Nonlinear acoustic waves in a collisional self-gravitating dusty plasma*

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Using the reductive perturbation method, we investigate the small amplitude nonlinear acoustic wave in a collisional self-gravitating dusty plasma. The result shows that the small amplitude dust acoustic wave can be expressed by a modified Korteweg–de Vries equation, and the nonlinear wave is instable because of the collisions between the neutral gas molecules and the charged particles.

Keywords: nonlinear acoustic waves, collisional self-gravitating dusty plasma, modified Korteweg–de Vries equation, instability

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1. Introduction

A dusty plasma is a conglomeration of ions, electrons and micro-sized dust grains. It has been found that the presence of static charged dust grains can significantly change the behaviour of a plasma. The existence of dust acoustic waves (DAWs) was predicted theoretically in an un-magnetized dusty plasma at the presence of the inertia provided by the dust particle mass and the restoring force provided by the pressures of the inertialess electrons and ions.[1] And the existence of dust ion-acoustic waves (DIAWs) was also discovered by Shukla and Silin[2] at higher frequency, the existence of DAWs and DIAWs was also confirmed in the laboratory experiments.[3−5] There has been considerable interest in understanding different types of collective processes in dusty plasma, which has gained considerable amount of momentum over the last few decades. Importance of such a research is due to its relevance to space plasma, ionosphere. It is also important for the study of the formation of stars and planets, planetary rings, cometary surroundings. Furthermore, it is very significant for fabricating microchips and hardening of metals in industry.[6]

Recently, nonlinear waves, especially solitary waves, have been intensively studied in collisionless dusty plasma.[7−13] The presence of negatively charged dust grains can significantly modify the dispersion properties of DIAWs by the grains as well as the grain charge fluctuations because of the collection of plasma particles. In addition, gas discharge plasmas at low temperature, various collision processes such as ionization, ion-neutral collision, ion-dust elastic and inelastic collisions play an important role and even may lead to instabilities of the waves.[14−16] However, the self-gravitation of plasma particles is not taken into account in all those studies. In fact, plasmas both in laboratory and in space exist in gravitation field. The linear waves propagating in a collisional dusty plasma existing in a gravitational field has been investigated on condition that the gravitations of electrons, ions and neutral gas molecules were neglected.[17] And the nonlinear ion-acoustic waves propagating in collisionless self-gravitating plasma have been investigated theoretically by Paul *et al.*,[18] and it was demonstrated that the solitary waves could exist in the collisionless self-gravitating plasma. However, in the investigations of earlier authors, the effects of gravitational force and collision on the propagation of nonlinear acoustic waves in dusty plasma have not been fully explored.

Considering the self-gravity of the charged particles and the collisions that the neutral gas molecules act on the charged particles of plasma, the nonlinear waves in the plasma is investigated in the present paper. The rest of this paper is organized as follows.

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In Section 2, the equations describing collective behaviour of collisional self-gravitating dusty plasma is reduced using the reductive perturbation, and a modified Korteweg–de Vries (mKdV) equation for the small amplitude nonlinear wave is obtained. In Section 3, the evolution of the mKdV equation is studied using Runge–Kutta method. Finally in Section 4, some conclusions are drawn from the present study.

2. Formulation

We consider a collisional self-gravitating dusty plasma composed of electrons, ions, and charged dust grains in the background of neutral gas molecules. At equilibrium, the gravitational force among the grains is balanced by the electromagnetic force acting on them. The dusty plasma is assumed to satisfy the quasi-neutrality condition $n_{\rm i0}+n_{\rm e0}+Z_{\rm d}n_{\rm d0}=0$, where $Z_{\rm d}$ is the number of electronic charge residing on the grains and n_{s0} with s=e, i, d is the number density of the s species. For mathematical simplicity, all grains are assumed to have spherical shape and the same size a and density ρ with constant charge. The inter-grain distance r_0 is assumed smaller than the plasma Debye length ($a << r_0 << \lambda_{\rm D}$), where $\lambda_{\rm D}$ is the effective Debye length of the self-gravitating dusty plasma. The quantities T_s and m_s with s=e, i, d, N denote the temperature and the mass of the s species (N refers to neutral gas molecules), respectively. Electrons, ions, and dust grains collide with the neutral gas molecules at collision frequencies[17]

$$\nu_{\rm e}=n_{\rm N}\,\sigma_{\rm e}\,C_{\rm e},\quad \nu_{\rm i}=n_{\rm N}\,\sigma_{\rm i}\,C_{\rm i},$$
$$\nu_{\rm d}=\frac{4\,m_{\rm N}\,n_{\rm N}\,a^2\,C_{\rm N}}{m_{\rm d}}, \tag{1}$$

where $\sigma_{\rm e}$ ($\sigma_{\rm i}$) is the electron–molecules (ion–molecules) collision cross section, $C_s=\sqrt{\kappa\,T_s/m_s}$ is the thermal speed, s represents the species (i.e., s=e, i, d and N for electrons, ions, dust particles and neutral gas molecules, respectively). In this paper, all collisions, except collisions of electrons, ions and dust particles on neutral gas molecules, are neglected. The state of this dusty plasma system is described by

$$\frac{\partial n_s}{\partial t}+\frac{\partial(n_s u_s)}{\partial x}=0, \tag{2}$$
$$\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}=-C_s^2\frac{\partial n_s}{\partial x}-\frac{q_s}{m_s}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}-\nu_s\,u_s, \tag{3}$$
$$\frac{\partial^2\phi}{\partial x^2}=\frac{1}{\epsilon_0}\sum_{s=\rm e,i,d}q_s n_s,\quad \frac{\partial^2\psi}{\partial x^2}=4\pi G\sum_{s=\rm e,i,d}m_s n_s, \tag{4}$$

where m_s (s=e, i, d) is the mass, u_s is the velocity, q_s is the charge, ϕ is the electrostatic potential, ψ is the gravitational potential.

Now we use the reductive perturbation method to investigate the behaviour of small amplitude DAW. We assume that the expressions of collision frequency ν_s can be written as

$$\nu_{\rm e}=\epsilon^3\,\nu_{\rm e1},\quad \nu_{\rm i}=\epsilon^3\,\nu_{\rm i1},\quad \nu_{\rm d}=\epsilon^3\,\nu_{\rm d1}. \tag{5}$$

The independent variables are stretched as $\xi=\epsilon^{1/2}\,(x-\upsilon_0\,t)$ and $\tau=\epsilon^{3/2}\,t$, where υ_0 is the velocity of the linear dust acoustic wave, and ϵ is a small parameter characterizing the strength of nonlinearity. The dependent variables are expanded into a power series of ϵ as follows:

$$n_s=n_{s0}+\epsilon\,n_{s1}+\epsilon^2\,n_{s2}+\epsilon^3\,n_{s3}+\ldots, \tag{6}$$
$$u_s=\epsilon\,u_{s1}+\epsilon^2\,u_{s2}+\epsilon^3\,u_{s3}+\ldots, \tag{7}$$
$$\phi_s=\phi_0+\epsilon\,\phi_1+\epsilon^2\,\phi_2+\epsilon^3\,\phi_3+\ldots, \tag{8}$$
$$\psi_s=\psi_0+\epsilon\,\psi_1+\epsilon^2\,\psi_2+\epsilon^3\,\psi_3+\ldots. \tag{9}$$

The boundary conditions are at $x\to\pm\infty,\phi\to\phi_0,\psi\to\psi_0,u_s\to 0,n_s\to n_{s0}$. Substituting Eqs. (5)–(9) into Eqs. (2)–(4) and collecting the terms according to their powers of ϵ. For the lowest order in ϵ, we obtain

$$\sum_{s=\rm e,i,d}q_s n_{s0}=0,\quad \mu_s\frac{\partial\phi_0}{\partial\xi}=-\frac{\partial\psi_0}{\partial\xi}, \tag{10}$$

where $\mu_s=q_s/m_s$ is the ratio of charge to mass. For the higher order in ϵ, we obtain

$$n_{s1}=\frac{n_{s0}}{\upsilon_0^2-C_s^2}(\mu_s\phi_1+\psi_1), \tag{11}$$
$$u_{s1}=\frac{\upsilon_0}{\upsilon_0^2-C_s^2}(\mu_s\phi_1+\psi_1), \tag{12}$$
$$\sum_{s=\rm e,i,d}\frac{\mu_s q_s n_{s0}}{\upsilon_0^2-C_s^2}\,\phi_1+\sum_{s=\rm e,i,d}\frac{q_s n_{s0}}{\upsilon_0^2-C_s^2}\,\psi_1 + \epsilon_0\frac{\partial^2\phi_0}{\partial\xi^2}=0, \tag{13}$$
$$\sum_{s=\rm e,i,d}\frac{q_s n_{s0}}{\upsilon_0^2-C_s^2}\,\phi_1+\sum_{s=\rm e,i,d}\frac{m_s n_{s0}}{\upsilon_0^2-C_s^2}\,\psi_1 - \frac{1}{4\pi G}\frac{\partial^2\psi_0}{\partial\xi^2}=0. \tag{14}$$

Let the determinant of Eqs. (13) and (14) equal zero, we obtain the dispersion relation

$$\sum\frac{\mu_s q_s n_{s0}}{\upsilon_0^2-C_s^2}\left(\sum\frac{m_s n_{s0}}{\upsilon_0^2-C_s^2}\right)=\left(\sum\frac{q_s n_{s0}}{\upsilon_0^2-C_s^2}\right)^2. \tag{15}$$

From Eqs. (10)–(15), we obtain

$$\psi_1=A\phi_1,\quad \left(A=-\sum\frac{q_s n_{s0}}{\upsilon_0^2-C_s^2}\Big/\sum\frac{m_s n_{s0}}{\upsilon_0^2-C_s^2}\right). \tag{16}$$

For the next higher order in ϵ, we obtain a set of equations

$$\begin{aligned}
&\left[\sum \frac{\mu_s q_s n_{s0}}{\upsilon_0^2 - C_s^2}\right]\frac{\partial \phi_2}{\partial \xi} + \left[\sum \frac{q_s n_{s0}}{\upsilon_0^2 - C_s^2}\right]\frac{\partial \psi_2}{\partial \xi} \\
&= -2\upsilon_0\left[\sum \frac{q_s n_{s0}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2}\right]\frac{\partial \phi_1}{\partial \tau} - 3\upsilon_0^2\left[\sum \frac{q_s n_{s0}(A+\mu_s)^2}{(\upsilon_0^2 - C_s^2)^3}\right]\phi_1\frac{\partial \phi_1}{\partial \xi} \\
&\quad -\epsilon_0\frac{\partial^3 \phi_1}{\partial \xi^3} - \upsilon_0\left[\sum \frac{\nu_s q_s n_{s0}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2}\right]\phi_1, \\
&\left[\sum \frac{q_s n_{s0}}{\upsilon_0^2 - C_s^2}\right]\frac{\partial \phi_2}{\partial \xi} + \left[\sum \frac{m_s n_{s0}}{\upsilon_0^2 - C_s^2}\right]\frac{\partial \psi_2}{\partial \xi} \\
&= -2\upsilon_0\left[\sum \frac{m_s n_{s0}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2}\right]\frac{\partial \phi_1}{\partial \tau} - 3\upsilon_0^2\left[\sum \frac{m_s n_{s0}(A+\mu_s)^2}{(\upsilon_0^2 - C_s^2)^3}\right]\phi_1\frac{\partial \phi_1}{\partial \xi} \\
&\quad +\frac{1}{4\pi G}\frac{\partial^3 \phi_1}{\partial \xi^3} - \upsilon_0\left[\sum \frac{\nu_s m_s n_{s0}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2}\right]\phi_1. \qquad (17)
\end{aligned}$$

Substituting Eq. (15) into Eq. (17), we obtain the modified KdV equation

$$B\frac{\partial \phi_1}{\partial \tau} + C\phi_1\frac{\partial \phi_1}{\partial \xi} + D\frac{\partial^3 \phi_1}{\partial \xi^3} + E\phi_1 = 0, \qquad (18)$$

where

$$\begin{aligned}
B &= 2\upsilon_0\left[\sum \frac{q_s n_{s0}}{\upsilon_0^2 - C_s^2}\sum \frac{m_s n_{s0}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2} - \sum \frac{m_s n_{s0}}{\upsilon_0^2 - C_s^2}\sum \frac{q_s n_{s0}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2}\right], \\
C &= 3\upsilon_0^2\left[\sum \frac{q_s n_{s0}}{\upsilon_0^2 - C_s^2}\sum \frac{m_s n_{s0}(A+\mu_s)^2}{(\upsilon_0^2 - C_s^2)^3} - \sum \frac{m_s n_{s0}}{\upsilon_0^2 - C_s^2}\sum \frac{q_s n_{s0}(A+\mu_s)^2}{(\upsilon_0^2 - C_s^2)^3}\right], \\
D &= -\frac{1}{4\pi G}\sum \frac{q_s n_{s0}}{\upsilon_0^2 - C_s^2} - \epsilon_0\sum \frac{m_s n_{s0}}{\upsilon_0^2 - C_s^2}, \\
E &= \upsilon_0\left[\sum \frac{q_s n_{s0}}{\upsilon_0^2 - C_s^2}\sum \frac{m_s n_{s0}\nu_{s1}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2} - \sum \frac{m_s n_{s0}}{\upsilon_0^2 - C_s^2}\sum \frac{q_s n_{s0}\nu_{s1}(A+\mu_s)}{(\upsilon_0^2 - C_s^2)^2}\right]. \qquad (19)
\end{aligned}$$

It is obvious that the collision ν_s appears only in the polynomial E, indicating that the fourth term $E\phi_1$ of Eq. (18) represents the effect of collisions on the wave.

3. Results

3.1. Quasi-soliton solution

In fact, Eq. (18) has a quasi-soliton solution.[19] Letting

$$\tau = \frac{B}{D}\eta, \quad \phi_1 = \frac{6D}{C}\phi, \qquad (20)$$

Eq. (18) becomes

$$\frac{\partial \phi}{\partial \eta} + 6\phi\frac{\partial \phi}{\partial \xi} + \frac{\partial^3 \phi}{\partial \xi^3} + \frac{E}{D}\phi = 0. \qquad (21)$$

Equation (21) is a KdV-type equation, If $E = 0$, Eq. (21) is a KdV equation, and it has a soliton solution

$$\phi = a_0\,\mathrm{sech}^2\sqrt{\frac{a_0}{2}}\,[\xi - 2a_0\eta]\,, \qquad (22)$$

where a_0 is the amplitude, $\sqrt{2/a_0}$ is the width, and $2a_0$ is the velocity. If $E \neq 0$, we also assume that it has a quasi-soliton solution

$$\phi = a(\eta)\,\mathrm{sech}^2\sqrt{\frac{a(\eta)}{2}}\,[\xi - 2a(\eta)\eta]\,, \qquad (23)$$

whose amplitude can be expressed by the following equation:[19–23]

$$a(\eta) = a(\eta_0)\exp\left[-\frac{E}{D}(\eta - \eta_0)\right]. \qquad (24)$$

3.2. Numerical results

Letting $\zeta = k\xi + \upsilon_0\tau$, Eq. (18) becomes

$$B\upsilon_0\frac{\partial \phi_1}{\partial \zeta} + Ck\phi_1\frac{\partial \phi_1}{\partial \zeta} + Dk^3\frac{\partial^3 \phi_1}{\partial \zeta^3} + E\phi_1 = 0. \qquad (25)$$

Equation (25) can be investigated by Runge–Kutta method. Using the order 'ODE45(ODE, TSPAN, Y0, OPTIONS)' available from the mathematic software Matlab, Eq. (25) can be solved numerically. Setting

$$\text{options} = \text{odeset}\left('\text{RelTol}', 10^{-4}, '\text{AbsTol}', [10^{-4}\ \ 10^{-4}\ \ 10^{-5}]\right).$$

The values are $B = 0.2$, $C = 0.3$, $D = 0.2$, $k = 1$, $v_0 = 2.0$, and the initial values $(\phi_1,\ \phi_{1\zeta},\ \phi_{1\zeta\zeta})|_{\zeta=0} = (0, 0, \text{eps})$, where the eps denotes an infinitely small positive quantity.

The numerical results for variations of ϕ_1 and $\phi_{1\zeta}$ with ζ for $E = 0.1$ are given in Figs. 1(a) and 1(b), respectively. Meanwhile, the variations of $\phi_{1\zeta}$ with ϕ_1 are plotted in Fig. 2. From Fig. 1, it follows that the amplitudes of ϕ_1 and $\phi_{1\zeta}$ increase as ζ increases within $[0, 132 - 133)$. It is remarkable that ϕ_1 and $\phi_{1\zeta}$ vary exponentially when ζ increases within $(132 - 133, \infty)$. Figure 2 shows the same result. It is indicated there is a critical state in which the wave is instable extraordinarily as ζ increases. It seems that the initial conditions $(0, 0, \text{eps})$ are instable. In order to confirm this phenomenon, we use different initial conditions $(0, 0, \pm\text{eps})$, $(\pm\text{eps}, 0, 0)$ and $(0, \pm\text{eps}, 0)$ to draw the maps of ϕ_1 and $\phi_{1\zeta}$ within the same scope, respectively, showing that the variations of ϕ_1 and $\phi_{1\zeta}$ are similar to the case of initial conditions $(0, 0, \text{eps})$. It is concluded that the state (0, 0, 0) is an instable focus point.

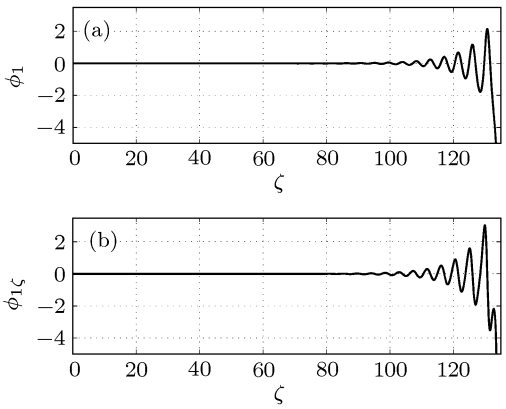


Fig. 1. Variations of ϕ_1 (a) and $\phi_{1\zeta}$ (b) with ζ.

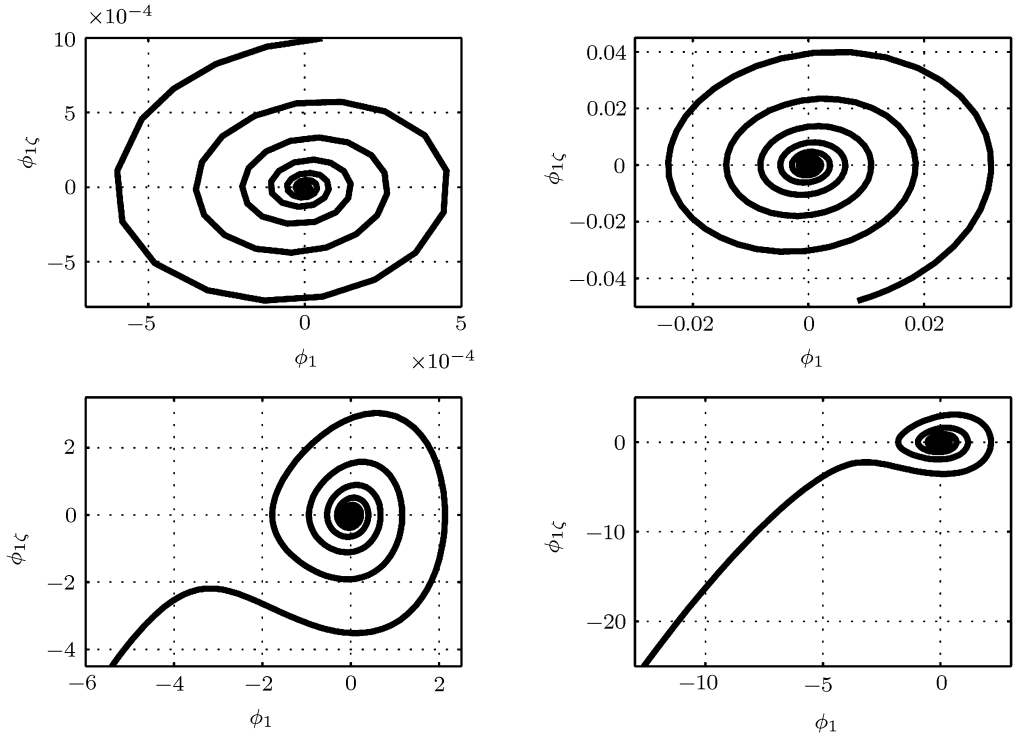


Fig. 2. Variations of $\phi_{1\zeta}$ with ϕ_1.

The numerical results for variations of ϕ_1 with ζ for different collisions $E = 0.05$, 0.075 and 0.1 are given in Fig. 3. It is found from Fig. 3 that the amplitude of ϕ_1 increases as E increases, furthermore, that the critical point of ζ, at which ϕ_1 (or $\phi_{1\zeta}$) varies extraordinarily, decreases as parameter E increases. The critical values of ζ are $216.6 - 216.7$, $159.8 - 159.9$, $132.8 - 132.9$ for $E = 0.05$, 0.075 and 0.1, respectively. It can be concluded that the degree of instability is dependent on collision. The more intensive the collision, the more instable will the dust acoustic wave become.

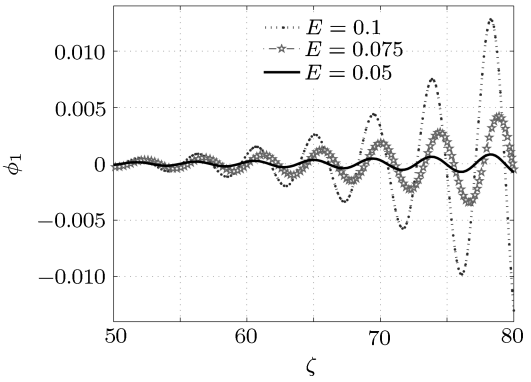


Fig. 3. The variation of ϕ_1 with respect to ζ for different E.

4. Conclusion

Using the reductive perturbation method, the nonlinear waves in a collisional self-gravitating dusty plasma composed of electrons, ions, dust and neutral gas molecules are studied, and the mKdV equation describing the small amplitude nonlinear dust acoustic waves is obtained. Furthermore, the quasi-soliton solution of the mKdV equation is discussed, and the amplitude of the quasi-soliton varies exponentially.

Numerical analysis shows that the small-amplitude nonlinear dust acoustic wave exhibits oscillatory behaviour with increase of amplitude at the beginning of the independent variable. When the independent variable reaches a certain value determined by collisions, the wave no longer oscillates but varies exponentially, which shows a collision instability arising from the collision in which the neutral molecules acts as the other components of plasmas. The different intensities of collision induce different degrees of instability. The stronger the collision intensity, the more the instable will the acoustic wave become.

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