_0. In Fig. 2, the azimuthal angles are $\pi/4$ and $\pi/3$. It is demonstrated that the inner electric field decreases as the electron density increases. This is caused by the strong electrical conductivity induced by the increase in electron density, as the field is shielded. It also can be seen from Fig. 3 that the inner field increases with the operating frequency. This is due to the fact that the variation of electric charges inside the plasma is not in agreement with that of outside electric field and the shielding effect is thus debased. When the frequency further

increases, the function $\varepsilon(\omega)^2 + 4\varepsilon(\omega) - \varepsilon_{\rm p}^2(\omega) + 4$ is nearly equal to zero, thus the electric field goes to its maximum. In physics, the dielectric parameter ε in S wave band is negative. After being polarised, the inner field in the plasma, i.e. the sum of the incident field and the field produced by the polarisation charge, is increased. Figure 4 shows the change of angle δ versus the frequency, in which the density $n = 4.5\times10^{17}$ m^{-3} and the outside magnetic fields B_0 are respectively 0.004 T and 0.005 T. It also demonstrates that the outside magnetic field has not great influence on the angle δ. Angle δ is greatly influenced by the frequency. It is well known that in isotropic medium, the angle δ is zero, so in the time-varying electromagnetic field, the electric charges, negative and positive, in the cold plasma cannot entirely agree with the outside field as the frequency is increased, which causes the angle's accretion.

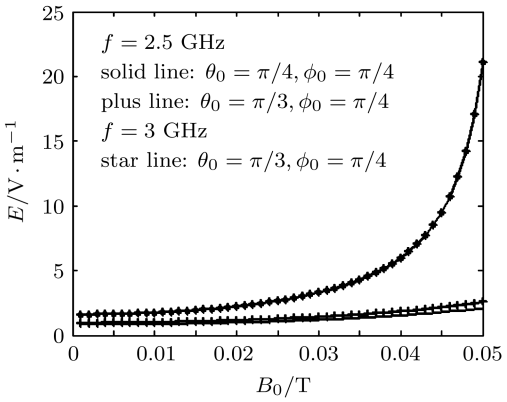


Fig. 1. Electric field as a function of outside magnetic field.

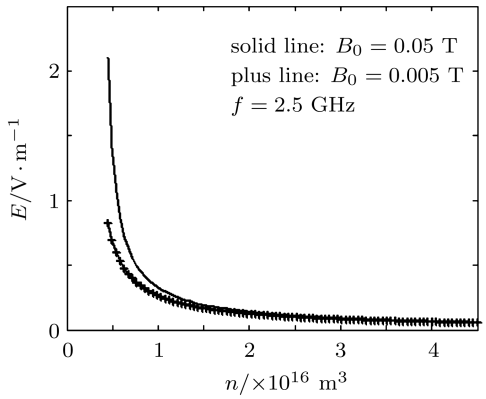


Fig. 2. Electric field as a function of electron density.

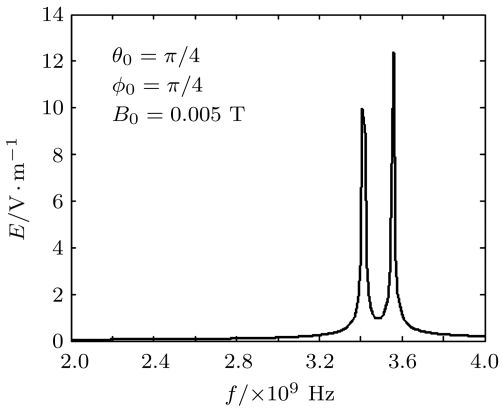


Fig. 3. Variation of electric field with frequency.

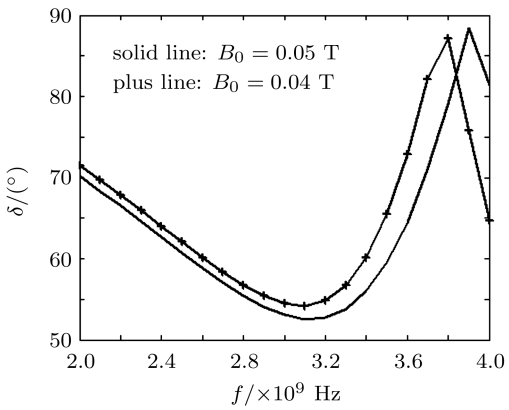


Fig. 4. Variation of angle δ changes with frequency.

3. Conclusion

In this paper, the electric fields inside and outside a magnetised cold plasma sphere are investigated. We use the scale transformation theory of the electromagnetic field to reconstruct the Laplace equation and then two analytical expressions for the electric potentials inside and outside the magnetised cold plasma sphere are obtained respectively. The results are consistent with those in the literature when the dielectric constant tensor is an isotropic medium. The angle between the two fields inside and outside the magnetised cold plasma sphere is derived. The effects induced by the incident direction, outside magnetic field, frequency etc. on the direction and the magnitude of the inside electric field are simulated. Since many particles such as sand-dust storm particulates, atomy particles and raindrops are generally anisotropic, the results obtained can provide a good theoretical foundation for studying those particles' scattering features and the magnetised cold plasma. How to use the scale

transformation theory to study the electromagnetic fields inside and outside a magnetised cold plasma target irradiated by the time-varying electromagnetic wave is our future research subject.

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