Two quantum oscillators coupled with a planar radio frequency ion trap*

Chen Liang(陈 亮)[a)b)c)†] and Gao Ke-Lin(高克林)[a)b)]

[a)] *State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan* 430071, *China*

[b)] *Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan* 430071, *China*

[c)] *Graduate School of the Chinese Academy of Sciences, Beijing* 100039, *China*

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In this scheme, two quantum oscillators in a planar radio frequency ion trap are coupled by the trap electrodes. The ions motional states encode the quantum bits (qubits), and a swap gate could be achieved. Under different conditions of the experiments, the intensity of the coupling between two quantum oscillators and the dissipation of the system are calculated. We compute fidelities for a quantum swap gate and discuss experimental issues.

Keywords: planar radio frequency ion trap, quantum oscillator, swap gate
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1. Introduction

Trapped ions are one of the ideal systems for quantum computation, where control of coherent quantum dynamics, state preparation, and measurement are achieved in the laboratory.[1−4] One impending question at the moment is how to build scalable ion trap quantum computing to perform quantum algorithms beyond the simple demonstration level.[5] One example is that the development of mesoscopic segmented ion traps is a key to a scalable ion trap quantum computing,[6−9] where such traps allow the moving of individual ions (qubits) between a storage and a processing unit. With these techniques, researchers have implemented quantum gates,[10,11] multiparticle entanglement[2,12,13] and even basic quantum algorithms.[14−16]

Surface electrode traps have the advantage of increased optical access over the traditional Paul traps. The interaction can be set up between neighbouring zones. Among the advantages of these 'wired' planar traps, the transmission line provides a stronger ion–ion interaction than the direct Coulomb interaction.[17] The use of electrical transmission lines as interconnecting channels for these quantum systems has been proposed previously,[18−23] assuming ideal and lossless connections between the qubit carriers, typically superconducting lines or cavities. Recently, transfer of quantum signals between solid-state qubits via a resonant superconducting transmission line has been achieved experimentally.[24,25]

In this work, the centres of the electrodes of two zones are connected by a metal transmission line. Two quantum oscillators in a planar radio frequency (RF) ion trap are coupled only by the trap electrodes. This scheme reduces the experimental difficulty and is helpful for building a scalable ion trap quantum computing. We consider that the transmission line is made of a conventional metal conductor and the characteristic frequencies of the trapped ions are far from the resonant frequencies of the line. So the ions are weakly coupled. This setting is similar to two atoms close to each other in free space. The coupling intensity between two quantum oscillators and the dissipation of the system are calculated by the method of the Green's function. Herein, in the dispersive regime, conventional conductors do not necessarily degrade the coherent dynamics of the confined ions on the timescale relevant for typical quantum manipulations.

In a planar RF trap, charged particles are confined by both the static and oscillating electric fields. The motion of a single ion in a planar RF trap can be described by two harmonic oscillators. Oscillating electric fields supply axial traps (plumbing up the electrodes), and the static electric field supplies ra-

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†Corresponding author. E-mail: smart9910@sohu.com

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dial trapping (perpendicular to the trap axis). In this work, we restrict ourselves to the axial motion of the ions.

2. System and Hamiltonian

We consider two zones connected by a transmission line as depicted in Fig. 1. Each zone stores a single ion.



Fig. 1. Two zones are connected with a connection line, which enables the induced charges to exchange energy between trapped ions. The electrodes are enbedded in a plane (black) made of isolator.

The Hamiltonian that describes the dynamics of the ions interacting via the transmission line is

$$\hat{H} = \hat{H}_{\rm a} + \hat{H}_{\rm tl} + \hat{H_{\rm int}}, \tag{1}$$

were $\hat{H}_{\rm a}$ represents the axial harmonic motion of the two trapped ions with the oscillating frequencies ω_1 and ω_2[26]

$$\begin{aligned}\hat{H}_{\rm a} &= \sum_{i=1}^{2}\left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega_i^2 z_i^2\right) \\ &= \sum_{i=1}^{2}\hbar\omega_i\left(a_i^+ a_i + \frac{1}{2}\right)\end{aligned} \tag{2}$$

with $a_i = \sqrt{m\omega_i/2\hbar}(z_i + {\rm i}\, p_i/m\omega_i)$, $\hbar$ is the reduced Planck's constant, m is the mass of the trapped ion, $a_i^+(a_i)$ is the annihilation (creation) operator for the ith ion. $\hat{H}_{\rm tl}$ is the Hamiltonian of the transmission line[27]

$$\hat{H}_{\rm tl} = \int_0^l \int_0^\infty \hbar\omega\hat{\phi^+}(y,\omega)\hat{\phi}(y,\omega){\rm d}\omega{\rm d}y, \tag{3}$$

here y is the coordinate along the line and $\hat{\phi}(y,\omega)$ and $\hat{\phi^+}(y,\omega)$ are bosonic fields that describe the electrical signals at frequency ω and position y along the lossy transmission line. They satisfy the commutation rule

$$[\hat{\phi}(y,\omega), \hat{\phi^+}(y,\omega)] = \delta(y-y')\delta(\omega-\omega'). \tag{4}$$

In both zones, the ion's motion induces a charge $Q_{1,2}$ on the electrode. This charge induces an electrical signal that propagates through the transmission line and influences the motion of the ion at the other end. The explicit expression for $H_{\rm int}$ is provided in appendix. Assuming that small-amplitude oscillations around the trap centre, the interaction Hamiltonian is

$$H_{\rm int} = \sum_{i=1}^{2}\frac{\alpha(z_0)}{C_i}Q_i q z_i, \tag{5}$$

where

$$\alpha(z_0) = -\frac{4a}{\pi(4z_0^2 + a^2)}, \tag{6}$$

C_i is the capacitance between the centre electrode and RF electrodes of the zone (i), z_0 is the equilibrium position of the trapped ion, a is the width of the centre electrode, and q is the charge of the ion.

3. Coupling of the two oscillators

In the following, we discuss experiments towards inter-trap coupling using $^{40}{\rm Ca}^+$ ions in both zones. The fabrication of these traps has been described in Refs. [28] and [29]. By coupling the ion to the transmission line, its natural oscillation frequency in the trap is modified. The new oscillation frequency is $\tilde{\omega}_0 = \omega_0 - \Delta\omega_0$. At the off-resonant modes of the transmission line, we find a frequency shift[30]

$$\begin{aligned}\Delta\omega_i = &\frac{g_i^2 C}{2m\omega_0}{\rm Re}[G(0,0,\omega_0)] \\ &- \frac{2}{\pi}\int_0^\infty \frac{{\rm Im}[G(0,0,\omega)]}{\omega+\omega_0}{\rm d}\omega,\end{aligned} \tag{7}$$

where $g_i = q\alpha(z_0)/C_i$, $C = \varepsilon_0(2h/w)$ is the capacitance of the transmission line per unit length. h is the thickness of the transmission line, and w is the space between the transmission lines. The structure of the two zones are the same, so $C_1 = C_2 = C_0 = \epsilon_0(2lh'/w')$ (l is the length of the trap's electrodes, h' is the thickness of the trap electrodes, w' is the space between the trap electrodes).

The coherent coupling between the ions arises in our approach from the off-resonant response of the charges on the transmission line. We adiabatically eliminate these modes and promote the resulting change in the average energy of the ion+line system to an effective Hamiltonian. This gives an effective coupling of the form[19]

$$\hat{H} = -\hbar\Omega_{12}(a_1^+ a_2 + a_2^+ a_1). \tag{8}$$

We focus on a weak coupling regime $\Gamma < \omega_0$. In the low-frequency regime, the Rabi frequency is

$$\Omega_{12} = \frac{g_i^2 C_0^2}{2m\omega_0(2C_0 + C_w)}. \tag{9}$$

The dissipation of the energy of each ion into the lossy transmission line is quantified by the rate[19]

$$\Gamma = \frac{g^2 R_w C_0^2}{6m}\left(1 - \frac{C_0(C_w + C_0)}{(2C_0 + C_w)^2}\right). \tag{10}$$

The energy that one ion has dissipated into the reservoir can also be absorbed by the other ions. This process is characterized by the rate[19]

$$\Gamma_{12} = \frac{g^2 R_w C_0^2}{6m}\left(\frac{1}{2} + \frac{C_0(C_w + C_0)}{(2C_0 + C_w)^2}\right), \tag{11}$$

where C_w is the total capacitance of the transmission line, R_w is the total resistance of the transmission line.

The Rabi frequency Ω_{12} characterises the coherent, capacitive coupling between the zones created by the line, so Rabi frequency Ω_{12} determines the speed of the energy exchange between two ions. Increasing Rabi frequency Ω_{12}, the energy exchange speed between two ions would increase and the time of the gate operation would decrease in the context of quantum computing architectures.

According to Eq. (11), Ω_{12} is decided by the structure of the ion trap, the transmission line, the ions' equilibrium position and oscillation frequency. Falling the ions' equilibrium position or decreasing the width of the centre electrode, the Rabi frequency Ω_{12} rapidly increases, but the system dissipation increases. Reducing the capacitance of the ion trap by decreasing the thickness of the trap's electrodes and increasing the space between the electrodes, the Rabi frequency Ω_{12} rapidly increases, but the system dissipation has a little increase, as shown in Fig. 2. Decreasing the thickness of the transmission line or increasing the space between the transmission lines, the capacitance of the transmission line is reduced. The Rabi frequency Ω_{12} rapidly increases, but the system dissipation has a little increase. Decreasing the length of the transmission lines, the capacitance and the resistance of the transmission lines are reduced. The Rabi frequency Ω_{12} rapidly increases, but the system dissipation is reduced, as shown in Fig. 3. So that in the experiment by choosing the structures of the ion trap and the transmission lines, the Rabi frequency Ω_{12} and the system dissipation could get a suitable parameter.

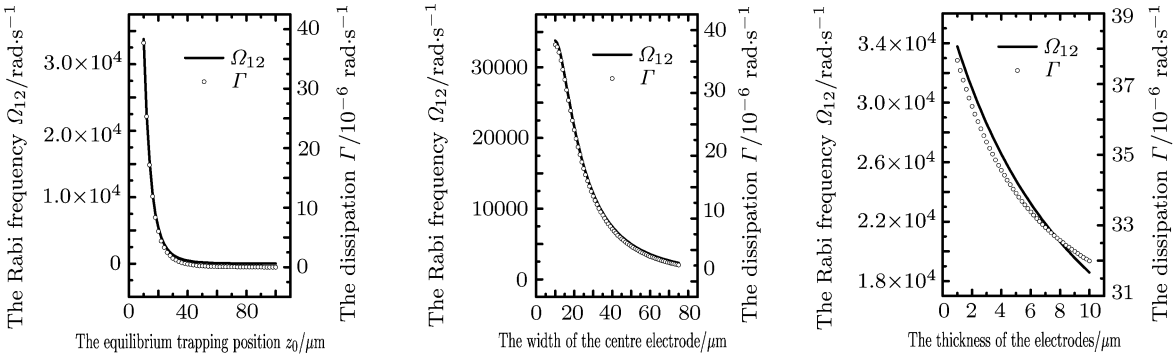


Fig. 2. The changes of the Rabi frequency and the dissipation of the system with the trap parameters of the ion trap.

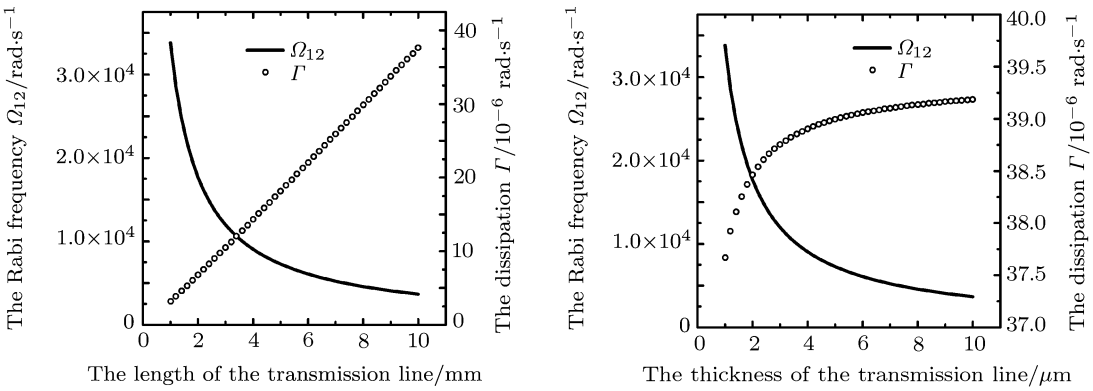


Fig. 3. The changes of the Rabi frequency and the dissipation of the system with the capacitance of the transmission line.

In Fig. 2 and Fig. 3, $^{40}Ca^{+}$ ions are trapped in two zones of the ion trap, where the length of the electrodes is 0.5 mm, the space between the centre electrode and the RF electrodes is 21 μm, the oscillating frequency is 1 MHz, and the space between the transmission lines is 21 μm. The resistance of the transmission line in unit length is 6 Ω/m. The length of the electrodes of the ion trap is less than the transmission lines in our simulation, so the resistance of the electrodes is ignored, and we only take into account the resistance of the transmission lines.

In order to reduce the impact from the potential field, the central electrode connects a notch circuit (LC resonant circuit), where the resonant frequency of the notch circuit is ω_0 which is ions' axial motion frequency. So only the signal with the frequency to be ω_0 could be transmitted in the transmission line, and the signals of the other frequencies are blocked. Oscillating ions induct charges in the RF electrodes. These inducted charges transmit to the other trap by the transmission lines, and affect the other. In order to reduce this effect, each RF electrode connects a notch circuit with the resonant frequency to be ω' (ω' is the frequency of the potential field). These inducted charges were blocked by the notch circuit.

4. Implementation efficiency and fidelity

As a similar direction with atoms in the cavity quantum electrodynamics (QED) and electrons in the Penning trap are discussed in Refs.[22] and [31]. Quantum information stored in the electronic degree of freedom of a single ion cooled to the motional ground state could be mapped onto the ion's motional degree of freedom by driving the motional sidebands of the electronic transition. A swap gate is implemented by considering the initial state $|01\rangle$ and target state $|10\rangle$, where $|n\rangle$ denotes the ion's motional state .

The temperature of the ions indicates the rate at which non-equilibrium states start to relax towards equilibrium. Rates for absorption and emission of one excitation quantum with the transmission line that scale with the product of Γ and Γ_{12} describe with $\bar{n}$ and $\bar{n}+1$ where we denote[19]

$$\bar{n} = \frac{1}{\mathrm{e}^{\frac{\hbar\omega_0}{k_{\mathrm{B}}T}} - 1}. \tag{12}$$

For the weak damping regime ($\Gamma : n\Gamma = \Omega_{12}$), $t = 1/\Gamma(1+4n)$, the fidelity can be approximated as[19]

$$F \approx \exp[-2(1+4\bar{n})\Gamma t]\sin^2(\Omega_{12}t). \tag{13}$$

According to the Eq. (13), the fidelity of the swap gate is decided by the temperature of the ions, the structure of the ion trap and the transmission lines. Decreasing the resistance of the transmission lines, the fidelity of the swap gate is rapidly increased. The resistivity of gold changes drastically with temperature, and we get a resistance per length $R \approx 5.6$ Ω/m below 10 K and $R \approx 122.5$ Ω/m at 80 K.[32] If putting the system into a liquid nitrogen environment, the resistance of the transmission lines is rapidly reduced. The fidelity can become as high as 0.97, as shown in Fig. 4.

Minishing the structure of the ion trap, the ions' heating rate is rapidly increased.[33] According to the Eq. (13), increasing the ions' temperature, the fidelity of a swap gate is rapidly reduced, as shown in Fig. 5.

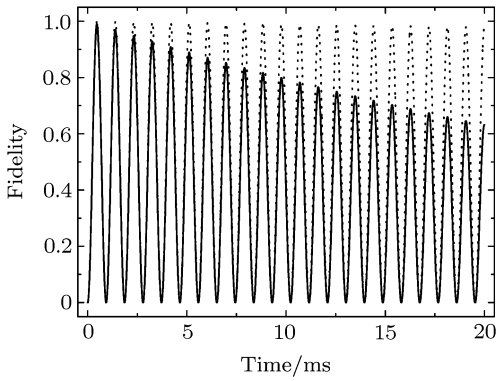


Fig. 4. Fidelity of a swap gate as a function of time for $R = 122.5$ Ω/m (solid line) and $R = 6$ Ω/m (dot line).

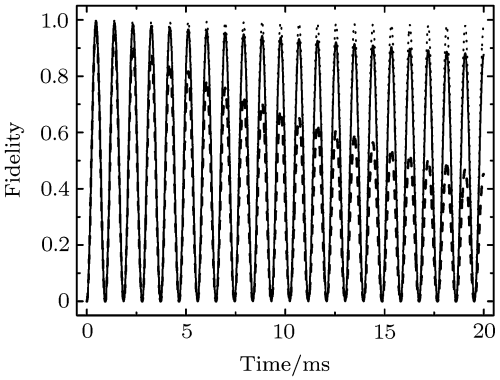


Fig. 5. Fidelity of a swap gate as a function of time for different ions' temperatures ($T = 170$ mK, dot line; $T = 1$ K, solid line; $T = 6$ K, dash line).

5. Conclusion

A theoretical framework has been presented without the need to move ions, but two trapped ions could be coupled in the planar RF ion trap. We have studied several issues in coupling the two confined ions in the planar RF ion trap joined by a lossy electrical transmission line: decoherence and coupling mechanism. We have also calculated the dissipation of the transmission lines, and presented a swap gate scheme that can overcome the charge noise. In our analyses, the couple is boosted up with reduced structure of the trap and the transmission line. The dissipation is reduced by minishing the resistance of the transmission line. Increasing the temperature of the ions, the fidelity of the swap gate is rapidly reduced. For example, when the temperature of the ions rises to 6 K, the fidelity reduces to below 0.5. If the system is put in the environment of the liquid helium, the heating rate of the ions and the resistance of the transmission line will be rapidly reduced. As a result, the dissipation of the system is decreased and the fidelity of the swap gate will keep very high.

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This work has benefited from fruitful discussions with Feng Mang and Yang Wan-Li.

Appendix

A thin rectangle slab and a concentric screen with a rectangle hole lie on the plane $z = 0$ (the gap between the slab and the screen is infinitesimal). The slab's width is a, and the length is l. The z-axis crosses the centre of the slab. If a charge q is located at position $r_{\rm s} = z_{\rm s}\boldsymbol{n}_z$ ($\boldsymbol{n}_z$ is the unit vector along the z-axis), then the potential obeys

$$\nabla^2\phi(r) = -\frac{q}{\varepsilon_0}\delta(r - r_{\rm s}) \tag{A1}$$

with boundary conditions: $\phi(r \in \text{slab}) = V$ and $\phi(r \in \text{screen}) = 0$. To solve Eq. (1) with the given boundary conditions, we can apply Green's Theorem

$$\oint [g(r', r)\frac{\partial\phi(r')}{\partial n'} - \phi(r')\frac{\partial}{\partial n'}g(r', r)]\mathrm{d}a' = \int [g(r', r)\nabla^2\phi(r') - \phi(r')\nabla^2 g(r', r)]\mathrm{d}^3r', \tag{A2}$$

where $g(r', r)$ is Green's function that satisfies

$$\nabla^2 g(r', r) = -\delta(r' - r). \tag{A3}$$

The normal derivative $\partial/\partial n'$ is taken with respect to the out side normal vector of the integration volume with the boundary of the slab and the screen. $g(r', r)$ is the potential of a point charge at r with the boundary condition that $g(r', r) = 0$ for r' on the plane $z = 0$. If the Green function and the potential decay sufficiently fast at infinity, the surface integral runs only over the slab. In this case, Green's theorem gives

$$\phi(r) = \phi_1(r) + \phi_2(r), \tag{A4}$$

where

$$\phi_1(r) = V\int_{\rm centre}\frac{\partial}{\partial z'}g(r', r)\mathrm{d}a', \tag{A5}$$

$$\phi_2(r) = \frac{q}{\varepsilon_0}g(r_{\rm s}, r). \tag{A6}$$

The potential $\phi_1(r)$, involving the value V on the slab, is the result of a pure boundary value problem without sources [$\nabla^2\phi_1(r) = 0$]. If the slab is very long ($a << l$), the potential $\phi_1(r)$ along the z-axis is[34]

$$\phi_1(r) = V\tilde{\alpha}(Z), \tag{A7}$$

where

$$\tilde{\alpha}(z) = \frac{2}{\pi}\arctan\left(\frac{a}{2z}\right). \tag{A8}$$

The potential $\phi_2(r)$ is the image potential in front of a grounded screen and obeys

$$\nabla^2\phi_2(r) = -\frac{q}{\varepsilon_0}\delta(r - r_{\rm s}). \tag{A9}$$

Applying Gaussian's theorem, the charge on the slab is

$$Q = -\varepsilon_0\int\lim_{z\to 0}\frac{\partial}{\partial z}(\phi_1(r) + \phi_2(r))\mathrm{d}a' = C_0V - q\tilde{\alpha}(z), \tag{A10}$$

where C_0 is the capacitance of the slab. In Eq. (A10), we assume that the voltage V on the slab is induced by the source q

$$V = \frac{1}{C_0}[Q + q\tilde{\alpha}(z)] = \frac{\partial H}{\partial Q}, \tag{A11}$$

where

$$H = \frac{Q^2}{2C_0} + \frac{\tilde{\alpha}(z)}{C_0}Qq, \tag{A12}$$

Assuming the source q is a trapped ion in the planar ion trap, in the Lamb–Dicke limit, the displacement of the ion along the axial direction is very small. Expanding $\tilde{\alpha}$ around the equilibrium trapping position $\bar{z}$, we obtain the interaction Hamiltonian

$$H_{\rm int} = -\frac{Qq}{C_0}\alpha(\bar{z})(z-\bar{z}), \tag{A13}$$

where

$$\alpha(\bar{z}) = \frac{4a}{\pi(4\bar{z}^2+a^2)}, \tag{A14}$$

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