System dynamics of behaviour-evolutionary mix-game models*

Gou Cheng-Ling(苟成玲)[a)†], Gao Jie-Ping(高洁萍)[b)], and Chen Fang(陈 芳)[a)]

[a)] *Physics Department, Beijing University of Aeronautics and Astronautics, Beijing* 100191, *China*
[b)] *Mathematics Department, Beijing University of Aeronautics and Astronautics, Beijing* 100191, *China*

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In real financial markets there are two kinds of traders: one is fundamentalist, and the other is a trend-follower. The mix-game model is proposed to mimic such phenomena. In a mix-game model there are two groups of agents: Group 1 plays the majority game and Group 2 plays the minority game. In this paper, we investigate such a case that some traders in real financial markets could change their investment behaviours by assigning the evolutionary abilities to agents: if the winning rates of agents are smaller than a threshold, they will join the other group; and agents will repeat such an evolution at certain time intervals. Through the simulations, we obtain the following findings: (i) the volatilities of systems increase with the increase of the number of agents in Group 1 and the times of behavioural changes of all agents; (ii) the performances of agents in both groups and the stabilities of systems become better if all agents take more time to observe their new investment behaviours; (iii) there are two-phase zones of market and non-market and two-phase zones of evolution and non-evolution; (iv) parameter configurations located within the cross areas between the zones of markets and the zones of evolution are suited for simulating the financial markets.

Keywords: minority game model, mix-game model, behavioural evolution, system dynamics
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1. Introduction

Minority game (MG) was extended from Arthur's 'El Farol bar' problem[1] by Challet and Zhang.[2] MG model attracts a lot of following studies as a potential market model.[3−5] However, there are some weaknesses in the MG. First, the diversity of agents is limited, since agents all have the same historical memory and time-horizon. Second, in real markets, some agents are trend-followers, i.e. 'noise traders',[6−13] who effectively play a majority game; while the others are 'fundamentalists', who effectively play a minority game. Some researches on mixed minority/majority games have been done analytically and numerically.[14−18] In their models, time-horizon was infinite and all agents had the same abilities to deal with historical information and could switch over between majority side and minority side. They studied the stationary macroscopic properties and the dynamical features of the systems with different information structures, i.e. agents received the information that was either exogenous ('random') or endogenous ('real'). In order to create an agent-based model that more closely mimics a real financial market, Gou[19] proposed 'mix-game' model which is an extended version of MG model by dividing agents into two groups: each group has different historical memories and time-horizons; Group 1 plays the majority game, and Group 2 plays the minority game. The difference between the mix-game and the mixed minority/majority games studied by Martino *et al.*[18] is that the two groups of agents in the mix-game had different bounded abilities to deal with historical information and to count their own performance. This feature of the mix-game implies that agents have bounded rationality.[7]

Gou[20] specified the spectra of parameters of mix-game models that fit financial markets by investigating the dynamic behaviours of mix-game models under a wide range of parameters. The main findings in Ref. [20] are (a) in order to improve efficiency, agents in a real financial market must be heterogeneous, boundedly rational and subject to asymmetric information; (b) an active financial market must be dominated by agents who play a minority game, otherwise, the market would die; (c) the system could be stable if agents who play a majority game have a faster learning rate than those who play a minority

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†Corresponding author. E-mail: gouchengling@buaa.edu.cn
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game; otherwise, the system could be unstable. Further work used mix-game models to forecast financial markets.[21−24]

However, the learning abilities of agents in both MG and mix-game are limited since agents cannot change their strategies or learn from others during the game. In real financial markets, participants can learn from others and change their strategies or their investment behaviours according to the situations of markets. Gou *et al.*[5] have studied the system dynamics of evolutionary mix-game models in which agents can change their strategies during the game. In the present paper, we investigate such a situation that some participants in real markets can change their investment behaviours by assigning the evolutionary abilities to agents: if the winning rates of agents are smaller than a threshold, they will join the other group; and agents will repeat such an evolution at certain time intervals. We refer to this modified model as behaviour-evolutionary mix-game model. It is investigated in this paper how the threshold of winning rate and the time interval of evolution affect the dynamics of behaviour-evolutionary mix-game model and specify the proper spectra of parameters for simulating financial markets. The remainder of the present paper is organised as follows. In Section 2, the model and simulation conditions are introduced, and in Section 3 the results are discussed. Finally, we draw some conclusions from the present study in Section 4.

2. Model and simulation conditions

In the mix-game, N is the total number of agents and N_1 is the total number of agents in Group 1; T_1 and T_2 are the time horizons of the two groups; m_1 and m_2 denote the history memories of the two groups, respectively. A strategy consists of a response, i.e., 0 or 1, to each possible history; hence there are $2^{2^{m1}}$ or $2^{2^{m2}}$ possible strategies for Group 1 or Group 2, respectively, which form full strategy space (FSS). At the beginning of the game, each agent is assigned to with s strategies and keeps them unchanged during the game. After each turn, an agent assigns one point (virtual) to a strategy which would have predicted the correct outcome; otherwise, the agent assigns zero to the strategy. For agents in Group 1, they will reward their strategies with one point if they are in the majority; for agents in Group 2, they will reward their strategies with one point if they are in the minority. The virtual points represent the satisfactory levels that agents have for their strategies. Agents collect the virtual points for their strategies over the time horizon T_1 or T_2, and they use their strategies which have the highest virtual points in each turn. If there are two strategies which have the highest virtual points, agents use coin toss to decide which strategy to be used.

In the behaviour-evolutionary mix-game, we introduce two new parameters: one is threshold of winning rate and the other is time interval of behavioral change. The parameter of the time interval denoted as F for both groups is to regulate a certain periodic time interval for agents to change their investment behaviours. If an agent's decision is correct, this agent will be rewarded with one point, which is accumulated as his/her real winning points at each turn. The winning rate is calculated as

$$\text{Winning rate} = \text{Real winning points/Current timestep}, \tag{1}$$

where 'Current timestep' means the number of time steps elapsed since the simulation starts. Q_1 and Q_2 denote the thresholds of the winging rates of agents in Group 1 and Group 2, respectively. If agents' winning rates are smaller than the thresholds, they join the other group and copy the best strategies and the corresponding virtual points of the newly-joining group but keep their real winning points. Therefore, N_1 and N_2 vary with time-steps but N remains the same in the behaviour-evolutionary mix-game.

Excess demand $D[(t)]$ is equal to the number of 1s (buy) that agents choose minus the number of 0s (sell) that agents choose, i.e.,

$$D[(t)] = n_{\text{buying}-\text{orders}}[t-1] - n_{\text{selling}-\text{orders}}[t-1]. \tag{2}$$

According to a widely accepted assumption that excess demand exerts an influence on the price of the

asset, and the change of price is proportional to the excess demand in a financial market,[5] the time series of price of the asset $P[(t)]$ can be calculated based on the time series of excess demand as

$$P[(t)] = D[t]/\lambda + P[t-1]. \tag{3}$$

Volatility of the time series of prices is represented by the variance of the increase of price. The local volatility Vol$[t]$ is calculated at each time-step by calculating the variance of the increase of price within a small time-window d. If $\lambda = 1$ for simplicity, the increase of price is just the excess demand. Therefore, we have

$$\mathrm{Vol}[t] = \frac{1}{d}\sum_{t-d}^{t} D[t]^2 - \left(\frac{1}{d}\sum_{t-d}^{t} D[t]\right)^2. \tag{4}$$

In simulations, the distribution of initial strategies of agents is randomly uniform in FSS. Each agent has two strategies i.e., $s = 2$. In the simulation, the number of time-steps is 3000. The window length of local volatility d is 5, i.e., $d = 5$. The remaining parameters are $m_1 = 3$, $m_2 = 6$, $N = 201$, $N_1(t = 0) = 40$, $N_2(t = 0) = N - N_1 = 161$, $T_1 = 12$, and $T_2 = 60$.

3. Results and discussion

First, we discuss the results of behaviour-evolutionary mix-game with $F = 1$. Then we investigate the differences in dynamic behaviour between systems with different values of F. Finally, we present the two-phase zones of markets and non-markets for different values of F.

3.1. Results with $F = 1$

Figure 1 indicates the relations between N_1 at $t = 3000$ and the threshold of Q_2 with $F = 1$ for different values of Q_1. We can find that N_1 increases sharply at $Q_2 = 0.05$ for all values of Q_1, then increases slowly to a maximum of 200 for $Q_2 = 0.05-0.4$ with all values of Q_1. Generally speaking, N_1 decreases with the increase of Q_1 except $Q_1 = 0$. Figure 2 shows the relations between mean volatility of system and the threshold of Q_2 with $F = 1$ for different values of Q_1. From Fig. 2, we can find that mean volatilities increase sharply at $Q_2 = 0.05$ for all values of Q_1 and increase very slowly for $Q_2 > 0.05$. Except $Q_1 = 0$, mean volatilities decrease with the value of Q_1 increasing.

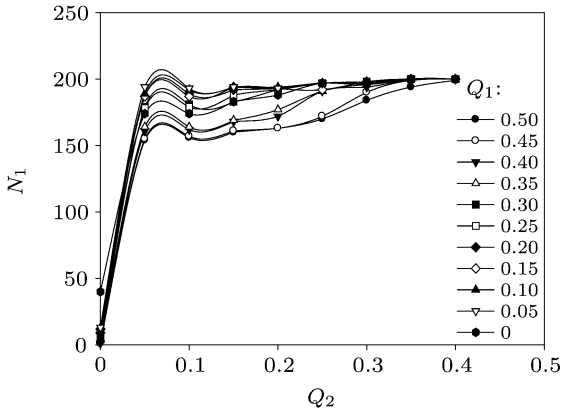


Fig. 1. Relations between N_1 ($t = 3000$) and the threshold Q_2 with $F = 1$ for different values of Q_1.

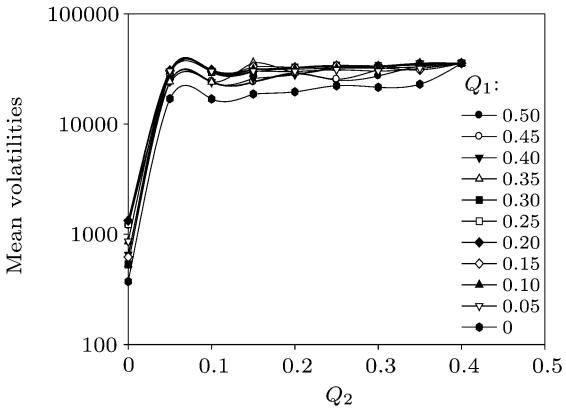


Fig. 2. Relations between mean volatility of system and the threshold Q_2 with $F = 1$ for different values of Q_1.

Comparing Fig. 1 with Fig. 2, we can naively say that mean volatility increases with the value of N_1 increasing. However, considering the situations of $Q_1 = 0$, we can find that the mean volatility is the smallest for $Q_1 = 0$, but the corresponding value of N_1 is not the smallest. So we can infer that mean volatility is also influenced by the time of behavioural evolution of agents because agents in Group 1 do not change their investment behaviours if $Q_1 = 0$. From Figs. 1 and 2, we can also infer that Q_2 has greater influence on N_1 and mean volatility than on Q_1.

3.2. Comparing results of different values of F

First, we make detailed comparisons between the results of $F = 30$ and between the results of $F = 1$ as shown in Figs. 3 and 4. Comparing Fig. 3 with Fig. 1, we find the first difference of N_1 for $F = 30$ is that N_1 does not change for Q_1=0–0.35 and Q_2=0–0.15. We

refer to this area as non-evolutionary zone as shown in Fig. 5, in which the area below the curve is non-evolutionary zone. This means that the winning rates of agents in this zone are greater than 0.35 for Group 1 and 0.15 for Group 2, respectively. Reference [12] shows a similar finding. The second difference in N_1 is that in the case of $F = 30$ N_1 increases slowly with the increase of Q_2 from 0.15–0.30 then increases rapidly from $Q_2 > 0.30$ with all values of Q_1. Comparing Fig. 4 with Figs. 3 and 2, we can conclude that mean volatility of $F = 30$ increases with the increase of N_1 and the time of behavioural evolution also, which is similar to the case of $F = 1$.

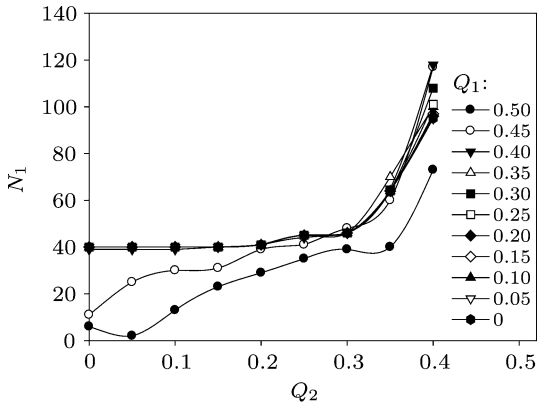


Fig. 3. Relations between N_1 ($t = 3000$) and the threshold Q_2 with $F = 30$ for different values of Q_1.

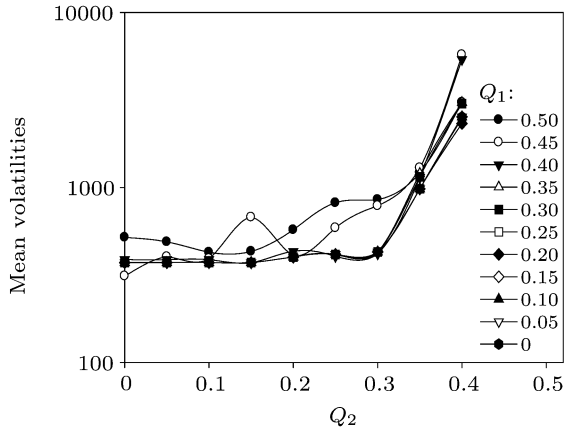


Fig. 4. Relations between mean volatility of system and the threshold Q_2 with $F = 30$ and different values of Q_1.

Then we discuss the results of $F = 10$ and 20. As shown in Fig. 5, the non-evolutionary zone in $F = 20$ is the same as that in $F = 30$ but greater than that in $F = 10$. For $F = 1$, agents do not change their investment behaviours only at the points of $Q_1 = 0$ and $Q_2 = 0$. This means that the winning rates of all agents increase with the increase of F within $F = 20$.

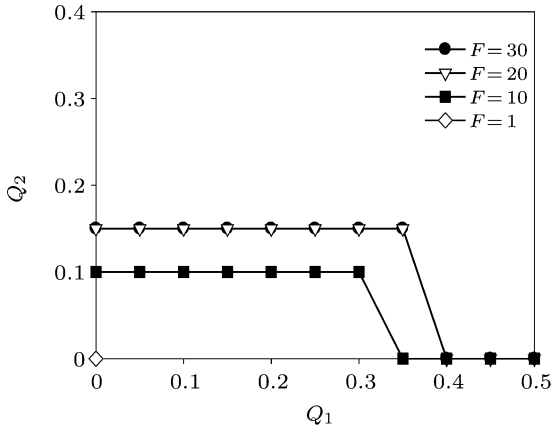


Fig. 5. Two-phase zones of evolution and non-evolution (the areas below curves represent non-evolutionary zones).

By carefully examining the simulation results, we find that system dynamic features of behaviour-evolutionary mix-game with $F = 10$ and 20 are qualitatively similar to but quantitatively different from that with $F = 30$. In order to observe the effects of F on N_1 and volatility, we pick out the typical data and plot them in Figs. 6 and 7, where shown are the variations of N_1 ($t = 3000$) and mean volatility of system with the value of F, respectively.

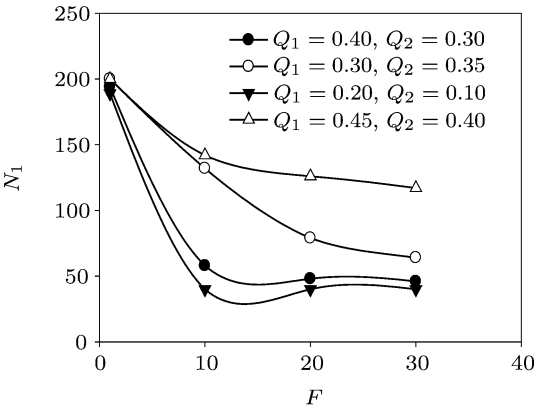


Fig. 6. Variations of N_1 ($t = 3000$) with the value of F for different values of Q_1 and Q_2.

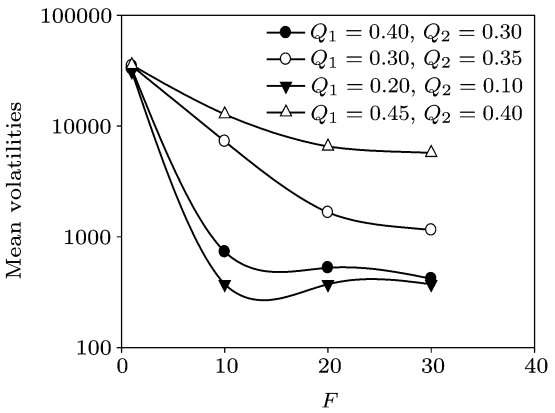


Fig. 7. Variations of mean volatility with the value of F for different values of Q_1 and Q_2.

From Figs. 6 and 7, we can find that both N_1 and the mean volatility decrease with the value of F decreasing within $F = 20$ then tend to become stable for $F > 20$.

3.3. Two-phase zones of markets and non-markets

The variations of price series and local volatility series with time shown in Fig. 8 are a typical example for non-market dynamic feature. In contrast, what is shown in Fig. 9 is a typical example for market dynamic feature.

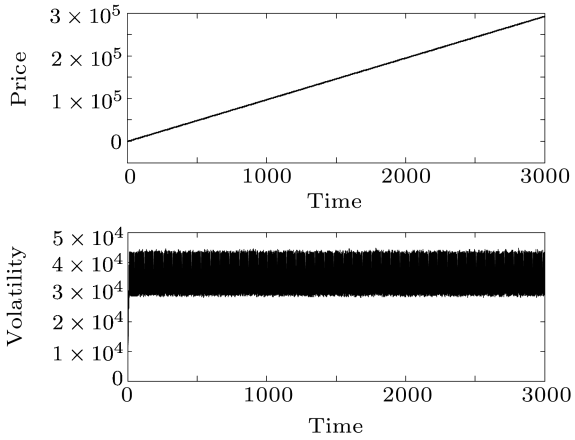


Fig. 8. Price series and local volatility series of non-markets reproduced by behaviour-evolutionary mix-game with configuration parameters of $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, $s = 2$, $Q_1 = 0.3$, $Q_2 = 0.2$, and $F = 1$.

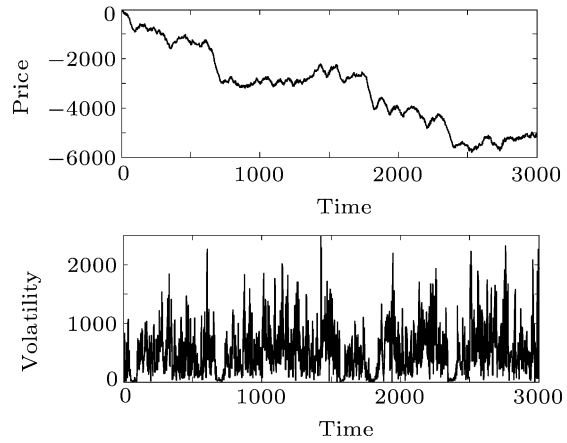


Fig. 9. Price series and local volatility series of markets reproduced by behaviour-evolutionary mix-game with configuration parameters of $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, $s = 2$, $Q_1 = 0.5$, $Q_2 = 0.1$, and $F = 10$.

Although the mix-game with the configuration parameters of $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, and $s = 2$ can reproduce the stylised facts of financial markets,[20] from the simulations we find that the behaviour-evolutionary mix-game with the same configuration parameters of $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, and $s = 2$, can reproduce the stylised facts of financial markets for some configuration parameters of Q_1 and Q_2, but it fail to do so for some other configuration parameters of Q_1 and Q_2. Figure10 shows the two-phase zones of markets and non-markets for different values of F in which the areas below the curves represent market zones. From Fig. 10, we can see that the bigger the value of F is, the greater the market zone is.

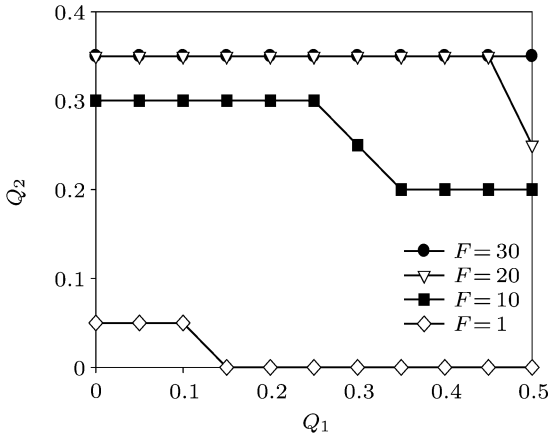


Fig. 10. Two-phase zones of markets and non-markets (the areas below the curves represent market zones).

Comparing Fig. 10 with Fig. 5, we find that the market zone is greater than the corresponding non-evolution zone for each value of F. This means that the behaviour-evolutionary mix-game can reproduce the stylised facts of financial markets if the configuration parameters are chosen within the cross areas between the zone of markets and the zone of evolutions. As an example, we plot the price series and the local volatility series reproduced by behaviour-evolutionary mix-game with the configuration parameters of $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, $s = 2$, $Q_1 = 0.5$, $Q_2 = 0.1$, and $F = 10$ as shown in Figs. 9(a) and 9(b) respectively.

4. Conclusion

The volatility of the system increases with the increase of the number of agents in Group 1 and the time of behavioural change. However, the performances of agents in both groups and the stabilities of systems become better if all agents take more time to observe their new investment behaviours. There are two-phase zones of market and non-market and two-phase zones of evolution and non-evolution. In order to simulate

financial market, we can choose configuration parameters: threshold of wining rate and the time interval within the cross areas between the zone of market and the zone of evolution.

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