Harmonic stochastic resonance-enhanced signal detecting in NW small-world neural network*

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The harmonic stochastic resonance-enhanced signal detecting in Newman-Watts small-world neural network is studied using the Hodgkin–Huxley dynamical equation with noise. If the connection probability p, coupling strength $g_{\rm syn}$ and noise intensity D matches well, higher order resonance will be found and an optimal signal-to-noise ratio will be obtained. Then, the reasons are given to explain the mechanism of this appearance.

Keywords: stochastic resonance, small-world, Hodgkin–Huxley, signal-to-noise ratio
PACC: 0590, 0540

1. Introduction

Noise-induced and resonance-enhanced weak signal detection and transduction in neural network with electrical or chemical synaptic coupling have become an attracting field in recent years, in which the phenomena of coherence resonance (CR)[1−3] and stochastic resonance (SR)[4−6] are two common subjects widely studied. SR or CR induced by noise in a neural network can not only promote the synchronisation of neurons but also enhance the ability of detecting weak signals. A neural network with many spontaneously oscillating neurons can show the action of spiking with the characteristics of threshold divided into two domains: subthreshold and suprathreshold, and this feature can be simulated by such theoretical models as: Hodgkin–Huxley (HH),[7,8] Chay,[9] Morris–Lecar (ML),[10] FitzHugh–Nagumo (FHN),[11] Hindmarsh-Rose (HR),[12] Integrate-and-Fire (IF),[13] etc. Researches showed that it was more efficient to detect weak signal by way of SR, and an optimal noise intensity $D_{\rm opt}$ with the best detecting ability can be achieved.[14,15]

It is generally believed that the network's topology with different coupling neurons is able to determine the detecting ability. The study of CR globally coupled HH neurons shows that the coherence and synchronisation of the system can be optimised by noise.[16] The authors have studied the SR globally coupled HH neurons, and given the frequency sensitivity in the neural network, by way of which they can enhance the detecting ability for weak signal.[17] The research of living neural networks with global coupling represents that the giant component disintegration is independent of the balance between excitatory and inhibitory neurons.[18] Wang and Lu[19] studied the Newman–Watts (NW) small-world[20] neural network theoretically and numerically, and found that the phase synchronisation can be obtained by increasing coupling strength.

From another point of view, harmonic stochastic resonance (HSR) in neural network has also attracted extensive attention. When the input signal frequency is f, if the stochastic resonance frequency is ($2f$, $3f$, ...), the phenomenon is HSR. Reference [21] studied a Watts–Strogtz (WS) small-world neural network by using a method of super-harmonic SR, and found an optimal amplitude. In a noisy environment with weak signal, special message can be found by HSR.

It is well known that our human brain is a system with many neurons connected with one another by way of small-world topology which can transmit a great deal of information.[22] When it comes to the neural network, for example, the NW small-world neural network, how it works and detects weak signal with HSR may give us some help to understand the subject

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stated above. The rest of the paper is organised as follows. In Section 2, we describe the HH neural network model, NW small-world and signal-to-noise ratio (SNR) in HSR. The results, discussions and reasons are explained in Section 3. Finally, the conclusions are given in Section 4.

2. Model and SNR

2.1. HH neural network model

In order to achieve an authentic result, the famous HH model, which can describe well the spiking behaviour and refractoriness of a real neuron, is employed in this work; and we use it as the basic dynamics of the cell node. The model is described by four nonlinear coupled equations for the membrane potential V, the gating variables: m, n and h; and they read

$$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = I_{\mathrm{ext}}(t) + I_{\mathrm{ion}}(t), \tag{1}$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = (m_\infty(V) - m)/\tau_m(V), \tag{2}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = (h_\infty(V) - h)/\tau_h(V), \tag{3}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = (n_\infty(V) - n)/\tau_n(V), \tag{4}$$

with

$$\begin{aligned} I_{\mathrm{ext}}(t) &= I_0 + I_1 \sin(2\pi f_{\mathrm{s}} t) + I_{\mathrm{noise}}, \\ I_{\mathrm{ion}}(t) &= -g_{\mathrm{na}} m^3 h (V - V_{\mathrm{na}}) - g_{\mathrm{k}} n^4 (V - V_{\mathrm{k}}) \\ &\quad - g_{\mathrm{l}} (V - V_{\mathrm{l}}). \end{aligned}$$

The ionic current $I_{\mathrm{ion}}(t)$ includes the usual currents for sodium (I_{na}), potassium (I_{k}) and leak (I_{l}). The parameters g_{na}, g_{k} and g_{l} are the maximal conductances for the ions and leakage channels; V_{na}, V_{k} and V_{l} are the corresponding reversal potentials. While m_∞, h_∞, n_∞, and τ_m, τ_h, τ_n represent the saturated values and the relaxation times of the gating variables respectively. Detailed values of parameters and meaning of variables can be found in Ref. [7].

The dynamical equations for the network consisting of HH neurons can be presented as follows:

$$C_m \frac{\mathrm{d}V^i}{\mathrm{d}t} = I^i_{\mathrm{ext}}(t) + I^i_{\mathrm{ion}}(t), \quad i = 1, 2, 3 \ldots, N, \tag{5}$$

where $I^i_{\mathrm{ext}}(t)$ is assumed to be composed of

$$I^i_{\mathrm{ext}}(t) = I_0 + I_1 \sin(2\pi f_{\mathrm{s}} t) + I_{\mathrm{noise}}(t) + I^i_{\mathrm{syn}}(t).$$

In this simulation, we have $I_0 = 2\ \mu\mathrm{A/cm}^2$ and $I_1 = 1\ \mu\mathrm{A/cm}^2$. All the neurons are beyond the firing threshold ($6.2\ \mu\mathrm{A/cm}^2$).[23]

Here I_{noise} represents the noisy component of the stimulus to a neuron from external fluctuations or intrinsic fluctuations of the neuron itself. The noisy current can be obtained from an Ornstein–Uhlenbeck (OU) process,

$$\tau_d \frac{\mathrm{d}I_{\mathrm{noise}}}{\mathrm{d}t} = -I_{\mathrm{noise}} + \sqrt{2D}\xi(t), \tag{6}$$

where $\xi(t)$ is Gaussian white noise with $\langle \xi(t)\xi'(t) \rangle = D\delta_{ij}\delta(t - t')$, D and τ_d are the noise intensity and the correlation time of the OU noise respectively, $\tau_d = 2$ ms is used in simulation. The synapse between two neurons is in effect if $(t^i(\mathrm{fire}) - t^j(\mathrm{fire})) \leq 2$ ms. The synaptic current[24] $I^i_{\mathrm{syn}}(t)$ reads

$$I^i_{\mathrm{syn}}(t) = -\frac{g_{\mathrm{syn}}}{N} \sum_{j=1, j\neq i}^{N} a_{ij}(V^i - V^j), \tag{7}$$

where g_{syn} is the coupling strength and a_{ij} is the coupling matrix with $a_{ij} = 1$ if there exists a connection between the ith and the jth neurons ($i \neq j$), and otherwise $a_{ij} = 0$. In simulation, we set up the NW small-world neural network with parameters $N = 101$, $K = 5$.

Average synaptic current input to the neurons is given by

$$\langle I_{\mathrm{syn}}(t) \rangle = -\frac{g_{\mathrm{syn}}}{N^2} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} a_{ij}(V^i - V^j). \tag{8}$$

The output of the network is defined as

$$V^{\mathrm{out}}(t) = \frac{1}{N} \sum_{i=1}^{N} \theta(V^i(t) - V^*), \tag{9}$$

where V^* is a membrane potential threshold, we set $V^* = -20$ mV. In Eq. (9), $\theta(x) = 1$ when $x > 0$ and $\theta(x) = 0$ when $x < 0$. In this way we can consider the average of the firing activities of all neurons as the output of the network, which may encode the input signal.

2.2. SNR of harmonic stochastic resonance

The SNR is a traditional physical term used to depict the stochastic resonance, which can reflect the detecting ability of a nonlinear system in noise environment. A high SNR value means strong detecting ability. Usually, the SNR used is defined as

$$\mathrm{SNR} = 10 \cdot \log_{10}(S(f_0)/B(f_0)), \tag{10}$$

where S and B represent the peak value of the input frequency and the average noise intensity in system's power spectrum density (PSD) respectively. In the same way, SNR of HSR can be defined as

$$\mathrm{SNR} = 10 \cdot \log_{10}(S(k \cdot f_0)/B(k \cdot f_0)), \quad (k = 2, 3, 4, \ldots), \tag{11}$$

where we use $k = 1, 2, 3, 4, \ldots$ to denote the 1st, 2nd, 3rd, 4th, ... order of HSR.

In order to achieve a good result, we make ten simulations and obtain an average PSD for $V^{\mathrm{out}}(t)$, we also use the second-order Runge–Kutta method[25] to solve the stochastic HH equations, taking 0.02 ms as the time step.

3. Simulation and result

3.1. The HSR

The HSR in the NW small-world neural network is determined by a few crucial parameters, for instance, noise intensity D, input frequency f_{s}, connection probability p, and coupling strength g_{syn}. In order to obtain the relationship between HSR and each parameter, we use sophisticated signal algorithm and carry out simulations by adjusting one parameter while fixing the other parameters.

First, the parameters f_{s}, p and g_{syn} are fixed. It is found that the spatiotemporal firings and the averaged synaptic current $\langle I_{\mathrm{syn}}(t)\rangle$ vary greatly when the noise intensity D changes. With such a noise intensity D, the HSR with the ordered spatiotemporal firings and the regular averaged synaptic current $\langle I_{\mathrm{syn}}(t)\rangle$ will appear, and lots of sharp peaks will appear in the PSD as shown in Figs. 1(a), 1(b), and 1(c) respectively. Hence, the noise intensity D plays an important role in the occurrence of HSR.

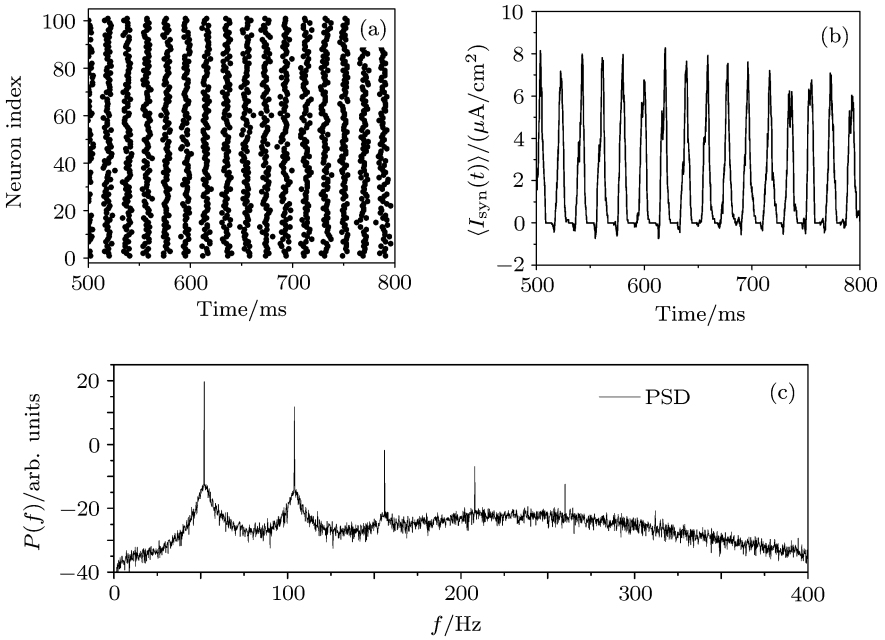


Fig. 1. For parameters $g_{\mathrm{syn}} = 1.0$ ms/cm^2, $p = 0.1$, $D = 5.0$, $f_{\mathrm{s}} = 52$ Hz, (a) the spatiotemporal firings, (b) the averaged synaptic current $\langle I_{\mathrm{syn}}(t)\rangle$, (c) the PSD (in $10 \cdot \log_{10}$ scale), represented by $P(f)$.

According to the classical stochastic resonance theory,[4] the maximal value of SNR will appear at the optimal noise intensity D_{opt}. For different orders of HSR, we calculate the SNR, represented by r_{SNR}, versus noise intensity D. It is found that all the curves are similar in Fig. 2. When $2 \leq D \leq 5$, the SNR increases significantly and when $D > 5$, the SNR decreases flatly. Apparently at $D = 5.0$, the SNR value is at its maximum, this noise intensity is called an optimal noise intensity D_{opt}.

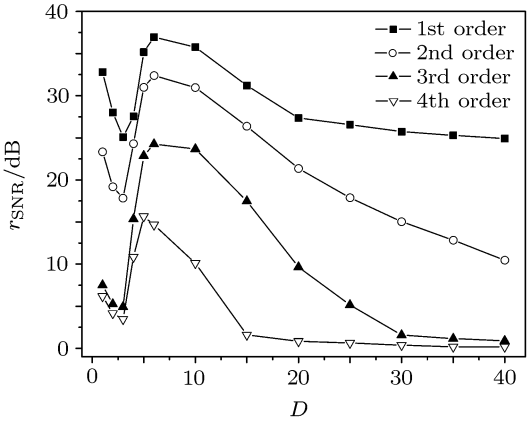


Fig. 2. The SNR versus the noise intensity D for different orders of HSR with parameters $g_{\rm syn} = 1.0$ ms/cm^2, $f_{\rm s} = 52$ Hz, and $p = 0.1$.

Secondly, as the neural network is sensitive to the input frequency, we then let the input frequency $f_{\rm s}$ increase from 0 to 80 Hz with fixed parameters of $D_{\rm opt}$, p and $g_{\rm syn}$. Figure 3 shows the PSDs with 4 different input frequencies. In Fig. 3(c), it can be seen, only when $f_{\rm s} = 52$ Hz, that the regular sharp peaks will appear in the PSD. So, 52 Hz is the resonance frequency($f_{\rm r}$). Only when the input frequency $f_{\rm s}$ gets close to or equals $f_{\rm r}$, will the HSR appear.

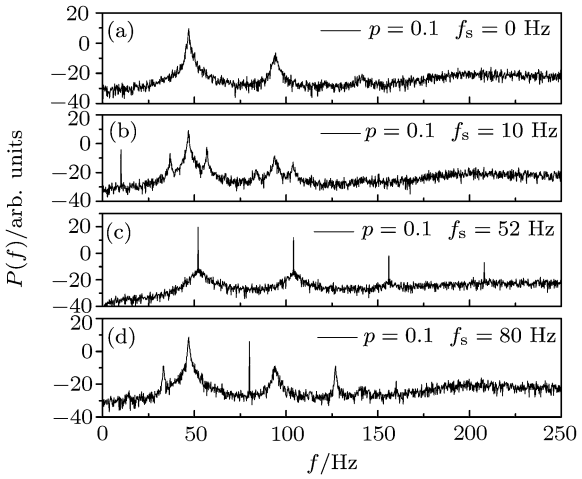


Fig. 3. The PSD (in $10 \cdot \log_{10}$ scale) of the system when the parameters are $g_{\rm syn} = 1.0$ ms/cm^2, $D = 5.0$, $p = 0.1$. From top to bottom, the input frequency $f_{\rm s}$ is (a) 0 Hz, (b) 10 Hz, (c) 52 Hz, (d) 80 Hz.

Finally, under the fixed condition of optimal noise intensity $D_{\rm opt}$ and resonance frequency $f_{\rm r}$, the effect of connection probability p is investigated through changing connection probability p. It is found that the number of sharp peaks in the PSD increases with p, e.g., when $p \geq 0.05$, the number is larger than 4, which can be seen in Fig. 4. This indicates that the connection probability p which determines the network topology plays a critically important role beyond the common sense.

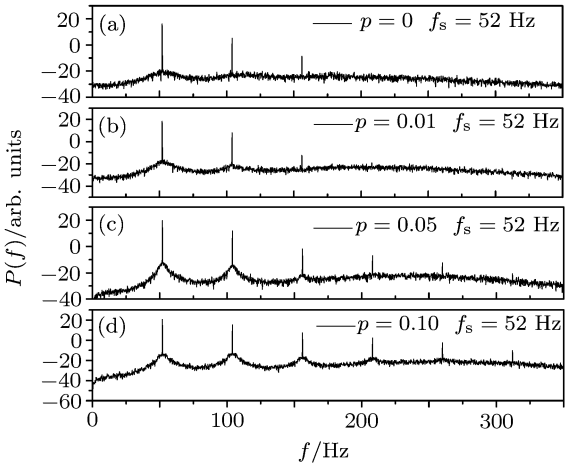


Fig. 4. The PSD (in $10 \cdot \log_{10}$ scale) of the system with parameters $g_{\rm syn} = 1.0$ ms/cm^2, $D = 5.0$, $f_{\rm s} = 52$ Hz. The connection probability is (a) $p = 0.0$, (b) $p = 0.01$, (c) $p = 0.05$, (d) $p = 0.1$.

Then the peaks in Fig. 4 are selected, and the peak value versus its corresponding frequency f is plotted within the skeleton of semi-logarithmic coordinate in Fig. 5. After numerical fitting, it is found that the curves follow the power law which plays an important role in the energy distribution.

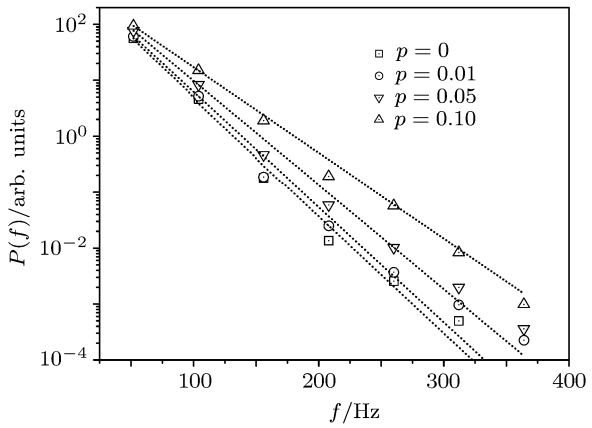


Fig. 5. Corresponding to Fig.4, the peak value versus the frequency. The symbol of square stands for $p = 0.0$, circle for $p = 0.01$, inverted triangle for $p = 0.05$, upward triangle for $p = 0.1$.

3.2. Optimal match

In addition, it is found that the parameter $g_{\rm syn}$ also plays a special role. When keeping other parameters fixed, one can see that the PSD changes greatly as $g_{\rm syn}$ changes from 0.1 to 2.0 as shown in Fig. 6.

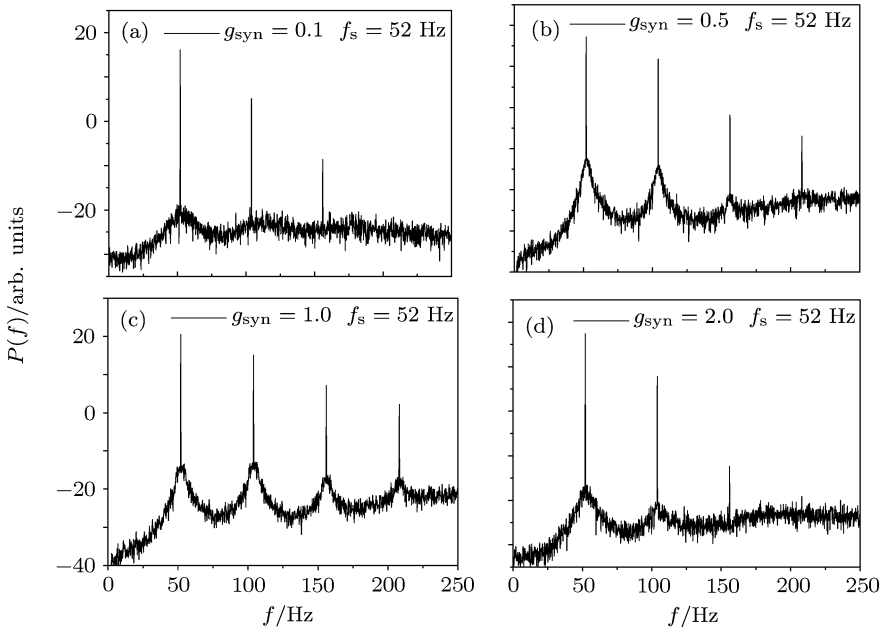


Fig. 6. The PSD (in $10\cdot\log_{10}$ scale) of the system versus frequency with parameters $p = 0.1$, $D = 5.0$, $f_s = 52$ Hz. for different coupling strength, (a) $g_{syn} = 0.1$ ms/cm^2, (b) $g_{syn} = 0.5$ ms/cm^2, (c) $g_{syn} = 1.0$ ms/cm^2, (d) $g_{syn} = 2.0$ ms/cm^2.

Then the dependence of SNR on the coupling strength g_{syn} for different order of HSR is displayed in Fig. 7. Though the connection probability p is different, the curves for the 2nd, 3rd, 4th orders of HSR have the same trend. When the coupling strength g_{syn} changes from 0.0 ms/cm^2 to 2.0 ms/cm^2, the SNR value increases first and then decreases. Compared with the study[21] in the WS small-world, the curves shown here can support part of its conclusions.

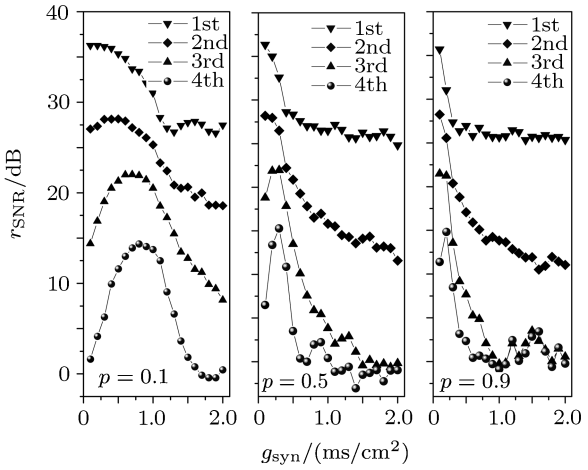


Fig. 7. The dependence of SNR of different orders on the coupling strength g_{syn} with parameters $D = 5.0$, $f_s = 52$ Hz. Figures from left to right stand for the connection probabilities $p = 0.1$, 0.3 and 0.5, respectively.

In order to study the behaviour of the different order of HSR in greater detail, the SNR values are plotted when p and g_{syn} changes at the same time. Without loss of generality, we choose the 3rd order HSR and calculate its SNR value versus p and g_{syn}. For $p = 0.1$, when 0 ms/cm^2 $< g_{syn} \leq 0.7$ ms/cm^2 the SNR value will increase and when $g_{syn} > 0.7$ ms/cm^2 it will decrease as g_{syn} increases. The largest SNR value (22.1 dB) can be gained when $g_{syn} = 0.7$ ms/cm^2. For this connection probability p, it is an optimal g_{syn}. More optimal g_{syn} can also be found in Fig. 8, e.g., when $p = 0.2$, the optimal g_{syn} is 0.5 ms/cm^2 and when $p = 0.3$, the optimal g_{syn} is 0.4 ms/cm^2. For a fixed p there exists a corresponding optimal g_{syn}.

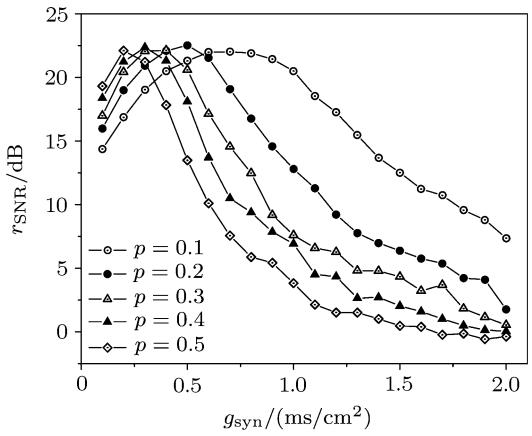


Fig. 8. The dependence of the 3rd order SNR on the coupling strength g_{syn} for $D = 5.0$, $f_s = 52$ Hz. The connection probability is $p = 0.1$, 0.2, 0.3, 0.4 and 0.5. There is an optimal g_{syn} for different p.

So, if the connection probability p and the coupling strength $g_{\rm syn}$ matches well, the largest SNR value, i.e. the best detecting ability, will be achieved in the neural network.

Furthermore, the SNR value of the 4th order HSR has been also calculated when p and $g_{\rm syn}$ varied as shown in Fig. 9. Obviously, their characteristics of the SNR are the same.

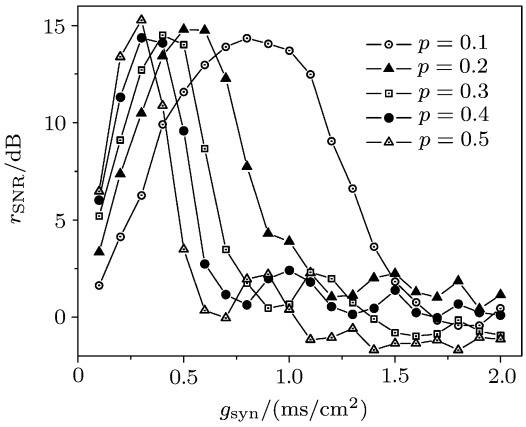


Fig. 9. The dependence of the 4th order SNR on the coupling strength $g_{\rm syn}$ for $D = 5.0$, $f_{\rm s} = 52$ Hz. For each connection probability p, there is an optimal $g_{\rm syn}$.

To summarise, the appearance of HSR in the neural network is due to the match of the system parameters. Under the appropriate condition of the optimal noise intensity $D_{\rm opt}$ and the resonance frequency $f_{\rm r}$, the subthreshold neurons in the neural network will start firing. As there is an cooperate mechanism for the small-world network in nature, for a proper connection probability p, most neurons are synchronised, so they can show an action of dynamical cluster synchronisation based on the resonance frequency, causing the input power to pile up to times of resonance frequency, and thus HSR happens. But when the connection probability p increases and the coupling strength $g_{\rm syn}$ changes, the small-world cooperating mechanism may be broken up. Then the network changes from temporal order to chaos, which may restrain the resonance. The results show that the power distribution for different orders is more coherent when connection between neurons is large, and the phenomenon will turn more complex. With more neurons, the result will become much more accurate.

4. Conclusion

We have studied the phenomenon of HSR in the NW small-world neural network system using the HH neuron as the basic node dynamics. Numerical simulation shows that there are four important parameters for the phenomenon of HSR, i.e. the optimal noise intensity $D_{\rm opt}$, resonance frequency $f_{\rm r}$ and the connection probability p, and the coupling strength $g_{\rm syn}$. As for the 3rd and 4th order harmonic, if the connection probability p and the coupling strength $g_{\rm syn}$ matches well, the largest value of SNR will be obtained. Although the result presented here have not been found in the realistic neural network, we believe it may provide another point of view for the complex neural network and enrich the study of HSR.

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