Scheme for teleportation of unknown single qubit state via continuous variables entangling channel*

Wang Zhong-Jie(王中结)†, Zhang Kan(张 侃), and Fan Chao-Yang(范朝阳)

Department of Physics, Anhui Normal University, Wuhu 241000, *China*

(Received 13 April 2010; revised manuscript received 18 May 2010)

A new scheme for quantum teleportation of single quantum bit state with using continuous variables entangling channel is presented. In our scheme two entangled light fields are employed. An outstanding characteristic of this scheme is that one atomic state is transmitted directly to another atom without using the third atom as the mediate.

Keywords: teleportation, entanglement state, quantum measurement
PACC: 0367, 4250

1. Introduction

Over the past few years, quantum teleportation, originally proposed by Bennett *et al.* in 1993, has aroused much interest due to its important applications in quantum calculation[1] and quantum communication.[2−10] In the original scheme, quantum teleportation is such a process that an unknown quantum state is transmitted from a sender to a receiver through a quantum channel with the help of some classical information. Since then, teleportation schemes for a variety of quantum states have been proposed, such as superposition of atom states or atomic Dicke states,[11,12] W states,[13] partially entangled states,[14] field states,[15−18] etc. The quantum channel is, generally speaking, composed of entangled qubits such as entangled Bell state,[2] three-particle entangled state,[7] etc. In Ref. [19] was investigated the teleportation based on dissipative channel. In Ref. [10] was proposed a scheme for quantum teleportation without quantum entanglement channel, which is based on a single photon detection technique. Experimental realization of quantum teleportation has also been demonstrated in optical system,[6] nuclear magnetic resonance,[20] and trapped ions.[21]

In the present letter, we give a new scheme for quantum teleportation of single qubit state using continuous variables entangling channel. In this scheme we employ two entangled light fields. By the interaction of atoms with light field in cavity, we can transmit one atomic state directly to another atom without using the third atom as a medium.

2. Model

We consider a three-level atom interacting with a single-mode cavity field and driven by a classical field. The atomic states are denoted by $|g\rangle$, $|e\rangle$, and $|i\rangle$. The transition frequency between states $|e\rangle$ and $|i\rangle$ is highly detuned from the cavity frequency, and thus the state $|i\rangle$ is not affected during the atom–cavity interaction. The Hamiltonian ($(\hbar = 1)$) is[9]

$$H = \frac{\omega_0}{2}\sigma_z + \omega_c a^+ a + g(\sigma_+ a + \sigma_- a^+) + \Omega(\sigma_+ \mathrm{e}^{-\mathrm{i}\,\omega t} + \sigma_- \mathrm{e}^{\mathrm{i}\,\omega t}), \quad (1)$$

where $\sigma^+ = |e\rangle\langle g|$, $\sigma^- = |g\rangle\langle e|$, and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, with $|e\rangle$ and $|g|\rangle$ being the excited state and the ground state of the atom, a^+ and a are the creation operator and the annihilation operator for the cavity mode, g is the atom–cavity coupling strength, and Ω is the Rabi frequency of the classical field. We assume that $\omega_0 = \omega$. Then the interaction Hamiltonian, in the interaction picture, is

$$H_{\mathrm{I}} = g(\mathrm{e}^{-\mathrm{i}\,\delta t}\sigma_+ a + \mathrm{e}^{\mathrm{i}\,\delta t}\sigma_- a^+) + \Omega(\sigma_+ + \sigma_-), \quad (2)$$

where $\delta = \omega_{\mathrm{c}} - \omega_0$ is the detuning between the atomic transition frequency and cavity frequency, here we set $\delta = 0$. We make a unitary transformation $H'_{\mathrm{I}} = TH_{\mathrm{I}}T^+ = \mathrm{e}^{-\mathrm{i}\,\pi\sigma_y/4}H_{\mathrm{I}}\mathrm{e}^{\mathrm{i}\,\pi\sigma_y/4}$, the transformed interaction Hamiltonian is represented as

$$H'_{\mathrm{I}} = \frac{g}{2}a^+(\sigma_z - \mathrm{i}\,\sigma_y) + \frac{g}{2}a(\sigma_z + \mathrm{i}\,\sigma_y) + \Omega\sigma_z. \quad (3)$$

*Project supported by the Natural Science Foundation of Anhui Province, China (Grant No. 090412060), and the Natural Science Foundation of the Education Committee of Anhui Province, China (Grant No. KJ2008A029).
†Corresponding author. E-mail: wuliwzj@mail.ahnu.edu.cn
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Making use of the rotating wave approximation, which is equivalent to the transformation $\mathrm{e}^{-\mathrm{i}\,2\Omega t\sigma_z}H_{\mathrm{I}}'\mathrm{e}^{-\mathrm{i}\,2\Omega t\sigma_z}$, the transformed interaction Hamiltonian H_{I}' becomes

$$H_{\mathrm{I}}' = \frac{g}{2}(a^+ + a)\sigma_z - \frac{\mathrm{i}\,g}{2}(\sigma_+\mathrm{e}^{\mathrm{i}\,2\Omega t} - \sigma_-\mathrm{e}^{-\mathrm{i}\,2\Omega t}) + \Omega\sigma_z. \tag{4}$$

Assuming that $2\Omega >> g$, we can neglect the fast oscillating terms. Then H_{I}' reduces to

$$H_{\mathrm{I}}' = \frac{g}{2}(a^+ + a)\sigma_z + \Omega\sigma_z. \tag{5}$$

Go on making the anti-transformation, $H_{\mathrm{I}} = T^+H_{\mathrm{I}}'T$, then we will obtain

$$H_{\mathrm{I}} = \frac{g}{2}(a^+ + a)\sigma_x + \Omega\sigma_x. \tag{6}$$

The evolution operator for H_{I} is represented as

$$U = \mathrm{e}^{-\mathrm{i}\,tH_{\mathrm{I}}} = \mathrm{e}^{-\mathrm{i}\,\Omega t\sigma_x}\mathrm{e}^{-\mathrm{i}\,gt(a^+ + a)\sigma_x/2}. \tag{7}$$

3. Teleportation of single qubit state

Suppose that the quantum channel is composed of two light cavities, which are prepared in the entangled state

$$|\psi\rangle_{\mathrm{c}} = (|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2)/\sqrt{N}, \tag{8}$$

where N is normalized constant, and α is assumed to be a real number. This kind of light entanglement state has been prepared theoretically.[22] Now there are two atoms 1 and 2, of which atom 1 belongs to a sender Alice, atom 2 to a receiver Bob. We assume the state of atom 1 to be teleported by Alice to Bob is an unknown quantum state

$$|\psi\rangle_1 = (a|e\rangle_1 + b|g\rangle_1), \tag{9}$$

where a and b are unknown coefficients, and $|a|^2 + |b|^2 = 1$. The initial state of the whole system is given by

$$|\psi(0)\rangle = \frac{1}{\sqrt{N}}(a|e\rangle_1 + b|g\rangle_1) \otimes |g\rangle_2 \otimes (|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2). \tag{10}$$

Step 1 Alice and Bob send atoms 1 and 2 into their cavities, respectively, at the same time, each atom being driven by a classical field. The evolution operator of each atom and cavity field is given by Eq. (7). By selecting the interacting time and the parameters Ω and g to satisfy the condition $\Omega t = m\pi$, and $gt/2 = \alpha$, with m being a large positive even integer, we can obtain the evolution of the system as follows:

$$\begin{aligned}|\psi(t)\rangle &= U_1U_2|\psi(0)\rangle = \mathrm{e}^{-\mathrm{i}\,\Omega t\sigma_{x1}}\mathrm{e}^{-\mathrm{i}\,gt(a_1^+ + a_1)\sigma_{x1}/2}\mathrm{e}^{-\mathrm{i}\,\Omega t\sigma_{x1}}\mathrm{e}^{-\mathrm{i}\,gt(a_2^+ + a_2)\sigma_{x2}/2}|\psi(0)\rangle \\ &= \mathrm{e}^{-\mathrm{i}\,g_1t(a_1^+ + a_1)\sigma_{x1}}\mathrm{e}^{-\mathrm{i}\,gt(a_2^+ + a_2)\sigma_{x2}}|\psi(0)\rangle,\end{aligned} \tag{11}$$

where the same coupling parameters Ω and g between atoms and light fields have been used. Computing Eq. (11) carefully, we can obtain

$$\begin{aligned}|\psi(t)\rangle = {} & \frac{1}{2\sqrt{N}}[a(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2) + b(|g\rangle_1|g\rangle_2 + |e\rangle_1|e\rangle_2)]|0\rangle_1|0\rangle_2 \\ & + \frac{1}{4\sqrt{N}}(a-b)(|e\rangle_1 - |g\rangle_1)(|g\rangle_2 - |e\rangle_2)|2\alpha\rangle_1|2\alpha\rangle_2 \\ & + \frac{1}{4\sqrt{N}}(a+b)(|e\rangle_1 + |g\rangle_1)(|g\rangle_2 + |e\rangle_2)|-2\alpha\rangle_1|-2\alpha\rangle_2 \\ & + \frac{1}{4\sqrt{N}}(a-b)(|e\rangle_1 - |g\rangle_1)(|g\rangle_2 + |e\rangle_2)|2\alpha\rangle_1|0\rangle_2 \\ & + \frac{1}{4\sqrt{N}}(a+b)(|e\rangle_1 + |g\rangle_1)(|g\rangle_2 - |e\rangle_2)|0\rangle_1|2\alpha\rangle_2 \\ & + \frac{1}{4\sqrt{N}}(a-b)(|e\rangle_1 - |g\rangle_1)(|g\rangle_2 + |e\rangle_2)|0\rangle_1|-2\alpha\rangle_2 \\ & + \frac{1}{4\sqrt{N}}(a+b)(|e\rangle_1 + |g\rangle_1)(|g\rangle_2 - |e\rangle_2)|-2\alpha\rangle_1|0\rangle_2.\end{aligned} \tag{12}$$

We define the even and the odd coherence states as

$$|\varphi^{\pm}\rangle_i = (|2\alpha\rangle_i \pm |-2\alpha\rangle_i)/\sqrt{N_1}, \quad (i = 1, 2), \tag{13}$$

where N_1 is normalized constant. The state of the whole system is then represented in terms of $|\varphi_i^{\pm}\rangle$ by

$$\begin{aligned}|\psi(t)\rangle = & \frac{1}{2\sqrt{N}}[a(|e\rangle_1|g\rangle_2+|g\rangle_1|e\rangle_2)+b(|g\rangle_1|g\rangle_2+|e\rangle_1|e\rangle_2)]|0\rangle_1|0\rangle_2\\ & +\frac{N_1}{8\sqrt{N}}[|e\rangle_1(a|g\rangle_2+b|e\rangle_2)+|g\rangle_1(b|g\rangle_2+a|e\rangle_2)|\varphi^+\rangle_1|\varphi^+\rangle_2\\ & +\frac{N_1}{8\sqrt{N}}[|e\rangle_1(-b|g\rangle_2-a|e\rangle_2)+|g\rangle_1(-a|g\rangle_2-b|e\rangle_2)|\varphi^+\rangle_1|\varphi^-\rangle_2\\ & +\frac{N_1}{8\sqrt{N}}[|e\rangle_1(-b|g\rangle_2-a|e\rangle_2)+|g\rangle_1(-a|g\rangle_2-b|e\rangle_2)|\varphi^-\rangle_1|\varphi^+\rangle_2\\ & +\frac{N_1}{8\sqrt{N}}[|e\rangle_1(a|g\rangle_2+b|e\rangle_2)+|g\rangle_1(b|g\rangle_2+a|e\rangle_2)|\varphi^-\rangle_1|\varphi^-\rangle_2\\ & +\frac{\sqrt{N_1}}{4\sqrt{N}}[|e\rangle_1(a|g\rangle_2-b|e\rangle_2)+|g\rangle_1(b|g\rangle_2-a|e\rangle_2)|\varphi^+\rangle_1|0\rangle_2\\ & +\frac{\sqrt{N_1}}{4\sqrt{N}}[|e\rangle_1(-b|g\rangle_2+a|e\rangle_2)+|g\rangle_1(-a|g\rangle_2+b|e\rangle_2)|\varphi^-\rangle_1|0\rangle_2\\ & +\frac{\sqrt{N_1}}{4\sqrt{N}}[|e\rangle_1(a|g\rangle_2-b|e\rangle_2)+|g\rangle_1(b|g\rangle_2-a|e\rangle_2)|0\rangle_1|\varphi^+\rangle_2\\ & +\frac{\sqrt{N_1}}{4\sqrt{N}}[|e\rangle_1(b|g\rangle_2-a|e\rangle_2)+|g\rangle_1(a|g\rangle_2-b|e\rangle_2)|0\rangle_1|\varphi^-\rangle_2. \qquad (14)\end{aligned}$$

Step 2 Alice and Bob perform a Bell-like measurement operation on the state of their cavities and inform each other of the results. The Bell-like measurement for field states (odd and even coherent states) has been studied in Ref. [23]. After measurement, two atoms stay in the entangled states.

Step 3 After Alice's receiving the information about the operation accomplishment, Alice then performs a measurement operation on the state of atom 1, and informs Bob.

Step 4 According to Alice's measurement result, Bob then performs a unitary operation on the state of atom 2 or not, so that he obtains the wanted state.

Table 1. Measurement results and the transformations made by Bob in teleportation process.

the measurement result on light fields made by Alice and Bob	the measurement result on atom 1 made by Alice	the states of the atom 2	the ransformation operated by Bob
$\|0\rangle_1\|0\rangle_2$	$\|e\rangle_1$	$a\|g\rangle_2+b\|e\rangle_2$	σ_x
	$\|g\rangle_1$	$b\|g\rangle_2+a\|e\rangle_2$	I
$\|\varphi_1^+\rangle_1\|\varphi_2^+\rangle_2$	$\|e\rangle_1$	$a\|g\rangle_2+b\|e\rangle_2$	σ_x
	$\|g\rangle_1$	$b\|g\rangle_2+a\|e\rangle_2$	I
$\|\varphi_1^+\rangle_1\|\varphi_2^-\rangle_2$	$\|e\rangle_1$	$b\|g\rangle_2+a\|e\rangle_2$	I
	$\|g\rangle_1$	$a\|g\rangle_2+b\|e\rangle_2$	σ_x
$\|\varphi_1^-\rangle_1\|\varphi_2^+\rangle_2$	$\|e\rangle_1$	$b\|g\rangle_2+a\|e\rangle_2$	I
	$\|g\rangle_1$	$a\|g\rangle_2+b\|e\rangle_2$	σ_x
$\|\varphi_1^-\rangle_1\|\varphi_2^-\rangle_2$	$\|e\rangle_1$	$a\|g\rangle_2+b\|e\rangle_2$	σ_x
	$\|g\rangle_1$	$b\|g\rangle_2+a\|e\rangle_2$	I
$\|\varphi_1^+\rangle_1\|0\rangle_2$	$\|e\rangle_1$	$a\|g\rangle_2-b\|e\rangle_2$	$\sigma_x\sigma_z$
	$\|g\rangle_1$	$b\|g\rangle_2-a\|e\rangle_2$	σ_z
$\|\varphi_1^-\rangle_1\|0\rangle_2$	$\|e\rangle_1$	$-b\|g\rangle_2+a\|e\rangle_2$	σ_x
	$\|g\rangle_1$	$-a\|g\rangle_2+b\|e\rangle_2$	$\sigma_x\sigma_z$
$\|0\rangle_1\|\varphi_2^+\rangle_2$	$\|e\rangle_1$	$a\|g\rangle_2-b\|e\rangle_2$	$\sigma_x\sigma_z$
	$\|g\rangle_1$	$b\|g\rangle_2-a\|e\rangle_2$	σ_z
$\|0\rangle_1\|\varphi_2^-\rangle_2$	$\|e\rangle_1$	$b\|g\rangle_2-a\|e\rangle_2$	σ_z
	$\|g\rangle_1$	$a\|g\rangle_2-b\|e\rangle_2$	$\sigma_x\sigma_z$

For example, in step 2, supposing that Alice and Bob obtain the measurement result $|0\rangle_1|0\rangle_2$, the state of the whole system collapses into

$$|\psi'\rangle = a(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2) + b(|g\rangle_1|g\rangle_2 + |e\rangle_1|e\rangle_2. \quad (15)$$

In step 3, supposing that the measurement result is $|e\rangle_1$, the state of atom 2 then becomes into

$$|\psi''\rangle = a|g\rangle_2 + b|e\rangle_2. \quad (16)$$

Thus the whole teleportation process ends. The other cases are represented in Table 1.

4. Preparation of entangled coherent state

In the following we propose a scheme for the preparation of entangled coherent state. Supposing that there are a pair of distant entangled atoms (atom 1 and atom 2) and distant light cavities (cavity 1 and cavity 2), we send atom 1 and atom 2 into light cavity 1 and cavity 2, respectively. The evolution of the atom 1(2) interacting with a single-mode cavity field and driven by a classical field is governed by (see Eq. (7))

$$U_1 = \mathrm{e}^{-\mathrm{i}\,\Omega t\sigma_{x1}}\mathrm{e}^{-\mathrm{i}\,gt(a_1^+ + a_1)\sigma_{x1}/2}, \quad (17a)$$

$$U_2 = \mathrm{e}^{-\mathrm{i}\,\Omega t\sigma_{x2}}\mathrm{e}^{-\mathrm{i}\,gt(a_2^+ + a_2)\sigma_{x2}/2}. \quad (17b)$$

We select the interacting time t and the parameter Ω such that the condition $\Omega t = 2\pi$ is satisfied, and assume that the initial state of the system is $|\psi(0)\rangle = (|g\rangle_1|g\rangle_2 + |e\rangle_1|e\rangle_2) \otimes |0\rangle_1|0\rangle_2/\sqrt{2}$, i.e., two cavities are in the vacuum state and the state in which two atoms stay is the entangled state, we can obtain the state of the system after interaction time t as follows:

$$\begin{aligned}|\psi(t)\rangle &= U_1U_2|\psi(0)\rangle \\ &= \frac{1}{4\sqrt{2}}(|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2)|g\rangle_1|g\rangle_2 \\ &\quad -\frac{1}{4\sqrt{2}}(|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2)|g\rangle_1|e\rangle_2 \\ &\quad -\frac{1}{4\sqrt{2}}(|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2)|e\rangle_1|g\rangle_2 \\ &\quad +\frac{1}{4\sqrt{2}}(|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2)|e\rangle_1|e\rangle_2, \end{aligned} \quad (18)$$

where $\alpha = gt/2$. After measuring the states of the two atoms, we will obtain the entangled state of two cavity fields as follows:

$$|\psi_{\mathrm{c}}\rangle = \frac{1}{\sqrt{N}}(|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2), \quad (19)$$

where N is a normalized constant.

Finally, we compare this scheme with the other schemes. The scheme presented in this paper has some advantages: 1) the continuous variable entangling channel is used; 2) it operates without the help of any other atoms. The state of atom 1 is directly transmitted to atom 2; 3) it can be realized in experiment.

References

[1] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* (London) **390** 575
[2] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[3] .Furusawa A, Sørensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[4] Yang C P and Guo G C 1999 *Chin. Phys. Lett.* **16** 628
[5] Lee H W *Phys. Rev.* 2001 A **64** 014302
[6] Browne D E, Plenio M B and Huelga S F 2003 *Phys. Rev. Lett.* **91** 067901
[7] Rigolin G 2005 *Phys. Rev.* A **71** 032303
[8] Liu J M and Guo G C 2002 *Chin. Phys. Lett.* **19** 456
[9] Zheng S B 2004 *Phys. Rev.* A **69** 064302
[10] Bose S, Knight P L, Plenio M B and Vedral V 1999 *Phys. Rev. Lett.* **83** 5158
[11] Tiegang D, Muthukrishnan A, Scully M O and Zubairy M S 2005 *Phys. Rev.* A **71** 062308
[12] Chen Q and Fang X M 2008 *Chin. Phys.* B **17** 1587
[13] Zhan Y B 2004 *Chin. Phys.* B **13** 1801
[14] Gilad Gour 2004 *Phys. Rev.* A **70** 042301
[15] Satyabrata Adhikari, Majumdar A S and Nayak N 2008 *Phys. Rev.* A **77** 012337
[16] Marian P and Marian T A 2006 *Phys. Rev.* A **74** 042306
[17] Adesso G and Illuminati F 2005 *Phys. Rev. Lett.* **95** 150503
[18] Dell'Anno F, Siena S D, Albano L and Illuminati F 2007 *Phys. Rev.* A **76** 022301
[19] Özdemir S K, Barkiewicz K, Liu Y X and Miranowicz A 2007 *Phys. Rev.* A **76** 042325
[20] Nielsen M A, Knill E and Laflamme R 1998 *Nature* (London) **396** 52
[21] Riebe M, Häffner H, Roos C F, Hänsel W, Benhelm J, Lancaster G P T, Körber T W, Becher C, Schmidt-Kaler F, James D F V and Blatt R 2004 *Nature* (London) **429** 734
[22] Gerry G C 1996 *Phys. Rev.* A **54** R2529
[23] de Oliveira M C and Miburn G J 2002 *Phys. Rev.* A **65** 032304