Controlled decoherence of floating flux qubits*

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In Born–Markov approximation, this paper calculates the energy relaxation time T_1 and the decoherence time T_2 of a floating flux qubit by solving the set of Bloch–Redfield equations. It shows that there are two main factors influencing the floating flux qubits: coupling capacitor in the circuit and the environment resistor. It also discusses how to improve the quantum coherence time of a qubit. Through shunt connecting/ series connecting inductive elements, an inductive environment resistor is obtained and further the reactance component of the environment resistor is improved, which is beneficial to the enhancement of decoherence time of floating flux qubits.

Keywords: decoherence time, floating flux qubit, control
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1. Introduction

Over the past decade, a variety of qubits have been theoretically proposed and experimentally realized based on superconducting Josephson junctions because of its compatibility with current state of the art fabrication methods for solid state devices.[1−4] Superconductor qubit is macroscopical quantum qubit, having a large size and having many merits, for example, it can be easily controlled, easily coupled and easily measured.[5−7] It is known that the building blocks of quantum computers are single-qubit logic gates and two-qubit gates.[8] One particular scheme is based on magnetic flux states in superconducting quantum interference devices (SQUIDs).[9−11] In some of these schemes, the SQUID quantum bit, or qubit, is based on a two-level system. The conventional configuration of SQUID qubit utilizes the lowest level in each of the double well as the computational basis. Unitary transformations required to accomplish one-qubit rotations, such as the Hadamard and NOT gates, are implemented by controlling the pulse area of the microwaves with frequency tuned to the level separation.[12−15] In parallel, coherence times of qubits have increased from tens of nanoseconds to a few microseconds through an improved understanding of microwave engineering and materials research.[16,17]

However, the disadvantage of superconducting qubit is that it is easily affected by environment. Compared with other qubit candidates (such as trapped ions, nuclear spins, and cavity QED), decoherence presents a much more formidable challenge to superconducting qubits. The interaction between quantum system and environment will cause two demolishment processes, e.g., quantum dissipation and quantum decoherence. The former will cause energy dissipation, and the latter will make the system degenerate from coherent state to classical state. Then, how to keep a long coherence time is a key problem in superconductor quantum qubit research. In order to overcome the influence of decoherence, one of the tactics proposed by the electrical engineers and researchers is to improve the decoherence time of the qubits by advancing the hardware design of the circuits. Electrically floating qubit is such an efficient design scheme. For electrically floating qubit, such dissipation mechanism has been well understood for charge qubits. However, Steffen *et al.* pointed out that the research results of charge qubits cannot be extended to the flux qubits simply.[18] For floating flux qubits, capacitive coupling easily occurs in the grounded capacitance and distributed capacitance between lead wires, which factually result in decoherence and then reduce the decoherence time. In this paper, this problem is further studied by adopting the multilevel quantum description method in Born–Markov approximation. The results indicate that for floating flux qubits, decoherence time decreases owing to the efficient coupling formed by the environment and the finite grounded capacitance and distributed capacitance. It is also theoretically pointed out that decoherence time of the floating flux

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qubits can be efficiently enhanced through shunt connecting/ series connecting inductive elements in the environment circuit (external controlling circuit).

2. Model

We investigate the decoherence property of superconducting flux qubit coupled the environment which is shown in Fig. 1.[19] The RF (radio frequency)-SQUID is made up of a superconducting ring interrupted by a Josephson tunnel junction. In the circuit, we use an impedance $Z(\omega)$ to imitate the phase relaxation which arises from environment. The $Z(\omega)$ is a dominate source of dissipation, which contains the grounded capacitors and the distributed capacitance between the wires. In previous studies, the impact of coupling capacitance on decoherence is often omitted. In fact, the coupling capacitance has a significant impact on decoherence of a floating flux qubits. In the following analysis this will be further reflected.

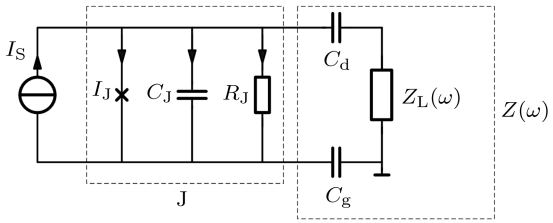


Fig. 1. The graph representation of the flux qubit. The impedance $Z(\omega)$ is used to imitate the phase relaxation which arises from environment.

The decoherence time T_2 is related to the relaxation time T_1 via

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}, \tag{1}$$

where T_ϕ denotes the dephasing time. Based on circuit theory and Bloch–Redfield equation, the relaxation time T_1 and the dephasing time T_ϕ of the flux qubit in the semiclassical approximation are given by[20,21]

$$\frac{1}{T_1} = \frac{4}{\hbar} |\langle 0| \varphi |1\rangle|^2 \left(\frac{\Phi_0}{2\pi}\right)^2 J(\omega) \times \coth\left(\frac{\hbar\omega_{01}}{2k_B T}\right)\Big|_{\omega=\omega_{01}}, \tag{2}$$

$$\frac{1}{T_\phi} = \frac{1}{\hbar} |\langle 0| \varphi |0\rangle - \langle 1| \varphi |1\rangle|^2 \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{J(\omega)}{\omega} \times \coth\left(\frac{\hbar\omega}{2k_B T}\right)\Big|_{\omega\to 0}, \tag{3}$$

where

$$J(\omega) = \mathrm{Im}\left(\frac{\mathrm{i}\omega}{Z(\omega)}\right). \tag{4}$$

and $\langle 0| \varphi |1\rangle = 2\pi/\Phi_0\sqrt{\hbar/2\omega C}$. For simplicity, in this paper, we only consider a symmetric double well, i.e. $\langle 0| \varphi |0\rangle - \langle 1| \varphi |1\rangle = 0$, then $T_\phi \to \infty$. What should be pointed out is that the states $|L\rangle$ and $|R\rangle$ in the left and the right wells respectively actually overlap locally, that is the overlap integral between the left and right states $\langle L|R\rangle \neq 0$. In semiclassical approximation, we take $\langle L|R\rangle = 0$ without considering the effect of such overlap locally on the decoherence of the system.

We briefly analyse the results mentioned above. (1) The construction of the circuit affects directly on the energy relaxation time T_1 and the decoherence time T_2. Only coming from the external impedance $Z(\omega)$, we obtain these results. The external impedance $Z(\omega)$ is taken as the elementary impedance in the quantum circuit and the fluctuation of the external current source. (2) Energy relaxation time and decoherence time decrease with the increase of the coupling matrix element. In order to ensure the accuracy of quantum calculation, the speed of gate operation must be improved under certain decoherence time. Apparently, it seems that we can improve gate speed by decreasing the potential barrier, namely improving φ_{01}. In fact, however, it does not work. For two level superconducting qubits, the results indicate that the opposite result will be achieved by decreasing the potential barrier.[22] (3) For the flux qubit described by the circuit shown in Fig. 1, suppose that $Z(\omega)$ is pure resistor, it is quite apparent that the dephasing time T_ϕ is proportional to the $Z(\omega)$; the energy relaxation time T_1 is also proportional to the $Z(\omega)$ which transits from high level to low level and the decoherence time T_2 has an identical property.

Expression for spectral density is closely related to the circuit structure and circuit elements. Appropriate circuit structure and circuit elements are indispensable to the manipulation and reading of the superconducting qubit state. Impedance $Z(\omega)$ is a function of frequency, which consists of real part and imaginary part. In the following discussion, we can notice that the part of grounded capacitance C_g and distributed capacitance C_d is closely linked with quantum bit. We need to distinguish them from other environmental resistance components.

The $Z_0(\omega)$ denotes the impedance formed by the C_g and C_d. The $Z_L(\omega)$ analogizes other environment

impedance.

$$Z(\omega) = Z_0(\omega) + Z_{\rm L}(\omega), \tag{5}$$

where

$$Z_0(\omega) = R_0(\omega) + {\rm i}\chi_0(\omega),$$
$$Z_{\rm L}(\omega) = R_{\rm L}(\omega) + {\rm i}\chi_{\rm L}(\omega).$$

Then the expression for spectral density

$$J(\omega) = \frac{\omega(R_0 + R_{\rm L})}{(R_0 + R_{\rm L})^2 + (\chi_0 + \chi_{\rm L})^2}. \tag{6}$$

When $T_\phi \to \infty$, from Eqs. (1), (2) and (6), we obtain

$$T_2 = \frac{\alpha}{2\omega}\frac{(R_0 + R_{\rm L})^2 + (\chi_0 + \chi_{\rm L})^2}{R_0 + R_{\rm L}}, \tag{7}$$

where

$$\alpha = \frac{\hbar}{|\langle 0|\,\varphi\,|1\rangle|^2}\left(\frac{2\pi}{\Phi_0}\right)^2 \tanh\left(\frac{\hbar\omega}{k_{\rm B}T}\right).$$

When other conditions are invariant, we can see from Eq. (7) that under ohmic impedance environment, $\chi_0 + \chi_{\rm L} = 0$, spectral density is inversely proportional to the environmental impedance, this fact reveals that decoherence time T_2 and relaxation time T_1 are proportional to the environmental impedance, which perfectly agrees with the results in Ref. [23].

It is well known that necessary input/readout circuits for qubit manipulation are indispensable to perform quantum information processing, where the input/readout circuits namely are external environment of the qubits. The entangling coupling between the qubit and environment will lead to decoherence. To overcome the influence of decoherence and improve decoherence time of the qubit, one of the tactics proposed by the electrical engineers and researchers is bettering the hardware design of the circuits. Electrically floating qubit is such an efficient design scheme.

For electrically floating qubit, such dissipation mechanism has been well understood for charge qubits. However, for floating flux qubits, capacitive coupling easily occurs in the grounded capacitance and stray capacitance between lead wires, which factually leads to decoherence and then decreases the decoherence time.

In Fig. 1, there are grounded capacitance $C_{\rm g}$ and distributed capacitance $C_{\rm d}$ in the bias circuit of the flux qubit, which also form into the impedance of the circuit. The $Z_{\rm L}$ is other analogical environment impedance. Based on classical circuit theory, we can obtain

$$J(\omega) = \frac{\omega^3 R_{\rm L} C_0^2}{(\omega R_{\rm L} C_0)^2 + (\omega\chi_{\rm L} C_0 - 1)^2}, \tag{8}$$

where $C_0^{-1} = C_{\rm g}^{-1} + C_{\rm d}^{-1}$. When other parameters and environmental temperature are invariant, we approximately have

$$T_2 = \frac{\alpha}{2\omega}\frac{(\omega R_{\rm L} C_0)^2 + (\omega\chi_{\rm L} C_0 - 1)^2}{\omega^2 R_{\rm L} C_0^2}. \tag{9}$$

Under pure ohmic environment ($\chi_{\rm L} = 0$), when $\omega R_{\rm L} C_0 << 1$,

$$T_2 = \frac{\alpha}{2\omega}\frac{1}{\omega^2 R_{\rm L} C_0^2}. \tag{10}$$

The above equation indicates that, when C_0 (i.e. $C_{\rm g}$ and $C_{\rm d}$) is certain, to enhance decoherence time, we should shunt connecting a lesser resistor to decrease resistance $R_{\rm L}$. In terms of classical circuit theory, the resistance of the lead wire is proportional to the length and inversely proportional to the cross-sectional area. When two resistors are shunt connected, the equivalent resistance mainly depends on the lesser resistor, suggesting that we should shorten the length and enlarge the cross-sectional area in qubit circuit design.

When $R_{\rm L}$ is certain, spectral density increases with the decrease of capacitor C_0. Namely, the bigger the capacitance is, the more obvious the decoherence phenomenon of the flux qubit is. Without considering other factors, we can obtain a longer decoherence time through decreasing the capacitance C_0. Different from the resistance, we should employ series connected capacitance to obtain lesser capacitance. When two capacitors are series connected, the equivalent capacitance mainly relies on the lesser capacitor. The existing technology is easily available for the capacitance of the magnitude of 10fF, while difficult for the capacitance of lower magnitude, which factually reflects from profile that for floating flux qubits of capacitive coupling, the decoherence time is hard to be enhanced because the coupling capacitance exists and the coupling capacitance has trouble in further decreasing.

Undoubtedly, realistic environment is reactive ($\chi_{\rm L} \neq 0$), namely, the environment can both exhibit capacitive and inductive characteristics. When $\omega\chi_{\rm L} C_0 \approx \omega R_{\rm L} C_0 << 1$, from Eq. (9), we have

$$T_2 = \frac{\alpha}{2\omega}\frac{1}{\omega^2 R_{\rm L} C_0^2},$$

which is similar to the pure resistance environment.

But if we can realize $\omega\chi_{\rm L}C_0 >> 1$, by designing the controlling circuit, then from Eq. (9) we have

$$T_2 = \frac{\alpha}{2\omega}\frac{R_{\rm L}^2+\chi_{\rm L}^2}{R_{\rm L}}. \tag{11}$$

From the above formula, we can see that when environmental impedance is larger, the grounded capacitance and distributed capacitance of the lead wires in quantum circuit have no impact on the decoherence time of the qubit. At this moment, decoherence time is mainly determined by the environmental impedance.

Next, we further analyse how to improve the flux qubits in a reactive environment. (i) When the imaginary part of the environmental impedance is certain, we can control the decoherence of the circuit by controlling the real part of the impedance. In fact $\chi_{\rm L} >> R_{\rm L}$ satisfies at this moment, decoherence time increases with the decrease of resistor $R_{\rm L}$, this fact is quite different from the influence of resistance on the decoherence, where the resistance reflects the fluctuation of current source. Apparently, resistor $R_{\rm L}$ can be lessened through shunt connecting a lesser resistor. Evidently, for a qubit circuit with bigger environmental dissipation, it is efficient to improve decoherence time through shunt connecting a lesser resistor. However, for a qubit circuit with smaller environmental dissipation, this method is not efficient even acts contrary. (ii) When the real part of the environmental impedance is certain, we can control decoherence of the circuit by controlling the imaginary part of the impedance. Because $\chi_{\rm L} >> R_{\rm L}$, the decoherence time approximates to

$$T_2 = \frac{\alpha}{2\omega}\frac{\chi_{\rm L}^2}{R_{\rm L}}. \tag{12}$$

Thus, we can see that decoherence time can be enhanced by increasing the imaginary part of the environmental impedance. Under the limit $\omega\chi_{\rm L}C_0 >> 1$, what technological line would be adopted to increase the reactive component of the environmental impedance to achieve the goal of enhancing the decoherence time?

If it is capacitance impedance environment, $\chi_{\rm L}^{-1} = \omega C_{\rm L}$, then $C_0/C_{\rm L} >> 1$ is required, which is difficult to realize for the moment. The best way to increase $\chi_{\rm L}$ is shunt connecting/ series connecting inductance elements to structure inductance environment. Let $\chi_{\rm L} = \omega L_{\rm L}$ and choose $\omega = 40$ GHz, $C_0 = 5$ fF, then we have $L_{\rm L} > 125$ nH for $\omega\chi_{\rm L}C_0 >> 1$, which is relatively easy to implement in realistic qubit circuits.

3. Conclusion

For floating flux qubits, capacitive coupling easily occurs in the grounded capacitance and stray capacitance between lead wires. Capacitor has the characteristics of alternating current and stopping direct current that low-frequency noise has little impact on the floating flux qubits with capacitance coupling. There are two main factors influencing decoherence of floating flux qubits: one is coupling capacitance in the circuit. Necessary but finite coupling capacitance will prevent the increase of decoherence time. The other is environmental impedance, which is the most significant source of decoherence. Increasing the environmental impedance, especially increasing the reactive component of the environmental impedance is beneficial to the enhancement of the decoherence time. While to achieve the goal of increasing the reactive component of the environmental impedance, the best way is to shunt connecting/ series connecting inductive elements to configure the inductance environment. Our research results may offer good instruction to the circuit design of the flux qubit.

References

[1] Chiorescu I, Bertet P, Semba K, Nakamura Y, Harmans C J P M and Mooij J E 2004 *Nature* (London) **431** 159

[2] Burkard G 2009 *Phys. Rev.* B **79** 125327

[3] Yoshihara F, Harrabi K, Niskanen A O, Nakamura Y and Tsai J S 2006 *Phys. Rev. Lett.* **97** 167001

[4] Astafiev A, Pashkin Y A, Nakamura Y, Yamamoto T and Tsai J S 2006 *Phys. Rev. Lett.* **96** 137001

[5] Ji Y H, Cai S H and Hu J J 2008 *Chin. Phys.* B **17** 440

[6] Wang Z W and Xiao J L 2007 *Acta Phys. Sin.* **56** 678 (in Chinese)

[7] Majer J B, Paauw F G, ter Haar A C J, Harmans C J P M and Mooij J E 2005 *Phys. Rev. Lett.* **94** 090501

[8] Ji Y H, Lai H F, Cai S H and Wang Z S 2010 *Chin. Phys.* B **19** 030310

[9] Amin M H S, Smirnov A Yu and Maassen van der Brink A 2003 *Phys. Rev.* B **67** 100508R

[10] Yang C P and Han S 2004 *Phys. Lett.* A **321** 273

[11] Kis Z and Paspalakis E 2004 *Phys. Rev.* B **69** 024510

[12] Muller C, Shnirman A and Makhlin Y 2010 *Phys. Rev.* B **80** 134517

[13] Hu X N and Li X Q 2006 *Acta Phys. Sin.* **55** 3259 (in Chinese)

[14] Zhu J M and Wang S J 2006 *Acta Phys. Sin.* **55** 5018 (in Chinese)
[15] Migliore R and Messina A 2004 *J. Opt.* B: *Quantum Semiclassical Opt.* **6** S136
[16] Chiarello F 2000 *Phys. Lett.* A **277** 189
[17] Averin D V, Friedman J R and Lukens J E 2000 *Phys. Rev.* B **62** 11802
[18] Steffen M, Brito F, DiVincenzo D, Kumar S and Ketchen M 2009 *New Journal of Physics* **11** 033030
[19] Feiedman J, Patel V, Chen W, Tolpygo S and Lukens J E 2000 *Nature* **406** 43
[20] Burkard G, Koch R H and DiVincenzo D P 2004 *Phys. Rev.* B **69** 064503
[21] Chirolli L and Burkard G 2006 *Phys. Rev.* B **74** 174510
[22] Zhou Z, Chu S I and Han S 2002 *Phys. Rev.* B **66** 054527
[23] Leggett A J, Chakravarty S, Dorsey A T, Fisher Matthew P A, Garg A and Zwerger W 1987 *Rev. Mod. Phys.* **59** 1