Unconventional geometric phase gate and multiqubit entanglement for hot ions with a frequency-modulated field*

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We present an alternative scheme for implementing the unconventional geometric two-qubit phase gate and preparing multiqubit entanglement by using a frequency-modulated laser field to simultaneously illuminate all ions. Selecting the index of modulation yields selective mechanisms for coupling and decoupling between the internal and the external states of the ions. By the selective mechanisms, we obtain the unconventional geometric two-qubit phase gate, multiparticle Greenberger–Horne–Zeilinger states and highly entangled cluster states. Our scheme is insensitive to the thermal motion of the ions.

Keywords: a frequency-modulated field, unconventional geometric phase gate, Greenberger–Horne–Zeilinger states, cluster states

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1. Introduction

Quantum computers, which are based on the fundamental quantum mechanical principle, have the potential to perform certain computational tasks more efficiently than classical computers.[1] The basic building blocks of a quantum computer are quantum logic gates. But decoherence arising from the coupling of the system with the environment, becomes a major obstacle in realising the quantum gates. Thus quantum computation must be operated in a built-in fault-tolerant manner. One possible approach to the achievement of fault-tolerant quantum logic gates is based on the geometric phase shifts,[2] which are determined only by the path area and are robust against the exact starting state distributions, the path shape, the orientation in phase space, and the dephasing.[3−5] However, in practice, geometric phases are often accompanied by dynamic phases that destroy the potential robustness against local inaccuracies and fluctuations. To remove or avoid the dynamic phases, two types of geometric quantum gates have been proposed, and they are called conventional geometric gates[6−10] and unconventional geometric gates,[11−19] respectively. The striking character of unconventional geometric gates is that the total phase with a nonzero dynamical phase is dependent only on global geometric features. In comparison with conventional geometric gates, unconventional geometric gates do not require additional operations to cancel the dynamical phases and so this simplifies the experimental efforts.

Recently, the ion-trap system has aroused a great deal of interest because of its potential suitability for scalable quantum computation.[20,21] In the ion-trap computers, qubits are encoded in the internal states of the ions. The number of qubits is equal to that of ions, and this system is scalable to the problem size. The four-particle Greenberger–Horne–Zeilinger (GHZ) state[22] and the eight-qubit W state[23] have been realised in this system. The implementation of quantum logic gates[18,24] has also been reported in the trapped ions. But multi-qubit cluster states, which are responsible for the one-way quantum computer via single-qubit projective measurements,[25,26] have not been demonstrated in such a system yet. In addition, for the above ion-trap computation,[17] two laser fields are required to illuminate the trapped ions and the Rabi frequencies of laser fields need to be much less than the vibration frequency of the centre-of-mass mode. It is the very reason why the operation speed is restricted.

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Here we propose to implement quantum phase gate, and prepare GHZ and cluster states by utilising a single frequency-modulated laser field to illuminate the hot ions. With a judicious choice of the index of modulation, we obtain quantum operations for coupling and decoupling between the internal and external states, respectively. Based on the above quantum operations, the unconventional geometric two-qubit phase gate, multiqubit GHZ states and the highly entangled cluster states are efficiently carried out by few operations. Therefore, a different kind of one-way quantum computation can be achieved via the generated cluster states. In our scheme, the quantum operations depend only on global geometric features and are insensitive to the thermal motion of the ions, and so lead to the high-fidelity information implementation. Furthermore, there are no restrictions on the intensity of the laser field, which allows to perform the quantum information processing at a high speed.

2. Dynamical model of ion-trap system with a frequency-modulated filed

We consider N two-level ions of mass m which are confined in a linear trap. They are cooled down to very low temperatures[21−23] and may perform small oscillations around their equilibrium positions. We simultaneously illuminate all ions with a frequency-modulated travelling-wave light field. In the optical rotating-wave approximation, the total Hamiltonian of the system is written as (assuming $\hbar = 1$)[27]

$$H = H_0 + V \tag{1}$$

with

$$H_0 = \nu a^{\dagger} a + \frac{\omega_a}{2} \sum_{j=1}^{N} \sigma_z^j, \tag{2}$$

$$V = \frac{g}{2} \sum_{j=1}^{N} \left[\mathrm{e}^{-\mathrm{i}\,\omega_0 t - \mathrm{i}\,M \sin(\Omega t)} \mathrm{e}^{\mathrm{i}\,\boldsymbol{k}x} \sigma_j^{\dagger} + \mathrm{H.c.} \right], \tag{3}$$

where $\sigma_z^j = |e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|$ and $\sigma_j^{+} = |e_j\rangle\langle g_j|$ $(j = 1 - N)$, with $|e_j\rangle$ and $|g_j\rangle$ being the excited and ground states for the jth ion of the transition frequency ω_a; a and $a^{\dagger}$ are, respectively, the annihilation and the creation operators associated with the collective centre-of-mass vibrational mode of frequency ν; g is the Rabi frequency of the travelling-wave field and ω_0 is its central frequency; Ω denotes the modulating frequency and M refers to the index of modulation; $\boldsymbol{k}$ is the wave vector and x is the position operator. Within the Lamb–Dicke regime $\eta\sqrt{\bar{n}+1} << 1$, where $\eta = k\sqrt{1/2N\nu m}$ is the Lamb–Dicke parameter and $\bar{n}$ is the mean phonon number of the centre-of-mass mode, we obtain the Hamiltonian in the interaction picture as

$$H_{\mathrm{I}} = \frac{g}{2} \left\{ \mathrm{e}^{-\mathrm{i}\,M \sin(\Omega t)} \left[1 + \mathrm{i}\,\eta \left(a^{\dagger} \mathrm{e}^{\mathrm{i}\,\nu t} + a \mathrm{e}^{-\mathrm{i}\,\nu t} \right) \right] \sum_{j=1}^{N} \sigma_j^{\dagger} + \mathrm{H.c.} \right\}, \tag{4}$$

where we have set the detuning $\Delta = \omega_a - \omega_0 = 0$ and have employed the relation $kx = \eta \left(a^{\dagger} + a \right)$. By using the generating function for the Bessel's function

$$\exp\left(\mathrm{i}\, z \sin\theta\right) = \sum_{m=-\infty}^{+\infty} \mathrm{e}^{\mathrm{i}\, m\theta} \mathrm{J}_m\left(z\right), \tag{5}$$

the Hamiltonian in Eq. (4) is rewritten as

$$H_{\mathrm{I}} = \frac{g}{2} \left\{ \sum_{m=-\infty}^{+\infty} \mathrm{e}^{-\mathrm{i}\, m\Omega t} \mathrm{J}_m\left(M\right) \left[1 + \mathrm{i}\,\eta \left(a^{\dagger} \mathrm{e}^{\mathrm{i}\,\nu t} + a \mathrm{e}^{-\mathrm{i}\,\nu t} \right) \right] \sum_{j=1}^{N} \sigma_j^{\dagger} + \mathrm{H.c.} \right\}, \tag{6}$$

where $\mathrm{J}_m\left(M\right)$ is Bessel's function. In what follows we focus on the case of the modulating frequency $\Omega = \nu - \delta$ with $\delta << \nu$. Discarding the rapidly oscillating terms in Eq. (6), we have

$$H_{\mathrm{I}} = \frac{g}{2} \left[\mathrm{J}_0\left(M\right) S_x + \mathrm{i}\,\eta \mathrm{J}_1\left(M\right) \left(a^{\dagger} \mathrm{e}^{\mathrm{i}\,\delta t} - a \mathrm{e}^{-\mathrm{i}\,\delta t} \right) S_x \right]. \tag{7}$$

Here $\mathrm{J}_{-1}(M) = -\mathrm{J}_1(M)$ has been used and the collective spin operator $J_x = \sum_{j=1}^{N} \sigma_x^j$ has been introduced. Interaction Hamiltonian (7) involves the central excitation and the sideband excitation, which depend strongly on the index of modulation M. For the case of $\mathrm{J}_1(M) = 0$ with the chosen M, the sideband excitation is absent and only the central excitation is present. The internal states decouple from the external state and are not affected by the external vibration mode. In this case each ion may be manipulated independently. For the case of $\mathrm{J}_0(M) = 0$, the central excitation is eliminated and only the sideband excitation is present. In this case, the internal state transition is accompanied by the creation and the annihilation of one phonon. We first focus on the case of $\mathrm{J}_0(M_0) = 0$. The interaction Hamiltonian reduces to

$$H_{\mathrm{I}} = \mathrm{i}\, G \left(a^{\dagger} \mathrm{e}^{\mathrm{i}\,\delta t} - a \mathrm{e}^{-\mathrm{i}\,\delta t}\right) S_x, \tag{8}$$

where we have taken $G = \mathrm{J}_1(M_0)\,\eta g/2$. Following the approach in Ref. [11], we show that the unconventional geometric two-qubit phase gate can be performed when two ions are involved in the above Hamiltonian (8). In terms of the computational basis $\{|+\rangle_1|+\rangle_2, |+\rangle_1|-\rangle_2, |-\rangle_1|+\rangle_2, |-\rangle_1|-\rangle_2\}$, where $|\pm\rangle_j = (1/\sqrt{2})\,(|g\rangle_j \pm |e\rangle_j)$ are the eigenstates of $\sigma_x^j\,(j = 1, 2)$ with eigenvalues ± 1 correspondingly, Hamiltonian (8) takes the form

$$H_{\mathrm{I}} = \mathrm{i}\, G \left(a^{\dagger} \mathrm{e}^{\mathrm{i}\,\delta t} - a \mathrm{e}^{-\mathrm{i}\,\delta t}\right) \mathrm{diag}\left[\lambda_{++}, \lambda_{+-}, \lambda_{-+}, \lambda_{--}\right], \tag{9}$$

where $\lambda_{jk}\,(j, k = +, -)$ are the eigenvalues of the operator $\left(\sigma_x^1 + \sigma_x^2\right)$ with $\lambda_{++} = -\lambda_{--} = 2$ and $\lambda_{+-} = \lambda_{-+} = 0$. It is clear that the populations of the two ions will not change when the system is governed by Hamiltonian (9). Correspondingly, the evolution operator of the system is written in the form

$$U(t) = \mathrm{diag}\left[U_{++}(t), 1, 1, U_{--}(t)\right], \tag{10}$$

where

$$\begin{aligned} U_{jk}(t) &= \hat{T} \exp\left(-\mathrm{i} \int_0^t H_{jk}(\tau)\, \mathrm{d}\tau\right) \\ &= \lim_{N\to\infty} \prod_{n=1}^{N} D\left(\Delta\alpha_{jk}(\tau_n)\right). \end{aligned} \tag{11}$$

Here $\hat{T}$ is the time ordering operator, and $H_{jk}(\tau)$ are the matrix elements of Hamiltonian (9), $\Delta\tau = t/N$ is the time interval, $\Delta\alpha_{jk}(\tau_n) = \lambda_{jk} G \mathrm{e}^{\mathrm{i}\,\delta\tau_n} \Delta\tau$, and $D(\alpha)$ is the displacement operator with the form $D[\alpha(t)] = \exp\left[\alpha(t)\, a^{\dagger} - \alpha^*(t)\, a\right]$. Using the relation $D(\alpha) D(\beta) = \mathrm{e}^{\mathrm{i}\,\mathrm{Im}(\alpha\beta^*)} D(\alpha + \beta)$, we obtain

$$U_{jk}(t) = \mathrm{e}^{\mathrm{i}\,\gamma_{jk}} D\left(\int \mathrm{d}\alpha_{jk}\right), \tag{12}$$

with

$$\gamma_{jk} = \mathrm{Im}\left(\int_c \alpha_{jk}^* \mathrm{d}\alpha_{jk}\right)$$

and

$$\alpha_{jk}(t) = \int_0^t \lambda_{jk} G \mathrm{e}^{\mathrm{i}\,\delta t'}\, dt' = \frac{\lambda_{jk} G}{\mathrm{i}\,\delta}\left(\mathrm{e}^{\mathrm{i}\,\delta t} - 1\right).$$

Under the condition of $\delta T = 2n\pi$ (n is an integer), the vibration mode returns to the original state along a closed path. This gives a geometric phase conditional on the electronic states

$$U(\gamma, T) = \mathrm{diag}\left[\exp(\mathrm{i}\,\gamma_{++}), 1, 1, \exp(\mathrm{i}\,\gamma_{--})\right], \tag{13}$$

which will result in a phase gate as shown below. According to the coherent-state path integral method,[11] the total phase, the dynamic phase and the geometric phase are, respectively, given by

$$\gamma_{jk} = \gamma_{jk}^{\mathrm{d}} + \gamma_{jk}^{\mathrm{g}}, \tag{14}$$

$$\gamma_{jk}^{\mathrm{d}} = -\int_0^T H_{jk}\left(\alpha_{jk}^*, \alpha_{jk}; t\right) \mathrm{d}t, \tag{15}$$

$$\gamma_{jk}^{\mathrm{g}} = \frac{\mathrm{i}}{2} \int_0^T \left(\alpha_{jk}^* \dot{\alpha}_{jk} - \dot{\alpha}_{jk}^* \alpha_{jk}\right) \mathrm{d}t. \tag{16}$$

Using the relations

$$D^{\dagger}(\alpha_{jk})\, a^{\dagger} D(\alpha_{jk}) = a^{\dagger} + \alpha_{jk}^*(t)$$

and

$$D^{\dagger}(\alpha_{jk})\, a D(\alpha_{jk}) = a + \alpha_{jk}(t)$$

with

$$\alpha_{jk}(t) = \int_0^t \lambda_{jk} G \mathrm{e}^{\mathrm{i}\,\delta t'}\, \mathrm{d}t',$$

we obtain

$$\begin{aligned} H_{jk}\left(\alpha_{jk}^*, \alpha_{jk}; t\right) &= \langle n_0 | D^{\dagger}(\alpha_{jk})\, H_{jk}(t)\, D(\alpha_{jk}) | n_0 \rangle \\ &= \mathrm{i}\,\lambda_{jk} G \left[\mathrm{e}^{\mathrm{i}\,\delta t} \alpha_{jk}^*(t) - \mathrm{e}^{-\mathrm{i}\,\delta t} \alpha_{jk}(t)\right]. \end{aligned} \tag{17}$$

Substitution of Eq. (17) into Eq. (15) gives the dynamic phase

$$\gamma_{jk}^{\mathrm{d}} = 2\frac{\lambda_{jk}^2 G^2}{\delta}\left[T - \frac{\sin(\delta T)}{\delta}\right]. \tag{18}$$

The geometric phase is

$$\gamma_{jk}^{\mathrm{g}} = -\frac{\lambda_{jk}^2 G^2}{\delta}\left[T - \frac{\sin(\delta T)}{\delta}\right]. \tag{19}$$

Thus we obtain

$$\gamma_{jk} = \frac{1}{2}\gamma_{jk}^{\mathrm{d}} = -\gamma_{jk}^{\mathrm{g}} = 2n\pi \left(\frac{\lambda_{jk} G}{\delta}\right)^2, \tag{20}$$

where the condition of the closed-path evolution for the vibration mode $T = 2n\pi/\delta$ with an integer n has been used. Therefore, cyclic evolution operator (13) is obtained as

$$U(\gamma, T) = \mathrm{diag}[\mathrm{e}^{\mathrm{i}\,\gamma}, 1, 1, \mathrm{e}^{\mathrm{i}\,\gamma}]. \tag{21}$$

As shown in the following section, such an operation can be used to implement two-qubit phase gate.

3. Two-qubit unconventional geometric phase gate

In this section, we construct a two-qubit unconventional geometric phase gate by the above-mentioned system for two ions interacting with a frequency-modulated field. We suppose that the quantum information is encoded by the states $|\pm_j\rangle$. As mentioned above, the vibration mode does not influence the quantum operations in the present scheme. Here we assume that it is initially prepared in the vacuum state $|0\rangle$. When the displacement parameter $\mathrm{d}\alpha$ moves along a closed path, returning to the original point in the phase space, a global geometric phase will appear, which is conditional on the external states of the ions. After a closed-path evolution, the system evolves to

$$\begin{aligned}
&|+_1\rangle|+_2\rangle \to \mathrm{e}^{\mathrm{i}\,\gamma}|+_1\rangle|+_2\rangle,\\
&|+_1\rangle|-_2\rangle \to |+_1\rangle|-_2\rangle,\\
&|-_1\rangle|+_2\rangle \to |-_1\rangle|+_2\rangle,\\
&|-_1\rangle|-_2\rangle \to \mathrm{e}^{\mathrm{i}\,\gamma}|-_1\rangle|+_2\rangle.
\end{aligned} \tag{22}$$

Note that the vibration mode remains in the original state $|0\rangle$ and so we omit it. Now the single-qubit operation is performed in the dressed state basis, i.e., $|+_1\rangle \to \mathrm{e}^{-\mathrm{i}\,\gamma}|+_1\rangle$ and $|-_2\rangle \to \mathrm{e}^{\mathrm{i}\,\gamma}|-_2\rangle$, the evolution turns to

$$\begin{aligned}
&|+_1\rangle|+_2\rangle \to |+_1\rangle|+_2\rangle,\\
&|+_1\rangle|-_2\rangle \to |+_1\rangle|-_2\rangle,\\
&|-_1\rangle|+_2\rangle \to |-_1\rangle|+_2\rangle,\\
&|-_1\rangle|-_2\rangle \to \mathrm{e}^{\mathrm{i}\,2\gamma}|-_1\rangle|+_2\rangle,
\end{aligned} \tag{23}$$

which represent an unconventional two-qubit geometric 2γ-phase gate. $2\gamma = \pi$ would yield a π-phase gate.[11] Such a phase gate possesses global geometric features and is robust against certain operation errors.

It is noted that geometric operation provides a promising way for implementing built-in fault-tolerant quantum phase gate since it is robust against the parameter fluctuations. However, a practical question is whether the geometric phases are generally accompanied by dynamic ones that are sensitive to the fluctuations. One approach is to remove the dynamic phases by canceling the dynamic phases accumulated in different loops, which is named the conventional geometric phase gate. But it requires some additional operations, which would result in additional errors. To avoid the unnecessary errors, the other method is presented and is called the unconventional geometric phase gate. Compared with the former, the unconventional geometric phase gate has the following advantages. (i) It not only possesses all geometric advantages owned by the former, but also is independent of the initial states of the system. (ii) The unconventional geometric phase gate does not require additional operations to cancel the effects of dynamic phases and so this simplifies the experimental setups. The simplification of operations will be helpful for improving the fidelity. (iii) A lot of theoretical schemes for the conventional geometric phase gate have been presented, but they have not been achieved experimentally. In contrast, for the unconventional geometric phase gate, the experiments have been done in ion-trap[18] and nuclear magnetic resonance.[19] In Ref. [18], the ions were driven by two laser beams, whose frequencies were far detuned from the electronic transition frequency. In this scheme, the main sources of gate error were the fluctuation of the relative detuning between the two laser beams, and the fluctuation of the laser beam intensity. While in the present scheme, only one single frequency-modulated field is involved. The reduction in the number of the lasers decreases not only the error caused by the fluctuation of relative detuning, but also the error induced by the intensity fluctuation.

4. Multi-qubit entanglement

Now we show how to use the above ion-trap system to prepare the multi-qubit GHZ state and the cluster state. For the case of N particles, we rewrite Eq. (21) in the bare-state basis as

$$U(\gamma, T) = \exp\left(\mathrm{i}\,\gamma \mathrm{J}_x^2\right). \tag{24}$$

First, we use quantum operation (24) to prepare multi-qubit GHZ entangled state in this system. Choosing $\gamma = \pi/2$ and using the initial state of the

system $|\Psi(0)\rangle = |g_1 g_2 \ldots g_N\rangle$, we obtain the GHZ state

$$|\Psi(T)\rangle = \frac{1}{\sqrt{2}}[\mathrm{e}^{\mathrm{i}\pi/4}|g_1 g_2 \ldots g_N\rangle + \mathrm{e}^{-\mathrm{i}\pi/4}(-1)^{N/2}|e_1 e_2 \ldots e_N\rangle], \quad (25)$$

when N is even. For odd N, the unitary operation (24) does not directly produce the maximally entangled state described in Eq. (25). However, it is obtained by adding the unitary operation $U = \exp(-\mathrm{i}(\pi/2)S_x)$. This evolution is performed by appropriately selecting the index of modulation M according to Hamiltonian (7). When $\mathrm{J}_1(M_1) = 0$ is satisfied, we obtain $H_\mathrm{I} = (\mathrm{J}_0(M_1)\, g/2)S_x$ and the corresponding operation

$$U_{M=M_1}(t) = \exp\left(-\mathrm{i}\frac{\mathrm{J}_0(M_1)\, gt}{2}S_x\right).$$

Choosing $\mathrm{J}_0(M_1)\, gt_1 = \pi$, where

$$g = \sqrt{\frac{\gamma}{2n\pi}}\frac{\delta}{\eta \mathrm{J}_1(M_0)}$$

with $\gamma = \pi/2$, we obtain GHZ state for odd N.

Next, we show that the multiqubit cluster state can be produced in terms of the unconventional geometric phase shift in the present system. It is well known that the cluster state serves as a universal resource for the one-way quantum computation, which is carried out by local measurements and feedback of their outcomes.[26] The striking advantage of the one-way computing strategy lies in the fact that it separates the processes of generating entanglement and executing the computation. Owing to their novel applications in quantum computation,[26,28,29] the generation of multiqubit cluster states has received considerable attention in recent years.[30–34] In the following we focus on the preparation of the highly entangled cluster states in the proposed system. Assume that all ions are initially prepared in the ground states $|\Psi(0)\rangle = |g_1 g_2 \ldots g_N\rangle$. We first utilise a travelling-wave laser field with the index of modulation $M = M_0$ to illuminate the trapped ions for the interaction time T. As a result, the long-range interactions among the ions occur and the corresponding Ising-like unitary operation is obtained as shown in Eq. (24), which is rewritten as

$$U_{M=M_0}(\gamma, T) = \exp\left[2\gamma \sum_{k>j=1}^{N} \sigma_x^j \sigma_x^k\right]. \quad (26)$$

Apparently, the couplings among the ions happen, which are independent of the external state. Then substituting the index of modulation $M = M_2$ for $M = M_0$, we have $\mathrm{J}_1(M_2) = 0$. So Hamiltonian (7) is simplified into $H_\mathrm{I} = \mathrm{J}_0(M_2)\, gS_x/2$, and the corresponding evolution operator is given by

$$U_{M=M_2}(t) = \exp\left[-\mathrm{i}\frac{\mathrm{J}_0(M_2)\, gt}{2}S_x\right]. \quad (27)$$

Here we can see that the internal and the external states of the ions are decoupled and the internal states are not affected by the external state. The interactions among the ions do not take place and each ion can be manipulated independently. We can perform two corresponding operations $U_{M=M_0}(\gamma, T)$ and $U_{M=M_2}(t_2)$ with the interaction times T and t_2 respectively and then obtain the wanted unitary operation. Setting $|\mathrm{J}_0(M_2)|\, gt_2 = (N-1)\gamma$, we have the total evolution operation

$$U(T+t_2) = U_{M=M_2}(t_2)\, U_{M=M_0}(\gamma, T) = \exp\left[\mathrm{i}\, 8\gamma \sum_{k>j=1}^{N} \frac{1+\sigma_x^j}{2}\frac{1+\sigma_x^k}{2}\right]. \quad (28)$$

It is found that the evolution operator $U(T+t_2)$ is periodic in time and generates 'entanglement oscillations' of the ion chain. When $\gamma = l\pi/4$ with l being an integer, the entanglement disappears, while for the other cases, the entanglement takes place. In particular, for the values $\gamma = (2l+1)\pi/8$, the entanglement is maximal in some sense. In this case, the generated cluster state is

$$|\Psi_N\rangle = \frac{1}{2^{N/2}}\bigotimes_{i=1}^{N}\left[|-\rangle_i (-1)^{N-i}\prod_{j=i+1}\sigma_x^{(j)} + |+\rangle_i\right], \quad (29)$$

where $\sigma_x^{(N+1)} = 1$ and $|\pm\rangle_i = \frac{1}{\sqrt{2}}(|g\rangle_i \pm |e\rangle_i)$ as shown before. In the present protocol, the effective anisotropic direction is along the x axis, rather than the z axis as the standard Ising model.[25] Here the cluster state is expressed in terms of the basis states $|\pm\rangle_i$, namely, the eigenstates of σ_x^j. To implement one-way quantum computation, one preforms the corresponding single-qubit projective measurements on the basis states $|e\rangle_i$ and $|g\rangle_i$.

Compared with the previous schemes for generating the cluster states, our scheme has the following merits. (i) We use a single travelling-wave laser to illuminate all ions. For the previous schemes,[30] in contrast, the ions are addressed individually, which remains an experimental challenge with much stronger traps used by the National Institute of Standards and

Technology (NIST) group.[35] (ii) This scheme is insensitive to the thermal motion of the ions and so the ions do not need cooling down to their vibrational ground states. This is due to the decoupling between the internal state and the external state, shown in Eq. (24). (iii) After generating the cluster state, the travelling-wave field is switched off and the interqubit coupling is also switched off, which is convenient for implementing one-way quantum computation via the single-qubit measurement. (iv) In our scheme, the couplings among different qubits are mediated by the vibration mode and no additional operations are required. The interaction of any qubit with the vibration mode is easily switched on or off by selecting the modulation index of the frequency-modulated laser field.

5. Discussion and conclusion

Now we briefly assess the feasibility of our scheme under the current experimental techniques. Here we take $\nu = 2\pi \times 500$ kHz and $\eta = 0.1$ as shown in Ref. [17]. To implement the unconventional geometric two-qubit phase gate, we require the condition $\delta T = 2n\pi$ to be satisfied, where n is an integer and $\delta << \nu$. The required time for this process is $T = 2n\pi/\delta$. The total time needed to generate GHZ state for odd N is added up to $t_{\rm GHZ} = T + t_1$, where $\mathrm{J}_0(M_1) g_1 t_1 = \pi$. For producing cluster states, the interaction time is given by $t_{\rm cluster} = T + t_2$, where

$$|\mathrm{J}_0(M_2)| g_2 t_2 = (N-1)\frac{(2l+1)\pi}{8}$$

with l being an integer. Choose the low values of n and l such that we may perform the fast quantum information processing. Note that there are no restrictions on the intensity of the laser field in our scheme and so the condition for the low values of n and l can be reached. Setting the parameters $\delta = 0.1\nu$, $n = 1$ and $l = 0$, we have the interaction times $T = 20$ μs,

$$t_1 = \frac{2\mathrm{J}_1(M_0)}{\mathrm{J}_0(M_1)}\ \mu\mathrm{s} \quad \text{and} \quad t_2 = \frac{\mathrm{J}_1(M_0)(N-1)}{2|\mathrm{J}_0(M_2)|}\ \mu\mathrm{s}.$$

Correspondingly, the total operation times $t_{\rm GHZ}$ and $t_{\rm cluster}$ are on the order of 10^{-5} s. To achieve the quantum information with high-fidelity, we consider $^9\mathrm{Be}^+$ ions of the experiments in Refs. [18] and [22]. Two ground-state hyperfine levels $|\downarrow\rangle = |F = 2, m_F = -2\rangle$ and $|\uparrow\rangle = |F = 2, m_F = -1\rangle$ are used to store the quantum information. The coupling between them is obtained by stimulating Raman transitions, which are induced by the frequency-modulating Raman lasers. Raman transition processes may suppress the spontaneous emission. Our scheme places no restrictions on the intensity of the laser field and thus the losses due to the spontaneous emission and phonon heating can be neglected.

In summary, we have proposed an alternative scheme for the implementation of the unconventional geometric two-qubit phase gate and the preparation of multiqubit entangled states in the trapped ions. A single frequency-modulated laser field is involved in the present scheme and no restrictions are placed on the intensity of the laser field. Simply by choosing the modulation index of the laser field, we achieve the unconventional geometric two-ion quantum phase gate, the multi-ion GHZ and the cluster states. Thus our scheme is quite simple, feasible and scalable. Moreover, the quantum operations are insensitive to the thermal motion of the ions and so lead to the high-fidelity quantum information processing.

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