Massive particle radiation from Gibbons–Maeda black hole*

Fang Heng-Zhong(方恒忠)†

Department of Physics, China University of Petroleum, Dongying 257061, *China*

(Received 12 April 2010; revised manuscript received 14 May 2010)

This paper investigated the massive particle radiation from Gibbons–Maeda black hole by using a semi-classical method. The calculations showed that, if the self-gravitation of the radiated particle is taken into account, the radiation spectrum deviates from exact black body spectrum and the rate of tunneling equals precisely the exponent of the difference of the black hole entropies before and after emission. The conclusion supports the viewpoint of information conservation.

Keywords: Hawking radiation, massive particle, Gibbons–Maeda black hole
PACC: 0470

1. Introduction

In a space–time background with an event horizon, the quantum effect of fields leads to the radiation known as Hawking radiation.[1,2] The famous calculation of Hawking radiation by using a semi-classical approximation showed that black holes have an exact thermal spectrum which is associated with the surface gravity of the black hole,[1,3] and this seems to combine quantum mechanics and gravitation to produce thermodynamics. There are also viewpoints that the near horizon phenomena such as the Hawking radiation and the reflection off the potential barrier outside the horizon have a conformal field theory interpretation in terms of one- and two-point functions in the boundary Liouville field theory.[4]

Thermal radiation, sourced at the black hole event horizon, is emitted into the surrounding space with the consequence that the semi-classical black hole slowly loses its mass and eventually evaporations. At the precise location of the event horizon the Hawking radiation is black body radiation. However, this radiation still has to traverse a non-trivial, curved space–time geometry before it eventually reaches an observer located at asymptotic infinity and is detected. The surrounding space–time works as a potential barrier for the radiation, giving a deviation from the black body radiation spectrum as seen by an asymptotic observer. Parikh and Wilczek[5,6] believed that black hole radiation can be regarded as particle tunneling, but there is not a barrier existing before the particle tunnels out of the black hole. Instead, the barrier is created by the outgoing particle itself. In this paper, we study the massive particle radiation from Gibbons–Maeda (GM)[7] dilaton black hole. The massless particle radiation from GM black hole has been calculated in different methods.[8−10] The radiation of massive particle from Schwarzschild–de Sitter were investigated by Zhang and Zhao.[11] To obtain the velocity of a massive particle, the authors used the Laudau's condition of the clock synchronisation and obtain an unreasonable result that the velocity of the massive particle is independent of its own energy. In our paper, we obtained the velocity of the particle by solving the normalised equation of the 4-velocity, then using Wenzel–Kramers–Brillouin (WKB) approximation method we obtained the radiation spectrum from GM black hole. The result shows that if the back reaction of the radiated particle is taken into account, the emission spectrum will not be precisely thermal, but yields corrections. The conclusion supports the viewpoint of information conservation.

2. Massive particle radiation from GM black hole

The line element of GM black hole is described as[7]

*Project supported by the National Natural Science Foundation of China (Grant No. 10847166).
†Corresponding author. E-mail: qdfang@163.com
© 2010 Chinese Physical Society and IOP Publishing Ltd

$$\begin{aligned} \mathrm{d}s^2 = & -\frac{(r-r_+)(r-r_-)}{R^2}\mathrm{d}t^2 \\ & + \frac{R^2}{(r-r_+)(r-r_-)}\mathrm{d}r^2 \\ & + R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right), \end{aligned} \tag{1}$$

where $R^2 = r^2 - D^2$, r_+ and r_- are the event horizon and the inner horizon, respectively, satisfying

$$r_\pm = M \pm \sqrt{M^2 + D^2 - P^2 - Q^2}, \tag{2}$$

and the axion dilaton charge D is

$$D = \frac{P^2 - Q^2}{2M}, \tag{3}$$

the parameters P and Q represent the black hole magnetic charge and the electric charge, respectively.

It is easy to find that one of the components of the metric in Eq. (1) is singular at $r = r_+$. In order to describe the particle tunneling across the event horizon, we need a coordinate system in which the components of the metric and inverse metric do not diverge at both sides of the horizon. Performing such a coordinate transformation as

$$\mathrm{d}T = \mathrm{d}t + \frac{R^2\sqrt{\dfrac{r_+}{r}}}{(r-r_+)(r-r_-)}\mathrm{d}r, \tag{4}$$

we obtain that in the new coordinate system (T, r, θ, φ), the line element of GM space–time is expressed as

$$\begin{aligned} \mathrm{d}s^2 = & -\frac{(r-r_+)(r-r_-)}{R^2}\mathrm{d}T^2 + 2\sqrt{\frac{r_+}{r}}\mathrm{d}T\mathrm{d}r \\ & + \frac{R^2}{r(r-r_-)}\mathrm{d}r^2 + R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right). \end{aligned} \tag{5}$$

The new coordinate system (T, r, θ, φ) has many attractive features. First, each component of the metric and inverse metric is well behaved at location $r = r_+$; this is important for the calculation of the particle tunneling across the horizon. Second, in the global space–time there exists a Killing vector $(\partial/\partial T)^a$, so we can construct the constant-time slices which are asymptotically Euclidean flat spaces.

Because the metric is spherical symmetry, regarding the outgoing particle as an s-wave i.e. a shell of energy is acceptable. Suppose that the outgoing wave be traced back towards the horizon, its wave-length, measured by local observers, would be blue-shifted, near the horizon, the radial wave number approaches infinity, so that the geometrical optics limit, or WKB approximation is valid. In the semi-classical limit, according to the WKB approximation, the s-wave of the outgoing positive energy particle can be expressed as $\Psi(r) = \mathrm{e}^{\mathrm{i}\,I(r)}$, and the tunneling probability Γ takes the form

$$\Gamma \sim |\Psi(r)|^2 = \exp(-2\,\mathrm{Im}\,I), \tag{6}$$

where I is the classical action, and can be obtained as

$$I = \int L\mathrm{d}t = \int p_\mathrm{r}\mathrm{d}r - \int H\mathrm{d}t, \tag{7}$$

in which L and H are respectively the Lagrangian and the Hamiltonian of the particle, p_r is the radial momentum.

The total energy (the Arnowitt–Deser–Misner mass M) of GM space–time is conserved during the emission, and this demands that, when a particle is radiated, the left mass of the black hole decreases, so the radius of the horizon contracts from its original radius r_i to a new, smaller radius r_f, and the scale of the contraction depends on the energy of the radiated particle, the separation between r_i and r_f is just the barrier to be tunneled.

Because H in Eq. (7) is real, it has no contribution to the amplitude of Γ, so the second term of the right hand side of Eq. (7) is ignored in the following discussion; then we have

$$I = \int_{r_\mathrm{i}}^{r_\mathrm{f}} p_\mathrm{r}\mathrm{d}r = \int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_0^{p_\mathrm{r}} \mathrm{d}p_\mathrm{r}'\mathrm{d}r. \tag{8}$$

Changing variable from momentum to energy by using Hamilton's equation $\mathrm{d}H/\mathrm{d}p\,|_r = \partial H/\partial p = \dot{r}$, we obtain

$$I = \int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_0^{H}\frac{\mathrm{d}H'}{\dot{r}}\mathrm{d}r = \int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{M_\mathrm{i}}^{M_\mathrm{f}}\frac{\mathrm{d}M}{\dot{r}}\mathrm{d}r, \tag{9}$$

where M_i and M_f are respectively the initial and final masses of the black hole.

Since the trajectory followed by a massive particle is not light-like, it does not obey the equation of radial null geodesic. But for a material particle, its 4-velocity has unit length. In relativity, the tangent vector U^a to a time-like curve parameterised by the proper time τ is called the 4-velocity, it is

$$U^a = \left(\frac{\partial}{\partial\tau}\right)^a. \tag{10}$$

In coordinate system (r, t, θ, φ), the energy–momentum 4-vector, p^a, of the particle with 'rest mass' m is defined by

$$p^a = mU^a, \tag{11}$$

and its covariant form (the dual vector) is

$$p_a = g_{ab}p^b = mg_{ab}U^b. \tag{12}$$

For a material particle moves in the space–time described by Eq. (1), the 'time component' of p_a is

$$p_0 = mg_{0\nu}U^\nu = mg_{00}U^t = -E', \tag{13}$$

where $g_{00} = -\dfrac{(r-r_+)(r-r_-)}{R^2}$, and E' is recognised to be the energy of the particle. From Eq. (13), we have

$$U^t = \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{-E'}{mg_{00}} = \frac{-E}{g_{00}}, \tag{14}$$

where E is the particle's energy (per unit mass).

According to the definition of the proper time τ, the 4-velocity has unit length, i.e.

$$U^aU_a = -1. \tag{15}$$

From Eq. (1), Eq. (15) can be written as

$$g_{00}\left(U^t\right)^2 + g_{11}\left(U^r\right)^2 = -1. \tag{16}$$

From Eqs. (14) and (16) we have

$$U^r = \frac{\mathrm{d}r}{\mathrm{d}\tau} = \sqrt{g_{00}+E^2}, \tag{17}$$

then

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{U^r}{U^t} = \frac{g_{00}\sqrt{g_{00}+E^2}}{-E}. \tag{18}$$

From Eq. (4) we obtain

$$\frac{\mathrm{d}T}{\mathrm{d}t} = 1 + \frac{R^2\sqrt{\dfrac{r_+}{r}}}{(r-r_+)(r-r_-)}\frac{\mathrm{d}r}{\mathrm{d}t}. \tag{19}$$

So we obtain the radial velocity in coordinate system (r, T, θ, φ)

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}T} = \frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}T} = \frac{g_{00}\sqrt{g_{00}+E^2}}{-E-\sqrt{\dfrac{r_+}{r}}\sqrt{g_{00}+E^2}}$$

$$= \frac{-(r-r_+)(r-r_-)\sqrt{g_{00}+E^2}}{R^2\left(-E-\sqrt{\dfrac{r_+}{r}}\sqrt{g_{00}+E^2}\right)}. \tag{20}$$

From the equation above, we find that the velocity of a massive particle in curved space–time is related to its own energy E, this is reasonable. This result is quite different from that obtained in Ref. [11].

Thus from Eq. (9), for the massive particle tunneling, the action

$$\begin{aligned} I &= \int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{M_\mathrm{i}}^{M_\mathrm{f}}\frac{\mathrm{d}M}{\dot{r}}\mathrm{d}r \\ &= \int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{M_\mathrm{i}}^{M_\mathrm{f}}\frac{R^2\left(-E-\sqrt{\dfrac{r_+}{r}}\sqrt{g_{00}+E^2}\right)}{-(r-r_+)(r-r_-)\sqrt{g_{00}+E^2}}\mathrm{d}M\mathrm{d}r \\ &= \int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{M_\mathrm{i}}^{M_\mathrm{f}}\frac{H(r)}{(r-r_+)}\mathrm{d}M\mathrm{d}r, \end{aligned} \tag{21}$$

where

$$H(r) = -\frac{R^2\left(-E-\sqrt{\dfrac{r_+}{r}}\sqrt{g_{00}+E^2}\right)}{(r-r_-)\sqrt{g_{00}+E^2}}.$$

It is easy to find that in the integral domain $r_\mathrm{i} \to r_\mathrm{f}$, the integrand has a pole at $r = r_+$. According to the Feynman prescription, the integral can be evaluated by deforming the contour around the pole. So we obtain the imaginary part of the action

$$\mathrm{Im}\, I = -\pi\int_{M_\mathrm{i}}^{M_\mathrm{f}} H(r_+)\,\mathrm{d}M\mathrm{d}r. \tag{22}$$

Because $g_{00} = 0$ at $r = r_+$, and the particle energy $E > 0$, we have $H(r_+) = \dfrac{2R^2}{(r_+ - r_-)}$. Then

$$\begin{aligned} \mathrm{Im}\, I &= -\pi\int_{M_\mathrm{i}}^{M_\mathrm{f}}\frac{2\left(r_+^2 - D^2\right)}{(r_+ - r_-)}\mathrm{d}M \\ &= -\int_{M_\mathrm{i}}^{M_\mathrm{f}} K(M)\,\mathrm{d}M, \end{aligned} \tag{23}$$

where

$$K(M) = \pi\frac{\left(M^2 - MD - Q^2 + M\sqrt{(M-D)^2 - 2Q^2}\right)}{\sqrt{(M-D)^2 - 2Q^2}}. \tag{24}$$

The integrand in Eq. (23) is quite complicate, it is not easy to integrate directly. Expanding $K(M)$ at the near field of M_i as a Taylor series

$$K(M) = K(M_\mathrm{i}) + K'(M_\mathrm{i})(M - M_\mathrm{i}) + \ldots, \tag{25}$$

we have

$$\operatorname{Im} I = K\left(M_{\mathrm{i}}\right)\omega - \frac{K^{'}\left(M_{\mathrm{i}}\right)}{2}\omega^{2} + \ldots, \tag{26}$$

where $\omega = M_{\mathrm{i}} - M_{\mathrm{f}}$ is the energy of the radiated particle. Then the tunnelling rate

$$\Gamma \sim \exp\left(-2\operatorname{Im} I\right) = \exp\left\{-2\left[K\left(M_{\mathrm{i}}\right)\omega - \frac{K^{'}\left(M_{\mathrm{i}}\right)}{2}\omega^{2} + \ldots\right]\right\}. \tag{27}$$

If only taking the first term while neglect the higher order terms in the expression (27), then we have

$$\Gamma \sim \exp\left[-2K\left(M_{\mathrm{i}}\right)\omega\right]. \tag{28}$$

We find that the the rate of tunneling, as is expected, does take the form of the Boltzmann factor $\exp\left(-\beta\omega\right)$ with the inverse of temperature $\beta \equiv 1/T = 2K\left(M\right)$. This is a familiar result and agrees with Hawking's theory of thermal radiation, so we confirm that Hawking radiation can be viewed as a process of particle tunneling. But if we take into account the higher order terms, then the emission spectrum is not precisely thermal, while yields corrections. This is an exciting news, because whether the information is lost during the black hole radiation depends in part on whether the spectrum is exactly thermal. That the spectrum is not thermal may open the way of looking for information-carrying correlations in the spectrum.

For Gibbons–Maeda black hole, the Bekenstein–Hawking entropy S is[12−14]

$$S = \pi\left(r_{+}^{2} - D^{2}\right). \tag{29}$$

So

$$\frac{\partial S}{\partial M} = \pi\left[2r_{+}\frac{\partial r_{+}}{\partial M} - 2D\frac{\partial D}{\partial M}\right] = \frac{2\pi\left(M^{2} - MD - Q^{2} + M\sqrt{\left(M-D\right)^{2} - 2Q^{2}}\right)}{\sqrt{M^{2} + D^{2} - P^{2} - Q^{2}}}. \tag{30}$$

Comparing Eq. (30) with Eq. (24), we are surprised to find that $\partial S/\partial M = 2K\left(M\right)$. So we obtain

$$\begin{aligned}\operatorname{Im} I &= -\frac{1}{2}\int_{M_{\mathrm{i}}}^{M_{\mathrm{f}}}\frac{\partial S}{\partial M}\mathrm{d}M \\ &= -\frac{1}{2}\int_{S_{\mathrm{i}}}^{S_{\mathrm{f}}}\mathrm{d}S = -\frac{1}{2}\Delta S.\end{aligned} \tag{31}$$

Then the rate of emission satisfies

$$\Gamma \sim \exp\left(-2\operatorname{Im} I\right) = \exp\left(\Delta S\right). \tag{32}$$

This is a familiar result which can be obtained by many other ways at the premise of information conservation.[15,16] For example, from quantum mechanics, the rate is expressed (Feynmn Golden-law) as

$$\Gamma\left(i \to f\right) = \left|M_{\mathrm{fi}}\right|^{2} \cdot (\text{phase space factor}), \tag{33}$$

where the first term on the right-hand side is the square of the amplitude for the process. The phase space factor is obtained by summing over final states and averaging over initial states. Since the number of initial (final) states is just the exponent of the initial entropy. So

$$\Gamma \sim \frac{\mathrm{e}^{S_{\mathrm{f}}}}{\mathrm{e}^{S_{\mathrm{i}}}} = \mathrm{e}^{\Delta S}. \tag{34}$$

The identity of Eqs. (34) and (31) implies that the radiation spectrum carries the microscopic information of the black hole, i.e. the information is conservational.

3. Conclusion

The calculation of the massive particle radiation from GM black hole gives the conclusion that the back reaction of the radiated particle causes the radiation spectrum to deviate from exact thermal one, and the tunneling rate satisfies $\Gamma \sim \exp\left(-2\operatorname{Im} I\right) = \exp\left(\Delta S\right)$. The result supports the viewpoint of information conservation.

References

[1] Hawking S W 1975 *Commun. Math. Phys.* **43** 199
[2] Hawking S W 1974 *Nature* **248** 30
[3] Zhou S W, Liu B, Huang J L and Liu W B 2010 *Chin. Phys.* B **19** 010403
[4] Solodukhin S N 2004 *Phys. Rev. Lett.* **92** 061302
[5] Parikh M K 2004 *Gen. Rel. Grav.* **36** 2419
[6] Parikh M K and Wilczek F 2000 *Phys. Rev. Lett.* **85** 5042
[7] Gibbons G W and Maeda K 1988 *Nucl. Phys.* B **298** 741
[8] Ren J 2008 *Chin. Phys. Lett.* **25** 1579
[9] Ren J and Zhao Z 2007 *Int. J. Theor. Phys.* **46** 3109
[10] Li G Q 2007 *Theore. Math. Phys.* **151** 586
[11] Zhang J and Zhao Z 2005 *Nucl. Phys.* B **725** 173
[12] Gao C J and Shen Y G 2002 *Phys. Rev.* D **65** 084043
[13] He F and Zhao F 2009 *Acta Phys. Sin.* **58** 740 (in Chinese)
[14] Jing J L and Yan M L 2000 *Chin. Phys.* **9** 389
[15] Massar S and Parentani R 2000 *Nucl. Phys.* B **575** 333
[16] Kraus P and Wilczek F 1995 *Nucl. Phys.* B **437** 231