An linear matrix inequality approach to global synchronisation of non-parameter perturbations of multi-delay Hopfield neural network*

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In this study, a successful linear matrix inequality approach is used to analyse a non-parameter perturbation of multi-delay Hopfield neural network by constructing an appropriate Lyapunov–Krasovskii functional. This paper presents the comprehensive discussion of the approach and also extensive applications.

Keywords: Hopfield neural network, LMI approach, global synchronisation, sliding mode control
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1. Introduction

The Hopfield neural network is a typical recurrent neural network, which has strong associative memory and optimization of computing power, so the dynamic properties of the model have received extensive attention.[1−3] The research and applications of the Hopfield neural network have penetrated into various fields and achieved rich results.[4−6] Hopfield neural networks (HNNs) were first proposed with an electrical circuit implementation as a Hopfield-type neural network.[7,8] HNNs have been extensively studied in the past few years and have found many applications in different areas such as pattern recognition, associative memory and combinatorial optimization. Many essential features of these networks, such as qualitative properties of stability, oscillation, and convergence issues have been investigated by several authors.[9,10] Hopfield neural networks with time delays (DHNNs) have been extensively investigated over the years, and various sufficient conditions for the stability of the equilibrium point of this class of neural networks have been presented in Refs. [8], [11]–[13]. Among the sufficient conditions, there are delay-independent and delay-dependent conditions. In general, the delay-independent result is simple and straightforward while a delay-dependent result is more complex but less restrictive and conservative. So far, most of these existing results for the global asymptotic stability of the DHNNs are independent of delay parameters. However, in some practical applications, system delays are non-independent, frequently fixed and their bounds are known. In addition, several famous neural networks are special cases of DHNNs, such as cellular neural networks (CNN).[14,15]

The main advantages of the sliding mode control (SMC) theory are as follows: fast response, good transient performance and robustness to variations of system parameters. Recently, many applications on the SMC approach were established.[16−18] A linear matrix inequality (LMI) technique is introduced in order to address the non-parameter perturbation multi-delay Hopfield neural networks where the delay independent and delay-dependent conditions are synchronised. The results obtained in the specified error system switching surface is globally asymptotically stable. Sliding-mode gain matrix can be obtained by solving LMI. The overall sliding mode controller is intended to ensure the specified sliding surface reachability. This study will contribute to the investigation of the synchronisation response for the drive system, and propose a model to solve the parameter mismatch problems.

The paper is organised as follows. In Section 2, system description and preliminaries are stated and some definitions and lemmas are listed. In Section 3, based on the Lyapunov stability theory, sliding mode controller for an LMI approach to global synchroni-

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sation of non-parameter perturbations of multi-delay Hopfield neural network will be designed. We give the conclusion of this study in Section 4.

2. System preliminaries and description

Considering the following non-parameter perturbations of the multi-delay Hopfield neural network model

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + T_0\boldsymbol{S}(\boldsymbol{x}(t)) + \sum_{k=1}^{K}\boldsymbol{T}_k\boldsymbol{S}(\boldsymbol{x}(t-\tau_k)) + \boldsymbol{I}, \quad (1)$$

where $\boldsymbol{x}(t) \in R^n$ is the state variable associated with neurons; $\boldsymbol{C} = \mathrm{diag}[c_1, c_2, \ldots, c_n]$, $c_i > 0$ is a diagonal matrix which indicates the feedback of neurons; $\boldsymbol{T}_0 \in R^{n\times n}$ is not associated with the delay of the interconnect $\tau_k (k = 1, 2, \ldots, K)$, $\boldsymbol{T}_K \in R^{n\times n}$ is associated with the delay of the interconnect τ_k, τ_k is the Kth delay and $0 < \tau_1 < \ldots < \tau_k < +\infty$; $\boldsymbol{I} = (I_1, I_2, \ldots, I_n) \in R^n$ is a constant input vector. The initial function of system is $\boldsymbol{x}(s) = \phi(s)$, where, $s \in [-\tau_k, 0]$, $\phi \in C([-\tau_k, 0], R^n)$, $\boldsymbol{S}(x) = [s_1(x_1), s_2(x_2), \ldots, s_n(x_n)]^{\mathrm{T}}$, $s_i(x_i)$ is activation and bounded function, and $s_i(0) = 0$.

Let (1) be the drive system. The response system is represented as follows:

$$\dot{\boldsymbol{y}}(t) = -\hat{\boldsymbol{C}}\boldsymbol{y}(t) + \hat{\boldsymbol{T}}_0\hat{\boldsymbol{S}}(\boldsymbol{y}(t)) + \sum_{k=1}^{K}\hat{\boldsymbol{T}}_k\hat{\boldsymbol{S}}(\boldsymbol{y}(t-\tau_k)) + \hat{\boldsymbol{I}} + \boldsymbol{u}(t), \quad (2)$$

where $\boldsymbol{u}(t) \in R^m$ is the control input to be designed and $\boldsymbol{y}(t) \in R^n$ is the state vector of the response system.

It is assumed that the measured output of system (1) is dependent on both the state and the delayed state with the following form:

$$\boldsymbol{z}(t) = \boldsymbol{D}\boldsymbol{x}(t) + \sum_{k=1}^{K}\boldsymbol{E}\boldsymbol{x}(t-\tau_k). \quad (3)$$

Let $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{y}(t)$, the purpose of this study is to design a controller $\boldsymbol{u}(t)$, such that $||\boldsymbol{e}(t)|| \to 0$ as $t \to +\infty$.

The error system can be obtained from (1) and (2) as follows:

$$\begin{aligned}\dot{\boldsymbol{e}}(t) = {} & -\boldsymbol{C}\boldsymbol{e}(t) + \boldsymbol{T}_0\phi(\boldsymbol{e}(t)) + \sum_{k=1}^{K}\boldsymbol{T}_k\phi(\boldsymbol{e}(t-\tau_k)) \\ & - \boldsymbol{u}(t) + (\hat{\boldsymbol{C}} - \boldsymbol{C})\boldsymbol{y}(t) - \hat{\boldsymbol{T}}_0\hat{\boldsymbol{S}}(\boldsymbol{y}(t)) \\ & - \sum_{k=1}^{K}\hat{\boldsymbol{T}}_k\hat{\boldsymbol{S}}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{T}_0\boldsymbol{S}(\boldsymbol{y}(t)) \\ & + \sum_{k=1}^{K}\boldsymbol{T}_k\boldsymbol{S}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{I} - \hat{\boldsymbol{I}}, \end{aligned} \quad (4)$$

where $\phi(\boldsymbol{e}(t)) = \boldsymbol{S}(\boldsymbol{x}(t)) - \boldsymbol{S}(\boldsymbol{y}(t))$. Now, let us formulate the problem dealt with in this study.

SMC design: for the neutral system in Eq. (1), a sliding mode controller is to be synthesized so that

(i) the sliding motion is asymptotically stable;

(ii) the state trajectory in Eq. (1) is globally driven onto the specified sliding surface, and maintained there for all subsequent time.

The following assumption is made throughout this paper.

Assumption 1 The neuron activation functions $S_i(\cdot)$ and $\hat{S}_i(\cdot)$ are Lipschitz continuous and monotonically nondecreasing. Specifically, there exist constants $\zeta_{1i} > 0$, $\zeta_{2i} > 0$ $(i = 1, 2, \ldots, n)$ such that for any $a \neq b \in R$

$$0 \leq \frac{S_i(a) - S_i(b)}{a - b} \leq \zeta_{1i}, \quad 0 \leq \frac{\hat{S}_i(a) - \hat{S}_i(b)}{a - b} \leq \zeta_{2i}.$$

For convenience, we denote $\boldsymbol{L}_1 = \mathrm{diag}(\zeta_{11}, \zeta_{12}, \ldots, \zeta_{1n})$ and $\boldsymbol{L}_2 = \mathrm{diag}(\zeta_{21}, \zeta_{22}, \ldots, \zeta_{2n})$.

3. The controller design of sliding mode

3.1. The stability analysis of sliding motion

The stability analysis problem is added to the sliding mode dynamics. Both delay-independent and delay-dependent criteria are derived so that the system is globally asymptotically stable. An LMI approach of the following theorems presents to determine the gain matrix $\boldsymbol{K}_1$.

Theorem 1 For given scalars $\tau_k > 0$ $(k = 1, 2, \ldots, K)$ and $\alpha > 0$, the sliding mode dynamics is globally asymptotically stable, if one of the following conditions is satisfied:

(i) (delay-independent condition) there exist real matrices $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, $\boldsymbol{\Lambda}_{1k} = \boldsymbol{\Lambda}_{1k}^{\mathrm{T}} > 0$, $\boldsymbol{\Lambda}_{2k} = \boldsymbol{\Lambda}_{2k}^{\mathrm{T}} > 0$, $(k = 1, 2, \ldots, K)$, $\boldsymbol{G} = \boldsymbol{P}\boldsymbol{K}_1$ and two diagonal matrices $\boldsymbol{\Gamma}_1 > 0$, $\boldsymbol{\Gamma}_2 > 0$ such that

$$\boldsymbol{\Pi}_k = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{GE} & \boldsymbol{\Phi}_{13} & \boldsymbol{PT}_k \\ * & -\boldsymbol{\Lambda}_{1k} & 0 & \boldsymbol{L}_1\boldsymbol{\Gamma}_2 \\ * & * & -2\boldsymbol{\Gamma}_1 + \boldsymbol{\Lambda}_{2k} & 0 \\ * & * & * & -\boldsymbol{\Lambda}_{2k} - 2\boldsymbol{\Gamma}_2 \end{bmatrix} < 0, \tag{5}$$

where asterisk "$*$" always denotes the symmetric block in a symmetric matrix, and

$$\boldsymbol{\Phi}_{11} = \frac{1}{\boldsymbol{K}}\sum_{k=1}^{K}(-\boldsymbol{P}(\boldsymbol{C}+\boldsymbol{K}_1\boldsymbol{D}) - (\boldsymbol{C}+\boldsymbol{K}_1\boldsymbol{D})^{\mathrm{T}}\boldsymbol{P}) + \boldsymbol{\Lambda}_{1k}, \quad \boldsymbol{\Phi}_{13} = \frac{1}{K}\sum_{k=1}^{K}(\boldsymbol{PT}_0 + \boldsymbol{L}_1\boldsymbol{\Gamma}_1);$$

(ii) (delay-dependent condition) there exist real matrices $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, $\boldsymbol{\Lambda}_{1k} = \boldsymbol{\Lambda}_{1k}^{\mathrm{T}} > 0$, $\boldsymbol{\Lambda}_{2k} = \boldsymbol{\Lambda}_{2k}^{\mathrm{T}} > 0$, $(k = 1, 2, \ldots, K)$, $\boldsymbol{R} = \boldsymbol{R}^{\mathrm{T}} > 0$, $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, $\boldsymbol{M}_3$, $\boldsymbol{M}_4$, $\boldsymbol{G} = \boldsymbol{PK}_1$ and two diagonal matrices $\boldsymbol{\Gamma}_1 > 0$, $\boldsymbol{\Gamma}_2 > 0$ such that

$$\boldsymbol{N}_k = \begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{\Psi}_{13} & \boldsymbol{\Psi}_{14} & \alpha\tau_k\boldsymbol{M}_1^{\mathrm{T}} & \boldsymbol{\Psi}_{16} \\ * & \psi_{22} & -\alpha\boldsymbol{M}_3 & \boldsymbol{\Psi}_{24} & \alpha\tau_k\boldsymbol{M}_2^{\mathrm{T}} & -\alpha\tau_k\boldsymbol{E}^{\mathrm{T}}\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{P} \\ * & * & \boldsymbol{\Psi}_{33} & 0 & \alpha\tau_k\boldsymbol{M}_3^{\mathrm{T}} & \alpha\tau_k\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P} \\ * & * & * & \boldsymbol{\Psi}_{44} & \alpha\tau_k\boldsymbol{M}_4^{\mathrm{T}} & \alpha\tau_k\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{P} \\ * & * & * & * & -\alpha\tau_k\boldsymbol{R} & 0 \\ * & * & * & * & * & \boldsymbol{\Psi}_{66} \end{bmatrix} < 0, \tag{6}$$

where

$$\begin{aligned}
\boldsymbol{\Psi}_{11} &= \frac{1}{\boldsymbol{K}}\sum_{k=1}^{K}(-\boldsymbol{P}(\boldsymbol{C}+\boldsymbol{K}_1\boldsymbol{D}) - (\boldsymbol{C}+\boldsymbol{K}_1\boldsymbol{D})^{\mathrm{T}}\boldsymbol{P}) \\
&\quad + \boldsymbol{\Lambda}_{1k} + \alpha\boldsymbol{M}_1 + \alpha\boldsymbol{M}_1^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{12} &= -\boldsymbol{PK}_1\boldsymbol{E} - \alpha\boldsymbol{M}_1^{\mathrm{T}} + \alpha\boldsymbol{M}_2, \\
\boldsymbol{\Psi}_{13} &= \frac{1}{\boldsymbol{K}}\sum_{k=1}^{K}(\boldsymbol{PT}_0 + \boldsymbol{L}_1\boldsymbol{\Gamma}_1) + \alpha\boldsymbol{M}_3, \\
\boldsymbol{\Psi}_{14} &= \boldsymbol{PT}_k + \alpha\boldsymbol{M}_4, \\
\boldsymbol{\Psi}_{16} &= -\alpha\tau_k\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P} - \alpha\tau_k\boldsymbol{D}^{\mathrm{T}}\boldsymbol{K}_1^{\mathrm{T}}\boldsymbol{P}, \\
\boldsymbol{\Psi}_{22} &= -\boldsymbol{\Lambda}_{1k} - \alpha\boldsymbol{M}_2 - \alpha\boldsymbol{M}_2^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{24} &= \boldsymbol{L}_1\boldsymbol{\Gamma}_2 - \alpha\boldsymbol{M}_4, \\
\boldsymbol{\Psi}_{33} &= -2\boldsymbol{\Gamma}_1 + \boldsymbol{\Lambda}_{2k}, \\
\boldsymbol{\Psi}_{44} &= -2\boldsymbol{\Gamma}_2 - \boldsymbol{\Lambda}_{2k}, \\
\boldsymbol{\Psi}_{66} &= -2\alpha\tau_k\boldsymbol{P} + \alpha\tau_k\boldsymbol{R}.
\end{aligned}$$

Proof Based on the Lyapunov theorem, the sliding surface's reaching condition is $\boldsymbol{S}(t)\dot{\boldsymbol{S}}(t) < 0$. If a control input can be chosen to satisfy this reaching condition, the control system will converge to the origin of the phase plane. The advantage of sliding mode control system is that it can overcome the uncertainty of the system, and has strong robustness to disturbance and unmodeled dynamics, especially has a good control effect for the control of the nonlinear system.

The dynamic behaviour of the error system (4) relies on the state $\boldsymbol{y}(t)$ of the response system (2) because of the difference between the drive and response systems. Therefore, complete synchronisation between non-identical non-parameter perturbations of multi-delay Hopfield neural networks can not be achieved only by utilising output feedback control. So an integral SMC approach is proposed to investigate the synchronisation problem of non-identical non-parameter perturbations of multi-delay Hopfield neural networks Eqs. (1) and (3). It is a designed integral sliding mode controller so that the sliding motion is globally asymptotically stable.

To utilise the information of the measured output $\boldsymbol{z}(t)$, a sliding surface is constructed as

$$\begin{aligned}
\tilde{\boldsymbol{S}}(t) &= \boldsymbol{e}(t) - \int_0^t [-\boldsymbol{Ce}(s) + \boldsymbol{T}_0\phi(\boldsymbol{e}(s)) \\
&\quad + \sum_{k=1}^{K}\boldsymbol{T}_k\phi(\boldsymbol{e}(s-\tau_k)) - \boldsymbol{K}_1(\boldsymbol{z}(s) - \boldsymbol{Dy}(s) \\
&\quad - \sum_{k=1}^{K}\boldsymbol{Ey}(s-\tau_k))]\mathrm{d}s,
\end{aligned} \tag{7}$$

where the gain matrix $\boldsymbol{K} \in R^{n\times n}$ is to be determined.

Then, from Eq. (4), one can get

$$\tilde{\boldsymbol{S}}(t) = \boldsymbol{e}(0) + \int_0^t \Big[\boldsymbol{K}_1\boldsymbol{D}\boldsymbol{e}(s) + \sum_{k=1}^{K}\boldsymbol{K}_1\boldsymbol{E}\boldsymbol{e}(s-\tau_k) - \boldsymbol{u}(s) + (\hat{\boldsymbol{C}}-\boldsymbol{C})\boldsymbol{y}(t) - \hat{\boldsymbol{T}}_0\hat{\boldsymbol{S}}(\boldsymbol{y}(t)) - \sum_{k=1}^{K}\hat{\boldsymbol{T}}_k\hat{\boldsymbol{S}}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{T}_0\boldsymbol{S}(\boldsymbol{y}(t)) + \sum_{k=1}^{K}\boldsymbol{T}_k\boldsymbol{S}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{I} - \hat{\boldsymbol{I}}\Big]\mathrm{d}s, \tag{8}$$

where $\boldsymbol{e}(0)$ is an initial condition of the error system Eq. (4).

According to the SMC theory, it is noted that $\boldsymbol{S}(t)$ varies finitely, which means that it is rational to take the time derivation of $\boldsymbol{S}(t)$: when the state trajectories of the system enter the sliding mode, we have $\boldsymbol{S}(t) = 0$ and $\dot{\boldsymbol{S}}(t) = 0$: thus from Eq. (8) and $\dot{\boldsymbol{S}}(t) = 0$ that an equivalent control rule can be designed as follows:

$$\boldsymbol{u}(t) = \boldsymbol{K}_1\boldsymbol{D}\boldsymbol{e}(s) + \sum_{k=1}^{K}\boldsymbol{K}_1\boldsymbol{E}\boldsymbol{e}(s-\tau_k) + (\hat{\boldsymbol{C}}-\boldsymbol{C})\boldsymbol{y}(t) - \hat{\boldsymbol{T}}_0\hat{\boldsymbol{S}}(\boldsymbol{y}(t)) - \sum_{k=1}^{K}\hat{\boldsymbol{T}}_k\hat{\boldsymbol{S}}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{T}_0\boldsymbol{S}(\boldsymbol{y}(t)) + \sum_{k=1}^{K}\boldsymbol{T}_k\boldsymbol{S}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{I} - \hat{\boldsymbol{I}}. \tag{9}$$

By substituting Eq. (9) into Eq. (4), the sliding mode dynamics can be obtained and described by

$$\dot{\boldsymbol{e}}(t) = -(\boldsymbol{C}+\boldsymbol{K}_1\boldsymbol{D})\boldsymbol{e}(t) - \sum_{k=1}^{K}\boldsymbol{K}_1\boldsymbol{E}\boldsymbol{e}(t-\tau_k) + \boldsymbol{T}_0\phi(\boldsymbol{e}(t)) + \sum_{k=1}^{K}\boldsymbol{T}_k\phi(\boldsymbol{e}(t-\tau_k)). \tag{10}$$

As the sliding mode can be designed and unrelated to the object parameters and disturbances, it makes sliding mode control has a fast response, corresponding to parameter variations and disturbance insensitive, and no system identification, implement easily in physics, and so on.

The role of sliding mode controller $\boldsymbol{u}(t)$ is based on the dynamic characteristics of the system expected to design the system of switching hyperplane, in attempting to make the system state close to the switching hyperplane with sliding mode controller $\boldsymbol{u}(t)$.

Because the characteristics and parameters of the system only depends on the design of the switching hyperplane, which has nothing to do with outside interference, so sliding mode control has strong robustness. Now, we proceed to the first task which analyses the stability of the sliding motion described by Eq. (10), and derive a sufficient condition by means of the LMI method. The proof of above theorem is divided into two parts. First, we prove that Eq. (10) is globally asymptotically stable under condition (i). The equivalent control rule can be designed as Eq. (9), so we design the Lyapunov function as

$$V_1(t) = \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t) + \sum_{k=1}^{K}\int_{t-\tau_k}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{\Lambda}_{1k}\boldsymbol{e}(s)\mathrm{d}s + \sum_{k=1}^{K}\int_{t-\tau_k}^{t}\phi^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{\Lambda}_{2k}\phi(\boldsymbol{e}(s))\mathrm{d}s.$$

We can get as follows:

$$\dot{V}_1(t) < \sum_{k=1}^{K}\boldsymbol{\Xi}_k^{\mathrm{T}}(t)\boldsymbol{\Pi}_k\boldsymbol{\Xi}_k(t),$$

where $\boldsymbol{\Xi}_k(t) = [\boldsymbol{e}^{\mathrm{T}}(t),\ \boldsymbol{e}^{\mathrm{T}}(t-\tau_k),\ \phi^{\mathrm{T}}(\boldsymbol{e}(t)),\ \phi^{\mathrm{T}}(\boldsymbol{e}(t-\tau_k))]^{\mathrm{T}}$, $(k = 1, 2, \ldots, K)$. It follows from Eq. (5) and $\boldsymbol{G} = \boldsymbol{P}\boldsymbol{K}_1$ and $\boldsymbol{\Pi}_k < 0$ $(k = 1,\ldots, K)$. There must exist a small scalar $\eta_k > 0$ $(k = 1, \ldots, K)$ such that $\sum_{k=1}^{K}(\boldsymbol{\Pi}_k + \mathrm{diag}\{\eta_k I, 0, 0, 0\}) \le 0$. It means that

$$\dot{V}_1(t) \le -K\delta\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) < 0,$$

where $\delta = \min_{0\le k\le K}\{\eta_k\}$. It means that the sliding mode dynamics Eq. (10) is globally asymptotically stable.

For case (ii), the purpose is to show that Eq. (10) is globally asymptotically stable, so we design Lyapunov function as follows:

$$V_2(t) = \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t) + \sum_{k=1}^{K}\int_{t-\tau_k}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{\Lambda}_{1k}\boldsymbol{e}(s)\mathrm{d}s + \sum_{k=1}^{K}\int_{t-\tau_k}^{t}\phi^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{\Lambda}_{2k}\varphi(\boldsymbol{e}(s))\mathrm{d}s + \alpha\sum_{k=1}^{K}\int_{-\tau_k}^{0}\int_{t+\sigma}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\sigma. \tag{11}$$

It is not difficult to derive that $\dot{V}_2(t) < 0$ for any $\boldsymbol{Z}_k(t) \neq 0$. It means that the sliding mode dynamics Eq. (10) is globally asymptotically stable. The proof is completed.

For the above results, the Ref. [18] also pointed out the corresponding conclusions, but our method can solve multi-delay compared with the result of Ref. [18], which is the proof idea of this study also. The principle of this paper's LMI approach is the same as Young's inequality, their purpose is to obtain the Lyapunov function less than zero with the space base under the influence of negative definite matrix. But this paper's method is more easily applied to practice.

Theorem 2 For given scalars $\tau_k > 0$ ($k = 1, 2, \ldots, K$) and $\alpha > 0$, the sliding mode dynamics (10) is globally asymptotically stable, if one of the following conditions is satisfied:

(i) (delay-independent condition) there exist real matrices $\boldsymbol{\Pi}_k$ shown by Eq. (5) as

$$\boldsymbol{\Pi} = \begin{bmatrix} \boldsymbol{\Pi}_1 & 0 & 0 & 0 & 0 & 0 \\ * & \boldsymbol{\Pi}_2 & 0 & 0 & 0 & 0 \\ * & * & \ddots & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Pi}_k & 0 & 0 \\ * & * & * & * & \ddots & 0 \\ * & * & * & * & * & \boldsymbol{\Pi}_K \end{bmatrix} < 0.$$

(ii) (delay-dependent condition) there exist real matrices $\boldsymbol{N}_k$ shown by Eq. (6) as

$$\boldsymbol{N} = \begin{bmatrix} \boldsymbol{N}_1 & 0 & 0 & 0 & 0 & 0 \\ * & \boldsymbol{N}_2 & 0 & 0 & 0 & 0 \\ * & * & \ddots & 0 & 0 & 0 \\ * & * & * & \boldsymbol{N}_k & 0 & 0 \\ * & * & * & * & \ddots & 0 \\ * & * & * & * & * & \boldsymbol{N}_K \end{bmatrix} < 0.$$

Proof For case (i), the equivalent control rule can be designed as Eq. (9), if we let $\boldsymbol{\Xi}^{\mathrm{T}}(t)\boldsymbol{\Pi}\boldsymbol{\Xi}(t) \leq 0$, where $\boldsymbol{\Xi}^{\mathrm{T}}(t) = [\boldsymbol{\Xi}_1^{\mathrm{T}}(t), \boldsymbol{\Xi}_2^{\mathrm{T}}(t), \ldots, \boldsymbol{\Xi}_k^{\mathrm{T}}(t), \ldots, \boldsymbol{\Xi}_K^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\boldsymbol{\Xi}_k(t) = [\boldsymbol{e}^{\mathrm{T}}(t), \boldsymbol{e}^{\mathrm{T}}(t-\tau_k), \phi^{\mathrm{T}}(\boldsymbol{e}(t)), \phi^{\mathrm{T}}(\boldsymbol{e}(t-\tau_k))]^{\mathrm{T}}$, $(k = 1, 2, \ldots, K)$. We can easily get the following:

$$\dot{V}_1(t) \leq \sum_{k=1}^{K} \boldsymbol{\Xi}_k^{\mathrm{T}}(t)\boldsymbol{\Pi}_k\boldsymbol{\Xi}_k(t) < 0.$$

For case (ii), if we let $\boldsymbol{Z}^{\mathrm{T}}(t)\boldsymbol{N}\boldsymbol{Z}(t) < 0$, we can easily get the following:

$$\dot{V}_2(t) \leq \sum_{k=1}^{K} \boldsymbol{Z}_k^{\mathrm{T}}(t)\boldsymbol{N}_k^1\boldsymbol{Z}_k(t) < 0,$$

where $\boldsymbol{N}_k^1 = \boldsymbol{\Sigma}_1 + \alpha\tau_k\boldsymbol{\Sigma}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{\Sigma}_2 + \alpha\tau_k\boldsymbol{\Sigma}_3^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\Sigma}_3$, $(k = 1, 2, \ldots, K)$. This completes the proof.

The global asymptotical stability of the sliding mode dynamics Eq. (10) is checked to the delay-independent and delay-dependent conditions in theorem 1. Though it is known that the delay-independent condition is usually more conservative than the delay-dependent one, the delay-independent condition can be applied to an arbitrarily large time delay because it is irrespective of the size of time delay, especially when the time delay is small. For the delay-dependent condition (ii), slack variables $\boldsymbol{M}_i$ $(i = 1, 2, 3, 4)$ are introduced to reduce its conservatism.

3.2. Reachability analysis

In order to guarantee the reachability of the specific switching surface, we design a suitable SMC law as follows.

Theorem 3 Consider the error system (4), assume that the sliding function is given by Eq. (7) with $\boldsymbol{G} = \boldsymbol{P}\boldsymbol{K}_1$, appeared in Eq. (5) or (6). Let $\varepsilon > 0$ be a constant scalar, if the SMC law is designed as follows:

$$\boldsymbol{u}(t) = \boldsymbol{K}_1\boldsymbol{D}\boldsymbol{e}(t) + \sum_{k=1}^{K} \boldsymbol{K}_1\boldsymbol{E}\boldsymbol{e}(t-\tau_k) - \omega(t)\mathrm{sgn}(\boldsymbol{S}(t))$$

with the switching gain $\omega(t)$ being taken as

$$\begin{aligned} \omega(t) = & -\varepsilon - [\|\hat{\boldsymbol{C}} - \boldsymbol{C}\| - \|\hat{\boldsymbol{T}}_0\|\|\hat{\boldsymbol{S}}\| - \|\boldsymbol{T}_0\|\|\boldsymbol{S}\|]\|\boldsymbol{y}(t)\| \\ & - \sum_{k=1}^{K} \{[\|\hat{\boldsymbol{T}}_k\|\|\hat{\boldsymbol{S}}\| + \|\boldsymbol{T}_k\|\|\boldsymbol{S}\|]\|\boldsymbol{y}(t-\tau_k)\|\} \\ & - \|\boldsymbol{I} - \hat{\boldsymbol{I}}\|. \end{aligned}$$

The trajectories of the error system can be globally driven onto the sliding surface $\boldsymbol{S}(t) = 0$.

Proof It can be shown that the reachability with probability of the switching surface $\boldsymbol{S}(t) = 0$ is guaranteed by the following SMC law, it follows from Eq. (8) that

$$\begin{aligned} \dot{\hat{\boldsymbol{S}}}(t) = & (\hat{\boldsymbol{C}} - \boldsymbol{C})\boldsymbol{y}(t) - \hat{\boldsymbol{T}}_0\hat{\boldsymbol{S}}(\boldsymbol{y}(t)) \\ & - \sum_{k=1}^{K} \hat{\boldsymbol{T}}_k\hat{\boldsymbol{S}}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{T}_0\boldsymbol{S}(\boldsymbol{y}(t)) \\ & + \sum_{k=1}^{K} \boldsymbol{T}_k\boldsymbol{S}(\boldsymbol{y}(t-\tau_k)) + \boldsymbol{I} - \hat{\boldsymbol{I}} + \omega(t)\mathrm{sgn}(\boldsymbol{S}(t)). \end{aligned}$$

To analyse the reachability, we choose the Lyapunov function candidate as follows:

$$V_3(t) = \frac{1}{2}\tilde{\boldsymbol{S}}^{\mathrm{T}}(t)\tilde{\boldsymbol{S}}(t),$$

then we can get $\dot{V}_3(t) < 0$, where $\tilde{\boldsymbol{S}}(t) \neq 0$, for all subsequent time, the trajectory of the error system (4) is globally driven onto the specified sliding surface and maintained, thus the proof is completed.

The designed SMC law can be regarded as an output error feedback control plus a high-gain compensation law which is used to cope with the effect brought by the difference between the nonidentical non-parameter perturbations of multi-delay Hopfield neural network.

We see that the bound of $\boldsymbol{S}(t)$ is required to synthesise the SMC law. In general, it is difficult to obtain the exact knowledge of the bound in practical application.

Similarly, we can get the following theorem.

Theorem 4 Consider the error system Eq. (4), assume that the sliding function is given by Eq. (7) with $\boldsymbol{G} = \boldsymbol{P}\boldsymbol{K}_1$ appeared in Eq. (5) or (6). If the SMC law is designed as follows:

$$\begin{aligned}\boldsymbol{u}(t) = {} & \boldsymbol{K}_1\boldsymbol{D}\boldsymbol{e}(t) + \sum_{k=1}^{K}\boldsymbol{K}_1\boldsymbol{E}\boldsymbol{e}(t-\tau_k) \\ & - \boldsymbol{\Xi}^{\mathrm{T}}(t)\boldsymbol{\Omega}\boldsymbol{\Xi}(t)(\boldsymbol{S}(t))^{-1} - \xi\end{aligned}$$

with the switching gain $\boldsymbol{\Omega} = \mathrm{diag}\{-l_1, -l_2, -l_3, -l_4, -m_1, \ldots, -m_K, -n_1, \ldots, -n_K\}$, $\boldsymbol{\Xi}(t) = [\tilde{\boldsymbol{S}}^{\mathrm{T}}(t), \boldsymbol{y}^{\mathrm{T}}(t), \boldsymbol{S}^{\mathrm{T}}(\boldsymbol{y}(t)), \hat{\boldsymbol{S}}^{\mathrm{T}}(\boldsymbol{y}(t)), \boldsymbol{S}^{\mathrm{T}}(\boldsymbol{y}(t-\tau_1)), \ldots, \boldsymbol{S}^{\mathrm{T}}(\boldsymbol{y}(t-\tau_K)), \hat{\boldsymbol{S}}^{\mathrm{T}}(\boldsymbol{y}(t-\tau_1)), \ldots, \hat{\boldsymbol{S}}^{\mathrm{T}}(\boldsymbol{y}(t-\tau_K))]$, l_1, l_2, l_3, l_4, $m_1, \ldots, m_K$, $n_1, \ldots, n_K$, ξ are large enough positive constants. The trajectories of the error system can be globally driven onto the sliding surface $\boldsymbol{S}(t) = 0$.

The principle of the theorem 4 is similar to the theorem 3, but the former is easily applied in practice.

4. Conclusions

In this study, the synchronisation problem has been studied for a non-parameter perturbation of multi-delay Hopfield neural network. Based on the LMI technique, a convenient solution can be obtained if the interior-point method in MATLAB environment is used. However, the conclusions of a lot of existed researches given by the system convergence criterion are algebraic inequality, there is no systematic way to verify the correctness of these inequalities and these documents are difficult to apply in practice. Therefore, the conclusions given by this study are more general.

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