The anomalous radiation force of light on a left-handed material*

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(Received 14 April 2010; revised manuscript received 13 May 2010)

This paper derives the force of the electromagnetic radiation on left-handed materials (LHMs) by a direct application of the Lorentz law of classical electrodynamics. The expressions of radiation force are given for TE-polarised and TM-polarised fields. The numerical results demonstrate that electromagnetic waves exert an inverse lateral radiation force on each edge of the beams, that is, the lateral pressure is expansive for TE-polarised beams and compressive for TM-polarised beams. The investigation of the radiation force will provide insights into the fundamental properties of LHMs and will provide to better understanding of the interaction of light with LHMs.

Keywords: left-handed material, radiation force, Lorentz law

PACC: 4225G, 4225J

1. Introduction

In the late 1960s, Veselago first introduced the concept of left-handed material (LHM) in which both the permittivity ε and the permeability μ are negative.[1] LHM did not receive much attention as it only existed in a conceptual form. After the first experimental observation using a metamaterial composed of split ring resonators (SRRs),[2−5] the study of such materials has received increasing attention over the last few years.[6−8] Comparing with right-handed materials(RHMs), LHMs have many inverse physical effects or parameters and the other anomalous physical effects, such as the reversed Doppler shift,[9] the backward Cerenkov radiation,[10,11] the negative refraction index[4,5] and anomalous self-steeping effect.[12] However, not all physical effect is inverse, for example, rotational Doppler effect and spin Hall effect of light.[13,14] In the present work, we will prove that the lateral radiation pressure also is reversed in LHMs.

In modern terms, the radiation pressure is equivalent to the carriage of a free-space momentum $\hbar\omega/c$ by a photon of energy $\hbar\omega$, as is expected from the quantum theory of radiation.[15] Poynting treated the problem of light incident from free space at an arbitrary angle on the surface of a transparent dielectric with a real refractive index greater than unity. Poynting pointed out that the pressure, or push, on the surface from the reflected component is exceeded by a pull from the refracted component. He predicted the existence of an outward force which is normal to the surface, irrespective of the angle of incidence.[15−17] Ashkin and Dziedzic observed the outward forces using focused pulsed laser light entering or leaving the liquid.[18]

The main purpose of the present work is to derive the force of electromagnetic radiation using the Lorentz law and obtain the analytical expressions of radiation force for TE-polarised and TM-polarised electromagnetic wave in LHMs. We find that the direction of lateral radiation forces are reversed in LHMs. To obtain a better physical picture of the radiation force on LHMs, we simulate a TE-polarised and a TM-polarised Gaussian beam of finite width propagating in LHMs, respectively. The numerical results show that a Gaussian beam exerts either an expansive or a compressive lateral force on its host medium, depending on the beam's polarisation state. All derivations are based directly on the Lorentz law of force in conjunction with Maxwell's equations, using the standard constitutive relations for homogeneous, isotropic, and linear LHMs.

*Project supported by the National Natural Science Foundation of China (Grant No. 10974063), the Research Foundation of Wuhan National Laboratory (Grant No. P080008), and the National "973" Project (Grant No. 2007CB310403).

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2. Maxwell equations and Lorentz law

To investigate the radiation pressure exerted by the electromagnetic fields in the LHM, we use the Maxwell's equations to determine the field distribution both inside and outside the LHM. We consider a monochromatic electromagnetic field $\boldsymbol{E}(\boldsymbol{r},t) = \mathrm{Re}[\boldsymbol{E}(\boldsymbol{r})\exp(-\mathrm{i}\omega t)]$ and $\boldsymbol{B}(\boldsymbol{r},t) = \mathrm{Re}[\boldsymbol{B}(\boldsymbol{r})\exp(-\mathrm{i}\omega t)]$ of angular frequency ω incident from the free space to the LHM. The field can be described by the following Maxwell's equations[17]

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}, \qquad \nabla\cdot\boldsymbol{B} = 0,$$
$$\nabla\times\boldsymbol{H} = \boldsymbol{J}_{\mathrm{c}} + \frac{\partial\boldsymbol{D}}{\partial t}, \quad \nabla\cdot\boldsymbol{D} = \rho_{\mathrm{c}}. \tag{1}$$

Here ρ_{c} and $\boldsymbol{J}_{\mathrm{c}}$ are the free charge density and current density, respectively. For external sources in free-space, $\boldsymbol{D} = \varepsilon_0\varepsilon\cdot\boldsymbol{E}$ and $\boldsymbol{B} = \mu_0\mu\cdot\boldsymbol{H}$. We can easily find that the wave propagation is only permitted in the medium with $\mathrm{Re}[\varepsilon]$, $\mathrm{Re}[\mu] > 0$ or $\mathrm{Re}[\varepsilon]$, $\mathrm{Re}[\mu] < 0$. In the former case, $\boldsymbol{E}$, $\boldsymbol{H}$ and the wave vector $\boldsymbol{k}$ form a right-handed triplet, while in the latter case, $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{k}$ form a left-handed triplet.

We first discuss the Lorentz force which can be written as[17]

$$\boldsymbol{F} = \rho\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}, \tag{2}$$

where $\boldsymbol{F}$ is the force density, ρ and $\boldsymbol{J}$ are the total charge density and current density respectively. We choose the z axis to be normal to the interface, the x and y axes locate at the plane of the interface. The total charge density $\rho = \rho_{\mathrm{b}} + \rho_{\mathrm{c}}$. The density of bound charges $\rho_{\mathrm{b}} = -\nabla\cdot\boldsymbol{P}$, where $\boldsymbol{P} = \varepsilon_0(\varepsilon-1)\boldsymbol{E}$ is the local polarisation density within the medium. Inside a homogeneous and isotropic medium, $\nabla\cdot\boldsymbol{P} = 0$ implys that $\rho_{\mathrm{b}} = 0$. It means that there are no bound charges which can exist inside such media.

At the interface between the two different media, the perpendicular component D_z, must be continuous. The implication is that E_z is discontinuous and, therefore, bound charges can exist at such interfaces. The interfacial bound charges will thus have an areal density $\sigma = \epsilon_0\Delta E_z$. Under the influence of the local electric field, these charges give rise to an electric Lorentz force $\boldsymbol{F} = \mathrm{Re}[\sigma\boldsymbol{E}^*]$, where $\boldsymbol{F}$ is the force per unit area of the interface. Since the tangential electric field $E_\perp$ is continuous across the interface, there is no ambiguity as to which field must be used in conjunction with the Lorentz law.

Since $\nabla\cdot\boldsymbol{B} = 0$ and $\boldsymbol{B} = \mu_0\mu\boldsymbol{H}$, the tangential component of magnetic field perpendicular to the interface must remain continuous. According to Maxwell's curl equation, we obtain an interfacial current density $\boldsymbol{J}_{\mathrm{c}} = \nabla\times\boldsymbol{H} - \partial\boldsymbol{D}/\partial t$. Such currents can exist on the surfaces of good conductors, where $E_\perp$ is negligible. Otherwise, the only sources of electrical currents are bound charges. Obviously, the bound current density is $\boldsymbol{J}_{\mathrm{b}} = \partial\boldsymbol{P}/\partial t = \varepsilon_0(\varepsilon-1)\partial\boldsymbol{E}/\partial t$, while the total current density $\boldsymbol{J} = \boldsymbol{J}_{\mathrm{b}} + \boldsymbol{J}_{\mathrm{c}}$ can be written as

$$\boldsymbol{J} = \nabla\times\boldsymbol{H} - \varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}. \tag{3}$$

The magnetic field of the electromagnetic wave then exerts a force on the total current according to the Lorentz law.[19,20]

In classic electromagnetic theory, a bulk material can be regarded as a collection of molecules (or atoms) embedded in the vacuum. Under the action of an incident field, the molecules oscillate as electric and magnetic dipoles and emit radiations. The radiation field and the incident field interact to form the new transmitted field in the material and the reflection field outside the material.[17] Let us carry this idea one step further, the metamaterial is structured into subunits. In the case of electromagnetic radiation this usually means that the subunits must be much smaller than the wavelength of radiation. Then the unit cells of metamaterials can be modelled as the molecules (or atoms) in ordinary materials.[21,22]

3. The radiation force exerted by TE-polarised light

In this section, we analyse the case of TE-polarisation, leaving a discussion of TM-polarised light in the next section. Figure 1 shows the case of oblique incidence with TE-polarised wave at the interface between the free-space and a LHM. For TE-polarised incident waves, matching the boundary conditions for each wave-vector component at the plane $z = 0$ gives the propagation field in the form

$$\boldsymbol{E}_{\mathrm{T}} = T_{\mathrm{TE}}E_0\boldsymbol{e}_y\exp[\mathrm{i}(q_x x + q_z z - \omega t)], \tag{4}$$

where $\boldsymbol{e}_y$ is the unit vector in the y-direction. The components of the $\boldsymbol{H}$ fields, which can be obtained from Maxwell's curl equation, are

$$\boldsymbol{H}_{\mathrm{T}} = \left[-\frac{T_{\mathrm{TE}}E_0 q_z}{\omega\mu_0\mu}\boldsymbol{e}_x + \frac{T_{\mathrm{TE}}E_0 q_x}{\omega\mu_0\mu}\boldsymbol{e}_z\right] \times \exp[\mathrm{i}(q_x x + q_z z - \omega t)], \tag{5}$$

where $\boldsymbol{e}_x$ and $\boldsymbol{e}_z$ are the unit vectors in the x-direction and z-direction respectively. The $T_{\rm TE}$ is the Fresnel transmission coefficient, which can be obtained as follows:

$$T_{\rm TE} = \frac{2\mu\cos\theta_{\rm I}}{\mu\cos\theta_{\rm I} - \sqrt{\varepsilon\mu - \sin^2\theta}}. \quad (6)$$

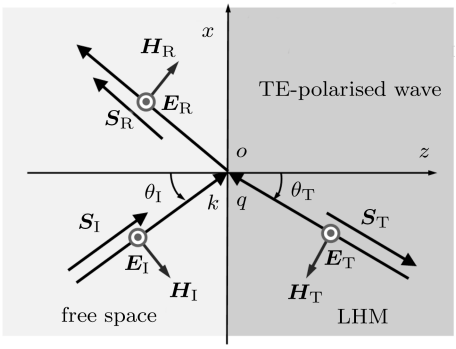


Fig. 1. Obliquely incident TE-polarised plane wave arrives at the surface of a semi-infinite LHM . Note that in free space, $\boldsymbol{E}_{\rm I}$, $\boldsymbol{H}_{\rm I}$ and $\boldsymbol{k}$ form a right-handed triplet, while in the LHM, $\boldsymbol{E}_{\rm T}$, $\boldsymbol{H}_{\rm T}$ and $\boldsymbol{q}$ form a left-handed triplet.

Since there are no free charges inside the medium (nor on its surface), the only relevant force here is the magnetic Lorentz force on the dipolar current density

$$\boldsymbol{J}_{\rm b}(x,z) = -{\rm i}\omega\varepsilon_0(\varepsilon-1)\boldsymbol{e}_y E_{{\rm T}y}(x,z). \quad (7)$$

Inside the medium, the distributions of radiation force are

$$F_x = \frac{1}{2}{\rm Re}[-{\rm i}\varepsilon_0(\varepsilon-1)q_x]\exp(-2{\rm Im}[q_z]z)|E_{\rm T}|^2, \quad (8)$$

$$F_z = \frac{1}{2}{\rm Re}[-{\rm i}\varepsilon_0(\varepsilon-1)q_z^*]\exp(-2{\rm Im}[q_z]z)|E_{\rm T}|^2. \quad (9)$$

Assuming that both ε and μ are complex, we allow them to approach a real number only after calculating the total force by integrating through the thickness of the medium. We find the net force components along the x- and z-axes to be

$$\langle F_x\rangle = \frac{1}{2}{\rm Re}[-{\rm i}\varepsilon_0(\varepsilon-1)q_x]\frac{1}{2{\rm Im}[q_z]}|E_{\rm T}|^2, \quad (10)$$

$$\langle F_z\rangle = \frac{1}{2}{\rm Re}[-{\rm i}\varepsilon_0(\varepsilon-1)q_z^*]\frac{1}{2{\rm Im}[q_z]}|E_{\rm T}|^2. \quad (11)$$

To obtain a better physical picture of the radiation force in LHMs, we investigate the radiation force produced by a Gaussian beam of finite width. Without any loss of generality, we assume that the input waist locates at the object plane $z = 0$. The fundamental Gaussian spectrum distribution can be written in the form

$$\tilde{E}(k_x,k_y) = \frac{w_0E_0}{\sqrt{2\pi}}\exp\left[-\frac{(k_x^2+k_y^2)w_0^2}{4}\right], \quad (12)$$

where w_0 is the spot size. The Rayleigh length is given by $z_{\rm R} = k_0w_0^2/2$. The field distribution can be obtained by the Fourier integral and angular spectrum representation.[23−25]

Figure 2 presents the numerical result of force density distribution for a TE-polarised Gaussian beam propagating in the semi-infinite LHM. We note that the lateral force exerted by the TE-polarised Gaussian beam on the LHM is orthogonal to the edge and produces expansive pressure toward the beam centre [Fig. 2(a)]. That is to say the direction of radiation force is opposite to the case in RHMs.[19,26] It is interesting that the direction of longitudinal force $\langle F_z\rangle$ still is in the propagation direction (Fig. 2(b)) and is the same as the case in RHMs. We also note that the value of $\langle F_z\rangle$ decreases with the propagation distance z due to the loss of LHM.

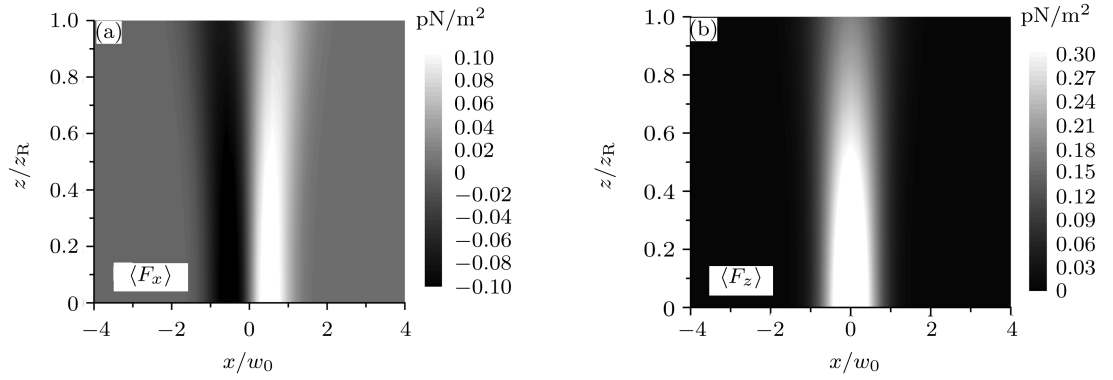


Fig. 2. Numerical computation of the force density of TE-polarised Gaussian beam propagating in semi-infinite LHM with $\varepsilon = -1+0.1{\rm i}$ and $\mu = -1+0.1{\rm i}$. (a) Force density distribution of the $\langle F_x\rangle$ component. (b) Force density distribution of the $\langle F_z\rangle$ component.

4. The radiation force exerted by TM-polarised light

In this section, we will discuss the radiation force exerted by TM-polarised light. Figure 3 shows the case of oblique incidence with TM-polarised light at the interface between the free-space and an LHM. For compactness, we assume that the magnetic field polarises along the y axis. For TM-polarised incident waves, the transmitted field can be written as

$$\boldsymbol{H}_{\mathrm{T}} = T_{\mathrm{TM}} H_0 \boldsymbol{e}_y \exp[\mathrm{i}(q_x x + q_z z - \omega t)]. \tag{13}$$

The components of the $\boldsymbol{E}$ fields, which can be obtained from Maxwell's curl equation, are

$$\boldsymbol{E}_{\mathrm{T}} = \left[\frac{T_{\mathrm{TM}} H_0 q_z}{\omega\varepsilon_0\varepsilon}\boldsymbol{e}_x - \frac{T_{\mathrm{TM}} H_0 q_x}{\omega\varepsilon_0\varepsilon}\boldsymbol{e}_z\right] \times \exp[\mathrm{i}(q_x x + q_z z - \omega t)]. \tag{14}$$

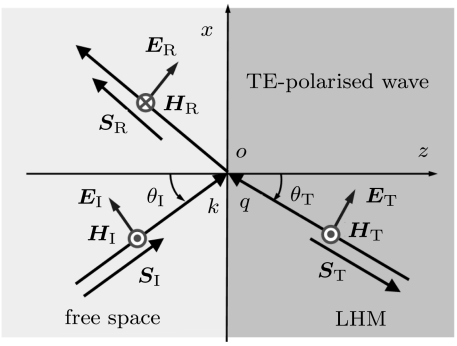


Fig. 3. Obliquely incident TM-polarised plane wave arrives at the surface of a semi-infinite LHM. Note that in free space, $\boldsymbol{E}_{\mathrm{I}}$, $\boldsymbol{H}_{\mathrm{I}}$ and $\boldsymbol{k}$ form a right-handed triplet, while in the LHM, $\boldsymbol{E}_{\mathrm{T}}$, $\boldsymbol{H}_{\mathrm{T}}$ and $\boldsymbol{q}$ form a left-handed triplet.

Based on the continuity of the tangential component of the electric field, we obtain the equation. A second equation can be found when we require continuity of the tangential components of the $\boldsymbol{H}$ fields, which can be obtained from Maxwell's curl equation. Equate the x component of the $\boldsymbol{E}$ vectors corresponding to the incident and reflected fields with transmitted fields. For TM-polarised incident waves, the transmission coefficient can be obtained similarly as

$$T_{\mathrm{TM}} = \frac{\varepsilon\cos\theta_{\mathrm{I}}}{\varepsilon\cos\theta_{\mathrm{I}} - \sqrt{\varepsilon\mu - \sin^2\theta}}. \tag{15}$$

Since there are no free charges inside the media, the only relevant radiation force is the magnetic Lorentz force on the dipolar current density given by

$$\boldsymbol{J}_{\mathrm{b}}(x,z) = -\mathrm{i}\omega\varepsilon_0(\varepsilon-1)\boldsymbol{E}_{\mathrm{T}}(x,z). \tag{16}$$

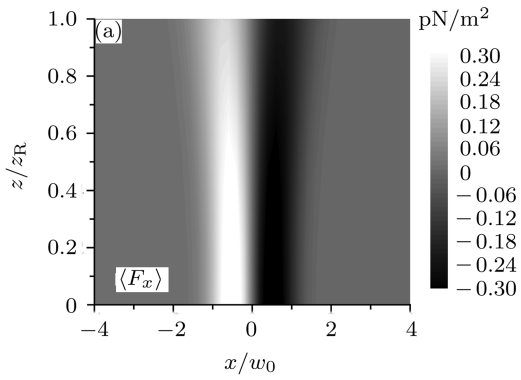
Next we consider the density of bound charges at the interface with the free space. In the interface associated with LHM the z-component of field is $E_z(x, z = 0^+) = E_{\mathrm{T}z}$. Immediately in the interface $z = 0^-$, the continuity of $D_z = \varepsilon_0\varepsilon E_{\mathrm{T}z}$ requires that $E_z(x, z = 0^-) = \varepsilon E_{\mathrm{T}z}$. The surface charge density σ, being equal to the discontinuity in E_z, is thus given by $\sigma = (1-\varepsilon)E_{\mathrm{T}z}$. As for the perpendicular component, the average $\boldsymbol{E}$ across the boundary, must be used in calculating the interfacial force. From the physical standpoint, the interfacial charges produce a local electric field which has the same magnitude but opposite directions on the two sides of the interface. It is this locally-generated electric field which is responsible for the field's discontinuity. Averaging electric field across the interface eliminates the local field, as it should be, since the charge cannot exert a force on itself.[19]

Following the same procedure as before, we find the net force components along the x- and z-axes to be

$$\langle F_x\rangle = \frac{1}{2}\mathrm{Re}[-\mathrm{i}(1-1/\varepsilon)q_x]\mu_0\mu^* \times \exp(-2\mathrm{Im}[q_z]z)|H_{\mathrm{T}}|^2, \tag{17}$$

$$\langle F_z\rangle = \frac{1}{2}\mathrm{Re}[-\mathrm{i}(1-1/\varepsilon)q_z]\mu_0\mu^* \times \exp(-2\mathrm{Im}[q_z]z)|H_{\mathrm{T}}|^2. \tag{18}$$

For providing a further evidence, we have simulated a TM-polarised Gaussian beam propagating in the LHM. Figure 4 presents the numerical results of the force density of a TM-polarised Gaussian beam propagating in the LHM. We note that the radiation force $\langle F_x\rangle$ turns out to be compressive (Fig. 4(a)). Hence, the lateral radiation force also is reversed in LHMs.[26] The distribution of longitudinal force $\langle F_z\rangle$ is shown in Fig. 4(b). Its direction accords with $+z$ and is unreversed.



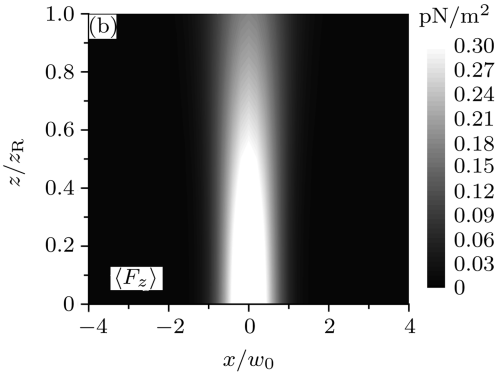


Fig. 4. Numerical computation of the force density of TM-polarised Gaussian beam propagating in LHM with $\varepsilon = -1+0.1\mathrm{i}$ and $\mu = -1+0.1\mathrm{i}$.(a) Force density distribution of the $\langle F_x \rangle$ component. (b) Force density distribution of the $\langle F_z \rangle$ component.

5. Conclusions

The radiation force experienced by the LHM has been calculated from a direct consideration of the charges and currents, and their interactions with the electric and magnetic components of the radiation field. We have established the analytical model of radiation force for TE- and TM-polarised electromagnetic waves on LHMs. As an practical example, we have obtained the distribution of radiation force for a TE- and TM-polarised Gaussian beam by direct numerical simulation. We have demonstrated that the TE-polarised Gaussian beam exerts an expansive force, while the TM-polarised Gaussian beam exerts a compressive lateral force to the LHM. The theoretical and numerical results have shown that the direction of lateral radiation force is reversed. However, the longitudinal component of radiation forces for TE- and TM-polarised wave all are unreversed. We believe that our results will provide insights into the fundamental properties of LHMs and will provide better understanding of the interaction of light with LHMs.

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