Feedback control of two-headed Brownian motors in flashing ratchet potential

Zhao A-Ke(赵阿可)[a)†], Zhang Hong-Wei(张红卫)[b)c)], and Li Yu-Xiao(李玉晓)[b)]

[a)] College of Physics and Electric Information, Luoyang Normal University, Luoyang 471022, China
[b)] School of Physics and Engineering, Zhengzhou University, Zhengzhou 450052, China
[c)] Department of Physics, Zhengzhou Normal University, Zhengzhou 450044, China

(Received 31 March 2010; revised manuscript received 21 June 2010)

We presented a detailed investigation on the movement of two-headed Brownian motors in an asymmetric potential under a feedback control. By numerical simulations the direct current is obtained. The current is periodic in the initial length of spring. There is an optimal value of the spring constant. And the dependence of the current on the opposing force is reversed. Then we found that when the change of the temperature and the opposing force have optimal values, the Brownian motors can also obtain the optimal efficiency.

Keywords: Brownian motors, ratchet, current, efficiency
PACC: 0540, 0250, 0520D

1. Introduction

Molecular motors are proteins or protein complexes which function as transporting engines in biological cells. To further study the property of Brownian motors, researchers have been seeking the physical mechanism of molecular motors. For example, some authors suggested that a position-dependent temperature-driven Brownian motor is studied in the presence of entropic and energy barriers.[1] Others put forward a simplified tether model which models the tether between motor and its cargo as a symmetric linear potential.[2] From the models which have been established, the two-headed Brownian motors model has particularly attracted tremendous attention, and the rectification of thermal fluctuation is becoming a major research topic in nonequilibrium statistical mechanics.[3] Roughly speaking, most of these Brownian ratchets work by introducing periodic or random time-dependent perturbation in an asymmetric system,[4,5] and all the control strategies are open-loop control methods.[6−8] But recently, the behaviours of many kinds of Brownian motors have been explained by a feedback-controlled perturbation. In some controllable experiments, the particles can be monitored in some systems, so a feedback control could be implemented in these systems. For example, the effect of a time delay in the implementation of feedback control in a closed-loop collective flashing ratchet has been studied.[9]

In this model, the positions of the two Brownian motors are monitored and the flashing ratchet potential can be switched on or off, depending on the positions of the two particles. Under this controlled condition, the single particle and an ensemble of particles had been studied in detail in Ref. [10]. In this paper, we focus on the two-headed Brownian motors. We study the current and the efficiency of the system which consists of two coupled particles connected via a spring.

2. The model

In the controlled situation, the dynamics of the system can be described by the Langevin equations as follows:

$$\gamma\dot{x}_1(t) = \alpha(t)F(x_1(t)) - k(x_1 - x_2 - a) + \zeta_1(t), \quad (1)$$

$$\gamma\dot{x}_2(t) = \alpha(t)F(x_2(t)) + k(x_1 - x_2 - a) + \zeta_2(t), \quad (2)$$

where $\zeta_1(t)$ and $\zeta_2(t)$ are Gaussion random noise with zero mean $< \zeta_i(t) >= 0$ $(i = 1, 2)$ and correlation $< \zeta_i(t)\zeta_j(t') >= 2D_{ij}\delta(t - t')$, $D = kT$ is the intensity of white noise; γ is the friction coefficient, k is the spring coefficient, and a is the initial length of the spring. The positions of particle 1 and particle 2 are $x_1(t)$ and $x_2(t)$ respectively.

$$F(x_i(t)) = -\frac{\mathrm{d}}{\mathrm{d}x_i}U(x_i),$$

†Corresponding author. E-mail: zzukeer@163.com
© 2010 Chinese Physical Society and IOP Publishing Ltd

where $U(x_i)$ is the asymmetric periodic potential which is taken as

$$U(x_i) = -\frac{1}{2\pi}\left[\sin(2\pi x_i) + \frac{1}{4}\sin(4\pi x_i)\right].$$

Finally, $\alpha(t)$ is a control parameter, which can take on the values 1 (attached state) and 0 (detached state) according to

$$\begin{aligned}\alpha(t) &= \Xi(f(t)),\\ f(t) &= \frac{1}{2}(F(x_1(t)) + F(x_2(t))),\end{aligned} \tag{3}$$

where $\Xi(f(t))$ is the Heaviside function. If $f(t) \geq 0$, then $\alpha(t)$ is set to 1 and if $f(t) < 0$, then $\alpha(t)$ is set to 0. Therefore, $\alpha(t)$ changes with the positions of the two coupled particles. Due to the feedback control, the state of the two particles are not independent.

3. Numerical simulation

3.1. No external opposing force

In order to find the situation of the two-headed Brownian motors under the feedback control, we study the current $\langle v\rangle$ of them. We use the following method to calculate: for $s = 3000$, the two-headed Brownian motors are at the same beginning spot to average; use 5000 time steps ($\Delta t = 0.01$) to separate the time, and use t_j to express the time corresponding to each time step. After using stochastic Runge–Kutta algorithms[11] to separate the time and displacement, we can calculate the position $x_i(t_j)$ of each Brownian motor at each time step $x_j = (1/s)\sum_{i=1}^{s} x_i(t_j)$. So we can obtain the current of the two-headed Brownian motors, according to the formula $\langle v\rangle = \lim\limits_{t\to\infty} < x(t) - x(t_0) > /(t - t_0)$.[12]

3.1.1. $\langle v\rangle$ vs a

We concentrate on the investigation of the average velocity $\langle v\rangle$ and the efficiency E as functions of various parameters of the model. In order to derive the interesting phenomena, we select some optimal values of the parameters. During the movement of the two-headed Brownian motors, a control of feedback is made in which the flashing of the asymmetry potential is determined by the position of the two heads. The average velocity as a function of the initial length of the spring is shown in Fig. 1, from which we can find that the currents of the two-headed Brownian motors show a periodic change when D is less than 0.2; moreover, the change of the temperature affects the current greatly. If the initial length of the spring reaches a value that approximately equals an integer times of the period of the potential, the two particles are trapped in the minimum of the potential, and the particles cannot escape from it easily, so the average velocity of the two coupled particles reaches a minimum. Furthermore, when D is zero, there is a very small current in this system due to lack of thermal fluctuation. When D is larger than 0.1, the change of the current is affected slightly by the initial length of the spring. We find that in order to obtain a larger current, there should be a coordination between the thermal fluctuation and the potential.

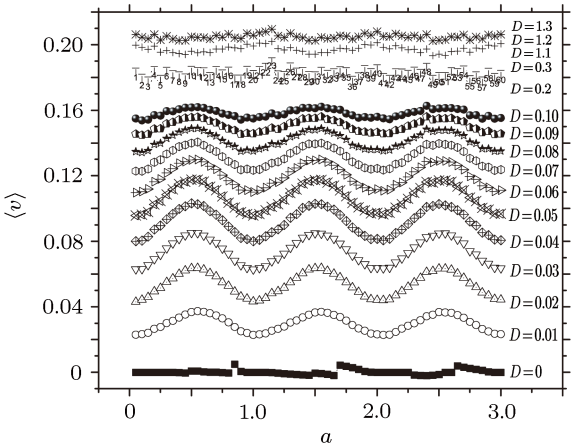


Fig. 1. The current $\langle v\rangle$ as a function of initial length a of spring for k=1.5 and different temperatures.

3.1.2. $\langle v\rangle$ vs k

First, we consider the two limiting values of the spring constant k. When k is zero, the two particles are in complete freedom and there is no relation between them. While, when $k \to \infty$, the two particles are equivalent to one particle. The current of the two coupled particles as a function of the spring coefficient is shown in Fig. 2. There is an optimal value of spring coefficient for the current when D is less than 0.08, and the current is so sensitive to the change of temperature that the current changes dramatically when the temperature varies slightly.

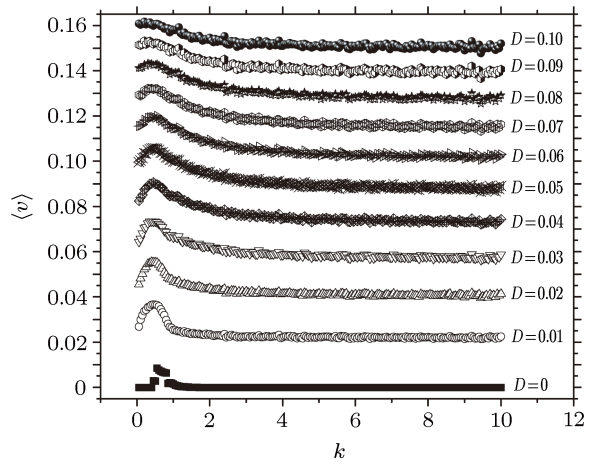
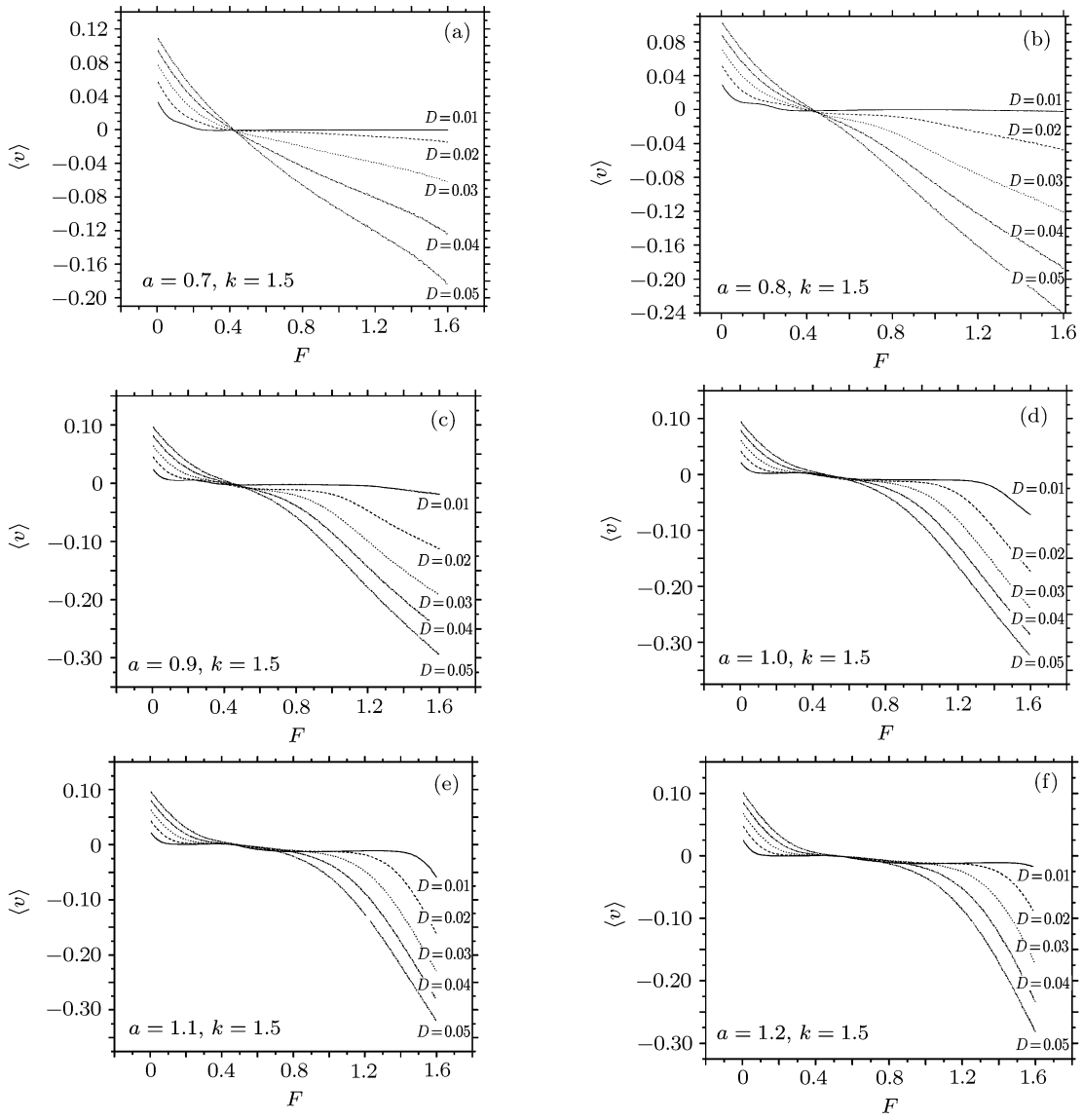


Fig. 2. The current $\langle v\rangle$ as a function of spring coefficient k for $a = 0.95$.

3.2. The effect of external opposing force

3.2.1. $\langle v\rangle$ vs F

When an external opposing force is applied ($F \neq 0$), we investigate the relationship between the current and the force. As shown in Fig. 3, we discover that the current decreases nolinearly with the increase of the opposing force. When the external opposing force is large enough, the current will be reversed. For small F, due to the large noise, the two-coupled particles can be pulled or pushed to climb up the energy barriers easily, so the velocity decreases with the decrease of D. When F is larger than a certain value, the current is reversed and becomes larger with the increase of D because of the common effect of the noise and the external opposing force.



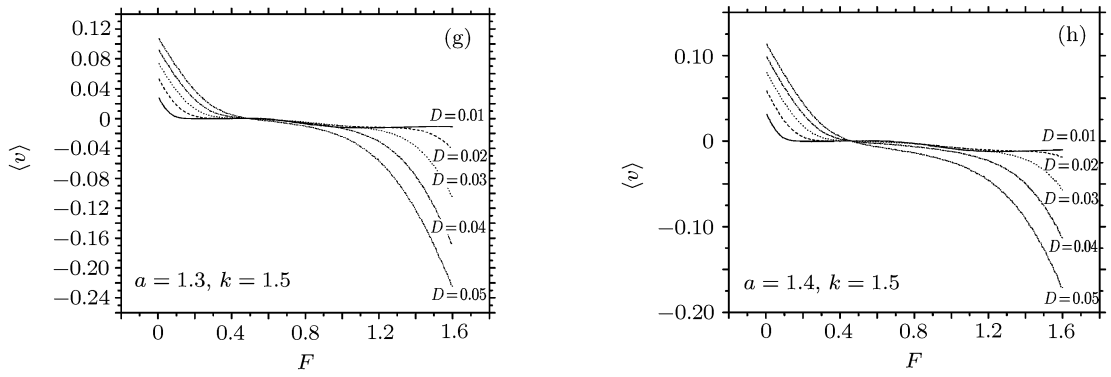


Fig. 3. The current $\langle v \rangle$ as a function of external force F.

3.2.2. The efficiency *E*

As an external opposing force is applied, the Brownian motor will do work against the opposing force and the input energy is given through potential transitions. The expression of the efficiency of the two-coupled particles is $E = E_{\rm out}/E_{\rm in}$, in which $E_{\rm out} = F\langle v \rangle$ is the work against external opposing force, $E_{\rm in}$ is the energy input by potential transition.

By a suitable choice of the system parameters, we can obtain two results: the efficiency can be expressed as a function of the temperature and the external opposing force, as shown in Figs. 4 and 5 respectively.

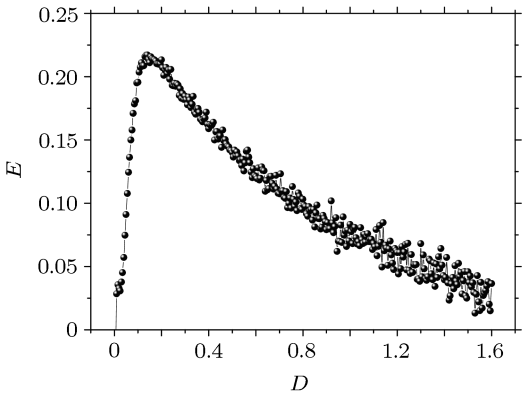


Fig. 4. The efficiency as a function of temperature D for $a = 0.95$, $k = 1.5$, $F = 0.32$.

There is an optimal value of temperature in Fig. 4. With the increase of temperature, thermal fluctuation dominates the potential, so the efficiency decreases and approaches zero, as is consistent with the fact of biology that the molecular motor produces the biggest biological activity at an optimal temperature. The efficiency as a function of the external opposing force is shown in Fig. 5. In this situation, that is to say in order to obtain a larger efficiency, a balance between the thermal fluctuation and the external opposing force must be properly chosen. When F is very small, the efficiency is also small; while with the increase of F, the efficiency begins to increase, then arrives at a peak when F is at a certain value. When F goes on increasing, the efficiency decreases until it reaches zero.

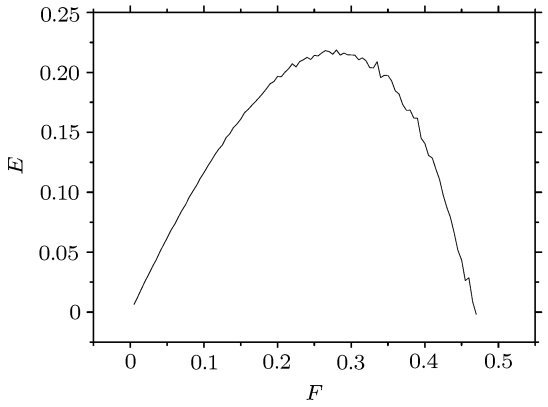


Fig. 5. The efficiency as a function of the external force F for $a = 0.95$, $k = 1.5$, $D = 0.12$.

4. Conclusion

In conclusion, we have mainly studied the movement of two-headed Brownian motors, which are in the asymmetric potential under a feedback control. By numerical simulations the direct current is obtained. In this respect, our result agrees with those of the investigation of other physical ratchet models by others.[7,8,13] The dependence of the current on the initial length of spring, spring constant and opposing force is discussed. It is shown that the current is periodic in the initial length of spring. There is an optimal value of spring constant. And the dependence of the current on the opposing force is reversed. The efficiency of the Brownian motor is evaluated. There are optimal values of temperature and the opposing force to obtain the better efficiency.

References

[1] Liu J L and He J Z 2010 *Chin. Phys.* B **19** 030504

[2] Li F Z and Jiang L C 2010 *Chin. Phys.* B **19** 020503

[3] Reimann P 2002 *Phys. Rep.* **361** 57

[4] Ajdari A and Prost J 1992 *C.R. Acad. Sci.*, Ser. IIb: *Mec. Phys. Chim. Astron.* **315** 1635

[5] Astumian R D and Bier M 1994 *Phys. Rev. Lett.* **72** 1766

[6] Li Y X, Wu X Z and Zhuo Y Z 2000 *Mod. Phys. Lett.* B **14** 479

[7] Akito Igarashi, Shinji Tsukamoto and Hiromichi Goko 2001 *Phys. Rev.* E **64** 051908

[8] Csahok Z, Family F and Vicsek T 1997 *Phys. Rev.* E **55** 5197

[9] Craig E M, Long B R, Parrondo J M R and Linke H 2008 *Europhys. Letter.* **81** 10002

[10] Franciso J Cao, Luis Dinis and Juan M R Parrondo 2004 *Phys. Rev. Lett.* **93** 040603

[11] Rebecca L Honeycutt 1992 *Phys. Rev.* A **45** 600

[12] Wang H Y and Bao J D 2004 *Physica* A **337** 13

[13] Wang H Y and Bao J D 2005 *Physica* A **357** 373