Positron-impact ionisation of atomic hydrogen in the presence of a bichromatic laser field*

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The positron impact-ionisation of atomic hydrogen in the presence of a linearly polarised bichromatic field is investigated in the first Born approximation. The field is composed of a fundamental frequency and its second harmonic. The state of positron in the field is described by the Volkov wavefunction, and the continuum state of the ejected electron is described by the Coulomb–Volkov wavefunction. The dressed ground state of target is a first order time-dependent perturbative wavefunction. The triple differential cross sections and their dependencies on laser field parameters are discussed and compared with the results modified by a monochromatic field. Numerical results show that the coherent phase control is significant and the laser-assisted ionisation cross sections caused by positron and electron are different.

Keywords: bichromatic laser field, positron, impact-ionisation

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Laser-assisted charged particle-impact ionisation of atoms or ions plays an important role in plasma confinement, chemical reactions, semiconductor physics, as well as in fundamental understanding of atomic structure and collision mechanisms. Especially the positron projectile provides a good testing ground for the 'scattering of electrons without exchange'. With great improvement in the intensity of positron beams, positron-atom collision has acquired more attention in many fields, such as the origin of astrophysical sources of annihilation radiation, positron emission tomography (PET) in the biomedical realm, the characterisation of material, and the formation of antihydrogen.[1−4] In the absence of laser, several experiments[5−7] and theoretical studies[8−14] have been carried out on positron-impact ionisation. Such a process is also investigated when a monochromatic laser field is applied.[15,16] In the previous investigations on the electron-impact ionisation [the (e,2e) reactions], a bichromatic field was used to control the reaction by changing the phase difference between the two frequency components. In the literature, this is referred to as the coherent phase control (CPC).[17−20] A summary on this topic can be found from the work of Giusti-Suzor *et al.*[21] and the review of Ehlotzky.[22]

Motivated by these studies, in this letter, we investigate the positron-impact ionisation of atomic hydrogen assisted by a bichromatic laser field. Consider the process $\mathrm{e}^+(E_\mathrm{i},\boldsymbol{k}_\mathrm{i}) + \mathrm{H(1s)} + \mathrm{l}\omega \rightarrow \mathrm{H}^+ + \mathrm{e}^+(E_\mathrm{f},\boldsymbol{k}_\mathrm{f}) + \mathrm{e}^-(E_\mathrm{e},\boldsymbol{k}_\mathrm{e})$, where $E_\mathrm{i,f}$ and $\boldsymbol{k}_\mathrm{i,f}$ respectively represent the energies and the momenta of positron in the initial and final channels. E_e and $\boldsymbol{k}_\mathrm{e}$ represent the energy and the momentum of the ejected electron. We describe the reaction in the Ehrhardt's asymmetric coplanar geometry.[23,24] The laser field is treated classically as a superposition of a fundamental frequency and its second harmonic, i.e., $\boldsymbol{\mathcal{E}}(t) = \boldsymbol{\mathcal{E}}_0[\sin(\omega t)+\sin(2\omega t+\varphi)]$, where $\boldsymbol{\mathcal{E}}_0$ is the electric field amplitude vector and φ is the relative phase that can be arbitrarily changed. The atomic units $\hbar = e = m = 1$ are used throughout unless otherwise especially stressed.

In the first Born approximation, the $\boldsymbol{S}$-matrix element for the bichromatic laser-assisted ionisation reads

$$\boldsymbol{S} = -\mathrm{i}\int_{-\infty}^{\infty} \mathrm{d}t \langle \chi_\mathrm{f}(\boldsymbol{r}_0,t)\chi_\mathrm{e}(\boldsymbol{r}_1,t)\left|\frac{Z_\mathrm{p}}{r_0}-\frac{Z_\mathrm{p}}{r_{01}}\right|\chi_\mathrm{i}(\boldsymbol{r}_0,t)\Phi_0(\boldsymbol{r}_1,t)\rangle, \quad (1)$$

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where $Z_{\rm p}$ is the charge of the projectile, $\boldsymbol{r}_0$ and $\boldsymbol{r}_1$ respectively denote the coordinates of positron and electron measured with respect to the target nucleus, and $\boldsymbol{r}_{01} = \boldsymbol{r}_0 - \boldsymbol{r}_1$. $\chi_{\rm i,f}(\boldsymbol{r}_0, t)$ are respectively the initial and the final states of the fast positron, which can be described by the Volkov wavefunctions[16,25]

$$\chi_{\rm i,f}(\boldsymbol{r}_0, t) = (2\pi)^{-3/2} \exp\{{\rm i}[\boldsymbol{k}_{\rm i,f} \cdot \boldsymbol{r}_0 + \boldsymbol{k}_{\rm i,f} \cdot \boldsymbol{\alpha}(t) - E_{\rm i,f} t]\}, \tag{2}$$

where $E_{\rm i,f} = k_{\rm i,f}^2/2$, and $\boldsymbol{\alpha}(t) = \boldsymbol{\alpha}_0[\sin(\omega t) + (1/4)\sin(2\omega t + \varphi)]$ with $\boldsymbol{\alpha}_0 = \boldsymbol{\mathcal{E}}_0/\omega^2$. $\chi_{\rm e}(\boldsymbol{r}_1, t)$ is the continuum states of the electron ejected in the final channel. Taking into account the laser effect and the attraction of the residual ion, the continuum state of electron can be described by the Coulomb–Volkov wavefunction as[26–28]

$$\chi_{\rm e}(\boldsymbol{r}_1, t) = (2\pi)^{-3/2} {\rm e}^{\pi\xi/2} \Gamma(1 + {\rm i}\xi)\, {}_1F_1(-{\rm i}\xi, 1, -{\rm i}(k_{\rm e} r_1 + \boldsymbol{k}_{\rm e} \cdot \boldsymbol{r}_1)) \exp\{{\rm i}[\boldsymbol{k}_{\rm e} \cdot \boldsymbol{r}_1 - \boldsymbol{k}_{\rm e} \cdot \boldsymbol{\alpha}(t) - E_{\rm e} t]\}, \tag{3}$$

where $E_{\rm e} = k_{\rm e}^2/2$, $\xi = 1/k_{\rm e}$ is the Sommerfeld parameter, and ${}_1F_1(a, b, z)$ is the confluent hypergeometric function.[29] In Eq. (1), $\Phi_0(\boldsymbol{r}_1, t)$ is a perturbative dressed wavefunction of the target atom which is initially in the ground state,[30] and expressed as

$$\Phi_0(\boldsymbol{r}_1, t) = \left\{1 - \frac{\rm i}{\omega} \boldsymbol{\mathcal{E}}_0 \cdot \boldsymbol{r}_1 \left[\cos(\omega t) + \frac{1}{2}\cos(2\omega t + \varphi)\right]\right\} \phi_0(\boldsymbol{r}_1) {\rm e}^{-{\rm i}E_0 t}, \tag{4}$$

where $\phi_0(\boldsymbol{r}_1) = \frac{1}{\sqrt{\pi}} {\rm e}^{-r_1}$ is the ground state of the atomic hydrogen, and $E_0 = -0.5$ a.u. is the ground state energy.

Inserting the wavefunctions (2)–(4) into the $\boldsymbol{S}$-matrix element (1) yields

$$\boldsymbol{S} = -2\pi{\rm i} \sum_l T(l) \delta(E_{\rm f} + E_{\rm e} - E_{\rm i} - E_0 + l\omega), \tag{5}$$

where

$$\begin{aligned} T(l) = {} & \frac{-1}{4\sqrt{2}\pi^3} {\rm e}^{\frac{\pi\xi}{2}} \Gamma(1 - {\rm i}\xi) \frac{Z_{\rm p}}{\boldsymbol{q}^2} \frac{\partial}{\partial\gamma} \left\{ \left[\frac{C_l}{\omega} \boldsymbol{\mathcal{E}}_0 \cdot \frac{\partial}{\partial \boldsymbol{\Delta}} + 4{\rm B}_l(\lambda, \lambda/4, \varphi)\right] \frac{[\boldsymbol{\Delta}^2 + (\gamma - {\rm i}k_{\rm e})^2]^{-{\rm i}\xi}}{[(\boldsymbol{k}_{\rm e} + \boldsymbol{\Delta})^2 + \gamma^2]^{1-{\rm i}\xi}} \right. \\ & \left. + \left[\frac{C_l}{\omega} \boldsymbol{\mathcal{E}}_0 \cdot \frac{\partial}{\partial \boldsymbol{q}} - 4{\rm B}_l(\lambda, \lambda/4, \varphi)\right] \frac{[q^2 + (\gamma - {\rm i}k_{\rm e})^2]^{-{\rm i}\xi}}{[(\boldsymbol{k}_{\rm e} - \boldsymbol{q})^2 + \gamma^2]^{1-{\rm i}\xi}} \right\} \Bigg|_{\gamma=1, \boldsymbol{\Delta}=\boldsymbol{0}}, \end{aligned} \tag{6}$$

in which ${\rm B}_l(\lambda, \lambda/4, \varphi) = \sum_{n=-\infty}^{\infty} {\rm J}_{l-2n}(\lambda) {\rm J}_n(\lambda/4) \exp^{-{\rm i}n\varphi}$ is the generalised Bessel function,[31] and $C_l = 2{\rm B}_{l-1}(\lambda, \lambda/4, \varphi) + 2{\rm B}_{l+1}(\lambda, \lambda/4, \varphi) + {\rm B}_{l-2}(\lambda, \lambda/4, \varphi)\, {\rm e}^{{\rm i}\varphi} + {\rm B}_{l+2}(\lambda, \lambda/4, \varphi)\, {\rm e}^{-{\rm i}\varphi}$. ${\rm J}_n(\lambda)$ is the ordinary Bessel function. In the arguments, $\lambda = (\boldsymbol{k}_{\rm e} - Z_{\rm p}\boldsymbol{q}) \cdot \boldsymbol{\alpha}_0$, where $\boldsymbol{q} = \boldsymbol{k}_{\rm f} - \boldsymbol{k}_{\rm i}$ is the momentum transfer.

The triple differential cross section (TDCS) resolved with the photon numbers reads

$$\frac{{\rm d}^3\sigma_l}{{\rm d}\Omega_{\rm f}\, {\rm d}\Omega_{\rm e}\, {\rm d}E_{\rm e}} = (2\pi)^4 \frac{k_{\rm e} k_{\rm f}}{k_{\rm i}} |T(l)|^2. \tag{7}$$

For numerical calculation, we choose the Nd:YAG laser ($\hbar\omega = 1.17$ eV) as the dressing field. The field polarisation is set to be parallel to the impact momentum, and the field amplitude is $\boldsymbol{\mathcal{E}}_0 = 5.0 \times 10^7\ {\rm V \cdot cm^{-1}}$. We denote the scattering angle of positron by $\theta_{\rm f}$, and the ejected angle of electron by $\theta_{\rm e}$.

In Fig. 1 we present the TDCSs as functions of the eject angle $\theta_{\rm e}$ of electron (solid curve), and compare them with the results for a monochromatic laser field (dashed curve). The scattering angle of the fast positron is $\theta_{\rm f} = 3^\circ$. We observe significant modifications of the second field component. For the $l = 0$ case (the colliding system exchanges no net photon with the laser field), the binary peak is considerably depressed while the two recoil peaks are hardly modified. For $l = 1$ (the colliding system releases a photon), the binary peaks are also greatly depressed, and a new binary peak appears. While the recoil peak is notably enhanced. The situation for $l = -1$ (the system absorbs a net photon) is quite the contrary: the binary peaks are prominently enhanced, and the recoil peak is prominently depressed. As we know, the binary peak is a result of the binary collision between the projectile positron and the target electron. The states of both particles are notably modified by both components of the laser field (see Eqs. (2)–(4)). The modification in TDCS is the superposition of the

dressing of both particles. Since the field tends to separate both light particles apart and the separation is stronger in the bichromatic assisting case, the results are more significantly modified in the binary peak. On the other hand, the recoil peaks are mainly governed by the electron–nucleus interaction, and the state of the nucleus is hardly modified by the field due to its heaviness. Therefore in this case, only the dressing of the positron state makes contribution, which causes the modification in TDCS in the recoil peaks to be not so prominent as in the binary peak.

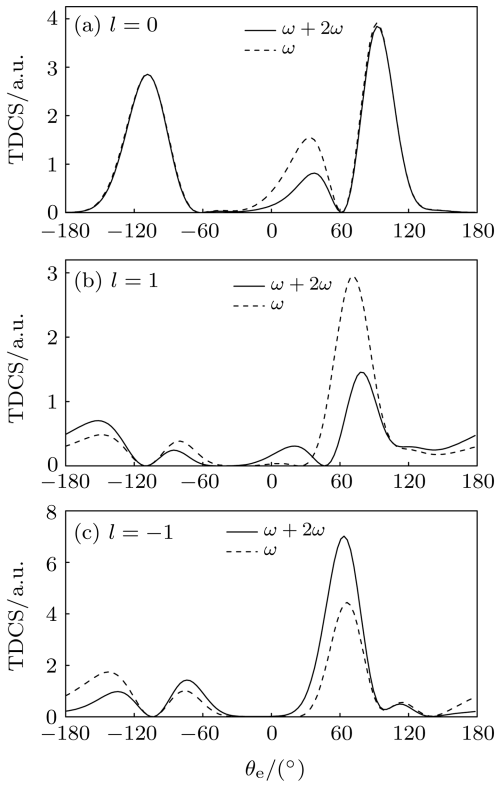


Fig. 1. TDCSs as functions of the ejected electron angle θ_e for the bichromatic (solid curve) and monochromatic laser field (dash curve). The laser polarisation is set to be parallel to the impact momentum ($\boldsymbol{\mathcal{E}}_0 \parallel \boldsymbol{k}_i$). The kinetic energies of the incident positron and the ejected electron are respectively $E_i = 250$ eV and $E_e = 5$ eV, and the scattering angle of the positron is $\theta_f = 3^\circ$. The laser electric amplitude is $\boldsymbol{\mathcal{E}}_0 = 5.0\times10^7$ V·cm^{-1}, and the frequency is $\omega = 1.17$ eV (Nd:YAG laser). The relative phase $\varphi = 0^\circ$.

In Fig. 2 we present the TDCSs for several phases. Significant difference is observed between the results of each phase. For the $l = 0$ case, the binary peaks change considerably, while the recoil peak is hardly changed. For $l = 1$, both the binary and the recoil peaks are changed. For $l = -1$, the structure of TDCS is richer than in other cases.

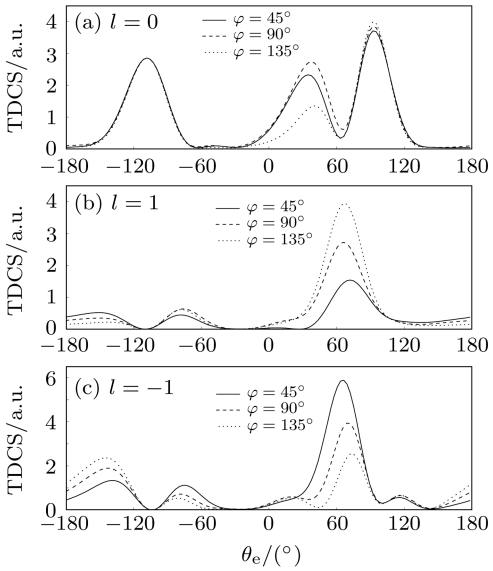


Fig. 2. TDCSs for different relative phases. The other parameters are the same as those in Fig. 1.

In Fig. 3 we display the phase dependence of TDCS at a fixed ejection angle $\theta_e = 80^\circ$. The result for $l = 0$ is less sensitive to phase than for $l = \pm1$. As can be found from Eq. (6), the results are periodic functions of φ. The variation tendency of TDCS for $l = -1$ is opposite to thar for $l = 1$. This may attribute to the symmetry relation of the generalised Bessel functions, $B_{-l}(\lambda, \lambda/4, \varphi) = (-1)^l B_l^*(\lambda, \lambda/4, \varphi - 180^\circ)$.

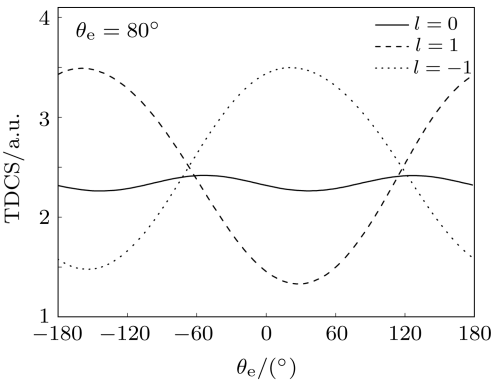


Fig. 3. TDCSs as funtions of relative phase φ for the ejected angle $\theta_e = 80^\circ$. The other parameters are the same as those in Fig. 1.

Figure 4 shows the TDCSs summed over all multiphoton processes. The differences in TDCS between the positron and the electron impacts are obvious at binary peak and recoil peak. The fact that can be understood as follows: in the absence of the laser, the TDCS is dependent on Z_p^2 , i.e., independent of the sign of the projectile charge. In the case of $\boldsymbol{\mathcal{E}}_0 \parallel \boldsymbol{k}_i$ in monochromatic laser field, there is no difference in

TDCS.[16] This is not so if a bichromatic laser field is present: with a fundamental frequency and its second harmonic involved, we expect the differences between positron and electron impacts to become more pronounced, even in the validity range of the first Born approximation.

In summary, we have investigated the positron-impact collision ionisation of atomic hydrogen in the presence of a bichromatic laser field. The numerical results show that the introduction of the second component of the laser field changes the triple differential cross section significantly, especially at the binary and recoil peaks. The opposite tendencies in phase dependence of result may be attributed to the symmetry property of the generalised Bessel function. In the end we show the laser-induced differences between particle and antiparticle impacts in the presence of a bichromatic laser field.

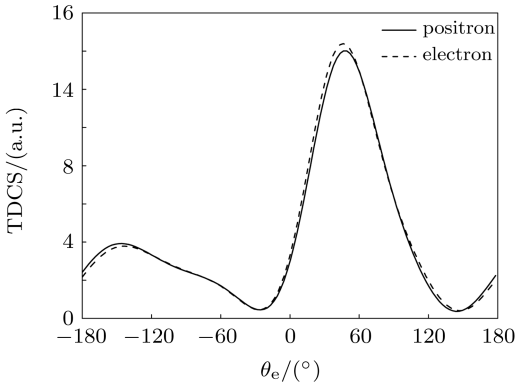


Fig. 4. Photonnumber-summed TDCSs versus the ejection angle θ_e of the positron and electron. The other parameters are the same as those in Fig. 1.

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