Entanglement dynamics of two distant atoms in two detuning cavities*

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(Received 6 January 2010; revised manuscript received 11 June 2010)

We investigate the entanglement dynamics via the concurrence of two distant atoms interacting off-resonantly with two cavity fields in Fock states, respectively. We find that the evolution of entanglement has sudden death and sudden birth phenomena, that with the increase of photon numbers in the two cavities, the alternate frequency of sudden death and sudden birth turns fast, and that the amplitude of concurrence oscillates regularly with oscillation frequency becoming slow when the cavity fields have the same photon numbers. While, the maximum of concurrence declines and the amplitude of concurrence oscillates irregularly when the two cavity fields have different photon numbers. In addition, we find the length of death time is dependent on the initial entanglement.

Keywords: entanglement dynamics, interacting off-resonantly, Fock state, sudden death and sudden birth

PACC: 0365, 4250

1. Introduction

Entanglement, a bizarrerie in quantum field, plays an important role in quantum physics. As is well known, the entanglement can not only be used to study nonlocal and nonclassical characteristics of quantum particles, but also, as a valuable resource, have extensive applications in quantum communication and quantum computation,[1,2] such as quantum key distribution,[3] quantum teleportation,[4] quantum error correction.[5] However, the decoherence processes caused by the influence of the environment often easily destroy entanglement and furthermore cause noise in the communication or errors in the computation results. Therefore, improving our understanding of entanglement dynamics and properties will help us to develop strategies for manipulating and quantifying the entanglement degree efficiently. Recently, Yu and Eberly[6] investigated the time evolution of entanglement of a bipartite qubit system undergoing various modes of decoherence. They found that for certain states the entanglement decays exponentially with time even if in the absence of interaction, while for other closely relevant states the entanglement vanishes completely in a finite time. This phenomenon, known as 'entanglement sudden death' (ESD), is so intriguing that it has stimulated a lot of theoretical[7−19] and experimental studies[20−22] recently. Yönac *et al.*[8,10] investigated the entanglement dynamics via the concurrence of atom–atom, atom–cavity and cavity–cavity in a system with two isolated atoms interacting with their vacuum cavities respectively. They showed that for the $|\Psi\rangle_{\rm AB}$ type of initial state, ESD does not occur, contrary to that for the $|\Phi\rangle_{\rm AB}$ type of initial state, which often has ESD phenomenon. Zhang *et al.*[16] studied the entanglement dynamics of strongly driven atoms off-resonantly coupled with cavity fields in vacuum states. They found that sudden death of entanglement always occurs if the atom–field coupling is larger than the atom–field detuning. Li *et al.*[9] also investigated the entanglement dynamics via concurrence and negativity of two space-separated atoms: one of them is in a cavity in Fock state, and the other is isolated completely. They showed that the length of the death time is not dependent on the degree of entanglement of the initial state. Moreover, they indicated a little inconsistency of the concurrence and the negativity in two-qubit states.[23]

In the present paper, we investigate the entanglement dynamics of two distant two-level atoms, trapped in two Fock-state cavities and interact off-resonantly with the two cavities, respectively. We

*Project supported by the National Natural Science Foundation of China (Grant No. 60667001).

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mainly investigate the influence of photon number, the detuning between the frequency of atom transition and the frequency of cavity field, and the initial entanglement, on the degree of entanglement of the two distant atoms. We also compare concurrence with negativity of their evaluation about the degree of entanglement of the two atoms.

2. The model

We consider that two entangled distant two-level atoms A and B, with configuration shown in Fig. 1, interact off-resonantly with two cavities a and b, respectively. The interaction Hamiltonian of the system can be written as

$$\begin{aligned} H &= H_{\rm A} + H_{\rm B} \\ &= \lambda_{\rm A}(a|e\rangle_{\rm A}\langle g|\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + a^{\dagger}|g\rangle_{\rm A}\langle e|\,{\rm e}^{{\rm i}\delta_{\rm A}t}) \\ &\quad + \lambda_{\rm B}(b|e\rangle_{\rm B}\langle g|\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + b^{\dagger}|g\rangle_{\rm B}\langle e|\,{\rm e}^{{\rm i}\delta_{\rm B}t}), \quad (1) \end{aligned}$$

where $\lambda_{\rm A}$ ($\lambda_{\rm B}$) is the coupling constant between the atom A (B) and the cavity a (b), a (b) and $a^{\dagger}$ ($b^{\dagger}$) are the annihilation operator and the creation operator of photons in a (b) mode, respectively. The detunings $\delta_{\rm A} = w_{e_{\rm A}} - w_{g_{\rm A}} - w_a$ and $\delta_{\rm B} = w_{e_{\rm B}} - w_{g_{\rm B}} - w_b$, where w_a (w_b) is the frequency of cavity field a (b) and $w_{e_{\rm A}}$ ($w_{e_{\rm B}}$) and $w_{g_{\rm A}}$ ($w_{g_{\rm B}}$) are the frequencies associated with the levels $|e\rangle$ and $|g\rangle$ of atom A (B), respectively. We assume that the two cavities are initially prepared in the Fock states $|n\rangle_a$ and $|n\rangle_b$.

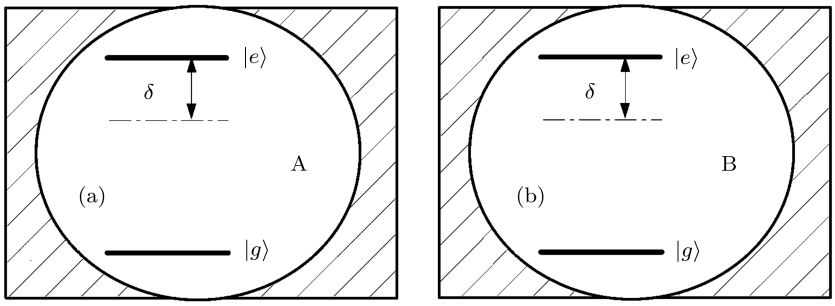


Fig. 1. Schematic setup of the system model.

In order to describe the entanglement dynamics we first need to introduce two kinds of measurements of entanglement, concurrence and negativity. For any two-qubit system, the concurrence is described by

$$C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (2)$$

where the quantities $\lambda_i (i = 1, 2, 3, 4)$ are the eigenvalues in decreasing order of the matrix $\boldsymbol{\rho}(\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y)\boldsymbol{\rho}^*(\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y)$, with $\boldsymbol{\rho}^*$ denoting the complex conjugation of the density matrix $\boldsymbol{\rho}$ and $\boldsymbol{\sigma}_y$ being the Pauli matrix. If a density matrix has the 'standard' form of two-qubit mixed state,[7]

$$\begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \quad (3)$$

the concurrence can be easily obtained as

$$C(\boldsymbol{\rho}) = 2\max\left\{0, |z| - \sqrt{ad}, |w| - \sqrt{bc}\right\}. \quad (4)$$

The rescaled negativity is given by

$$N(\boldsymbol{\rho}) = 2\max\left\{0, -\sum_j \mu_j\right\}, \quad (5)$$

where μ_j are the negative eigenvalues of $\boldsymbol{\rho}^{T_X}$, and T_X denotes the partial transpose with respect to the system X. If a density matrix has the 'standard' form of two-qubit mixed state as shown in Eq. (3), the negativity can be obtained as

$$\begin{aligned} N(\boldsymbol{\rho}) &= \max\left\{0, \left[-(b+c) + \sqrt{(b-c)^2 + 4|w|^2}\right]\right. \\ &\quad \left. + \left[-(a+d) + \sqrt{(a-d)^2 + 4|z|^2}\right]\right\}. \quad (6) \end{aligned}$$

If $z = 0$, then $N(\boldsymbol{\rho}) = \max\left\{0, -(b+c) + \sqrt{(b-c)^2 + 4|w|^2}\right\}$, while if $w = 0$, then $N(\boldsymbol{\rho}) = \max\left\{0, -(a+d) + \sqrt{(a-d)^2 + 4|z|^2}\right\}$.

3. Entanglement dynamics of atoms A and B

When atoms A and B are initially in the state $|\Psi\rangle_{\rm AB} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}$, the initial state of the system is

$$|\Psi(0)\rangle_{{\rm AB}ab} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}|nn\rangle_{ab}. \tag{7}$$

After the interaction time t, the total system of the atom and cavity will evolve into the state

$$\begin{aligned}|\Psi(t)\rangle_{{\rm AB}ab} = {} & (x_1(t)|e,e,n,n-1\rangle + x_2(t)|e,e,n-1,n\rangle + x_3(t)|e,g,n,n\rangle + x_4(t)|e,g,n-1,n+1\rangle \\ & + x_5(t)|g,e,n,n\rangle + x_6(t)|g,e,n+1,n-1\rangle + x_7(t)|g,g,n,n+1\rangle + x_8(t)|g,g,n+1,n\rangle)_{{\rm AB}ab},\end{aligned} \tag{8}$$

where $x_m(t),\ (m = 1, 2, \ldots 8)$ are probability amplitudes. Substituting Eqs. (1) and (8) into the time-dependent Schrödinger equation, we can obtain the differential equations for the probability amplitudes as follows:

$$\begin{aligned}
{\rm i}x_1'(t) &= \lambda_{\rm B}x_3(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_6(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t},\\
{\rm i}x_2'(t) &= \lambda_{\rm B}x_4(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_5(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t},\\
{\rm i}x_3'(t) &= \lambda_{\rm B}x_1(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_8(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t},\\
{\rm i}x_4'(t) &= \lambda_{\rm B}x_2(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_7(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t},\\
{\rm i}x_5'(t) &= \lambda_{\rm A}x_2(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_7(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t},\\
{\rm i}x_6'(t) &= \lambda_{\rm A}x_1(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_8(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t},\\
{\rm i}x_7'(t) &= \lambda_{\rm A}x_4(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_5(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t},\\
{\rm i}x_8'(t) &= \lambda_{\rm A}x_3(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_6(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t}.
\end{aligned} \tag{9}$$

Solving numerically the differential equations of Eq. (9) with initial conditions $x_1(0) = x_2(0) = x_4(0) = x_6(0) = x_7(0) = x_8(0) = 0, x_3(0) = \cos\alpha$, and $x_5(0) = \sin\alpha$, we can obtain the numerical solutions of the probability amplitudes $x_m(t), (m = 1, 2, \ldots 8)$. For simplicity, we assume that $\delta_{\rm A} = \delta_{\rm B} = \delta$, and $\lambda_{\rm A} = \lambda_{\rm B} = \lambda$. By tracing out the two cavity modes, we calculate the reduced density matrix $\boldsymbol{\rho}_{\rm AB}$. The reduced density matrix $\boldsymbol{\rho}_{\rm AB}$ can be written in the basis $\{|ee\rangle_{\rm AB}, |eg\rangle_{\rm AB}, |ge\rangle_{\rm AB}, |gg\rangle_{\rm AB}\}$ as

$$\begin{pmatrix}
|x_1(t)|^2 + |x_2(t)|^2 & 0 & 0 & 0\\
0 & |x_3(t)|^2 + |x_4(t)|^2 & x_3(t)x_5^*(t) & 0\\
0 & x_3^*(t)x_5(t) & |x_5(t)|^2 + |x_6(t)|^2 & 0\\
0 & 0 & 0 & |x_7(t)|^2 + |x_8(t)|^2
\end{pmatrix}. \tag{10}$$

We use the concurrence to measure the degree of entanglement. Because the matrix $\boldsymbol{\rho}_{\rm AB}$ has the 'standard' form of two-qubit mixed state, the concurrence can be calculated from

$$C_{\rm AB} = 2\max\left\{0, |x_3(t)x_5^*(t)| - \sqrt{(|x_1(t)|^2 + |x_2(t)|^2)(|x_7(t)|^2 + |x_8(t)|^2)}\right\}. \tag{11}$$

We first consider small detuning for $\delta = \lambda$ (dash line). From the dash lines of Figs. 2(a)–2(c) it can be seen that there are sudden death and sudden birth phenomena when the cavity fields are in Fock states. We can also see that with the increase of photon number in the two cavities, the alternate frequency of sudden death and sudden birth turns fast and the amplitude of concurrence oscillates regularly with oscillation frequency becoming slow when the photon numbers $n_a = n_b = n$. Figure 2(d) shows that when $n_a \neq n_b$, the maximum of concurrence declines and the amplitude of concurrence does not oscillate regularly.

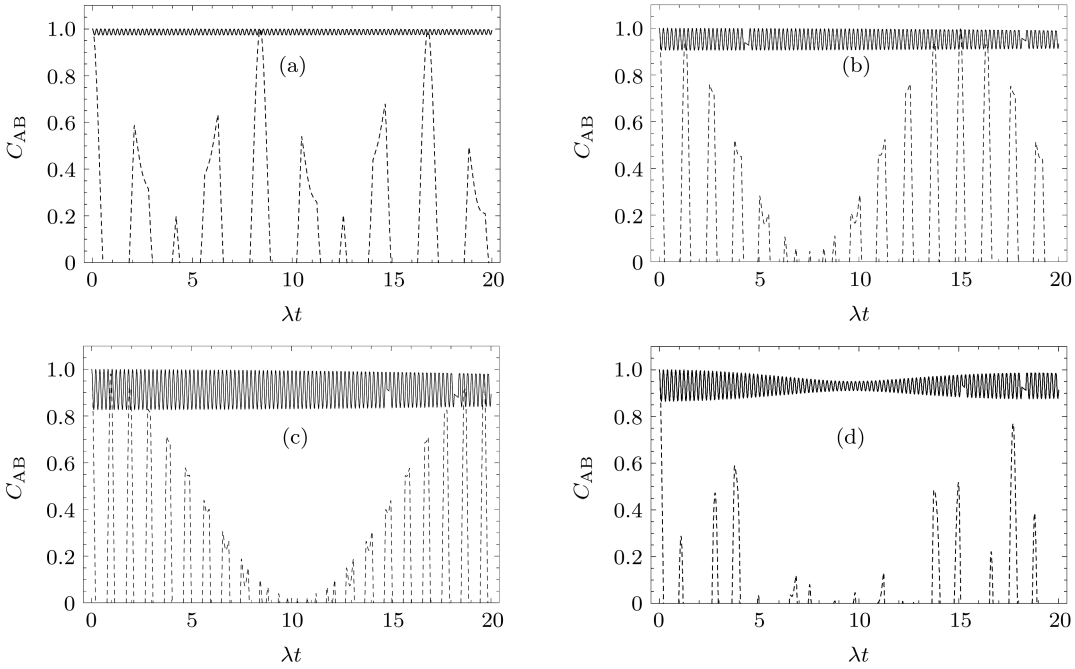


Fig. 2. Concurrences as a function of λt with the initial atomic state $|\Psi\rangle_{\rm AB} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}$ for $\delta = \lambda$ (dash line), $\delta = 30\lambda$ (solid line) and (a) $n_a = n_b = 1$, (b) $n_a = n_b = 3$, (c) $n_a = n_b = 5$ and (d) $n_a = 5, n_b = 10$ with $\alpha = \pi/4$.

For large detuning, that is, $\delta = 30\lambda$, it can be seen from the solid lines of Figs. 2(a)–2(d) that the sudden death and sudden birth disappear and the average concurrence increases obviously. That is to say, enlargement of the detuning can increase the degree of entanglement.

When atoms A and B are initially in the state $|\Phi\rangle_{\rm AB} = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{\rm AB}$, the initial state of the system is

$$|\Psi(0)\rangle_{{\rm AB}ab} = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{\rm AB}|nn\rangle_{ab}. \tag{12}$$

After the interaction time t, the total system of the atom and cavity will evolve into the state

$$\begin{aligned} &|\Psi(t)\rangle_{{\rm AB}ab} \\ =\ &(x_1(t)|e,e,n,n\rangle + x_2(t)|e,e,n-1,n-1\rangle + x_3(t)|e,g,n,n+1\rangle + x_4(t)|e,g,n-1,n\rangle \\ &+x_5(t)|g,e,n+1,n\rangle + x_6(t)|g,e,n,n-1\rangle + x_7(t)|g,g,n+1,n+1\rangle + x_8(t)|g,g,n,n\rangle)_{{\rm AB}ab}. \end{aligned} \tag{13}$$

Substituting Eqs. (1) and (13) into the time-dependent Schrödinger equation, we can obtain the differential equations for the probability amplitudes as follows:

$$\begin{aligned} {\rm i}x_1'(t) &= \lambda_{\rm B}x_3(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_5(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t}, \\ {\rm i}x_2'(t) &= \lambda_{\rm B}x_4(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_6(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t}, \\ {\rm i}x_3'(t) &= \lambda_{\rm B}x_1(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_7(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t}, \\ {\rm i}x_4'(t) &= \lambda_{\rm B}x_2(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t} + \lambda_{\rm A}x_8(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t}, \\ {\rm i}x_5'(t) &= \lambda_{\rm A}x_2(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_8(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t}, \\ {\rm i}x_6'(t) &= \lambda_{\rm A}x_1(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_7(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t}, \\ {\rm i}x_7'(t) &= \lambda_{\rm A}x_3(t)\sqrt{n_a+1}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_5(t)\sqrt{n_b+1}\,{\rm e}^{-{\rm i}\delta_{\rm B}t}, \\ {\rm i}x_8'(t) &= \lambda_{\rm A}x_4(t)\sqrt{n_a}\,{\rm e}^{-{\rm i}\delta_{\rm A}t} + \lambda_{\rm B}x_6(t)\sqrt{n_b}\,{\rm e}^{-{\rm i}\delta_{\rm B}t}. \end{aligned} \tag{14}$$

Solving numerically the differential equations of Eq. (14) with initial conditions $x_2(0) = x_3(0) = x_4(0) = x_5(0) = x_6(0) = x_7(0) = 0, x_1(0) = \cos\alpha$, and $x_8(0) = \sin\alpha$, we can obtain the numerical solutions of the

probability amplitudes $x_m(t), m = 1, 2, \ldots 8$. Here we also assume that $\delta_{\rm A} = \delta_{\rm B} = \delta, \lambda_{\rm A} = \lambda_{\rm B} = \lambda$. By tracing out the two cavity modes, we can obtain the reduced density matrix $\boldsymbol{\rho}_{\rm AB}$,

$$\begin{pmatrix} |x_1(t)|^2 + |x_2(t)|^2 & 0 & 0 & x_1(t)x_8^*(t) \\ 0 & |x_3(t)|^2 + |x_4(t)|^2 & 0 & 0 \\ 0 & 0 & |x_5(t)|^2 + |x_6(t)|^2 & 0 \\ x_1^*(t)x_8(t) & 0 & 0 & |x_7(t)|^2 + |x_8(t)|^2 \end{pmatrix} \tag{15}$$

in the same basis as that of Eq. (9). We also use the concurrence to measure the degree of entanglement. Similarly, the matrix $\boldsymbol{\rho}_{\rm AB}$ of Eq. (15) has the 'standard' form of two-qubit mixed state,[7] so the concurrence can be obtained as

$$C_{\rm AB} = 2\max\left\{0, |x_1(t)x_8^*(t)| - \sqrt{(|x_3(t)|^2 + |x_4(t)|^2)(|x_5(t)|^2 + |x_6(t)|^2)}\right\}. \tag{16}$$

Similarly, we consider the cases of small and large detunings. From Fig. 3 we can see that when $\delta = \lambda$, the evolutions of concurrence (dash lines) have similar forms to those shown in Fig. 2 with the atoms initially in state $|\Psi\rangle_{\rm AB} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}$. When $\delta = 30\lambda$, it can be seen from the solid lines in Fig. 3 that the sudden death and sudden birth disappear, which is the same as the scenario in Fig. 2. However, there is a small difference in the peak of the evolution curve of the concurrence between Figs. 2 and 3.

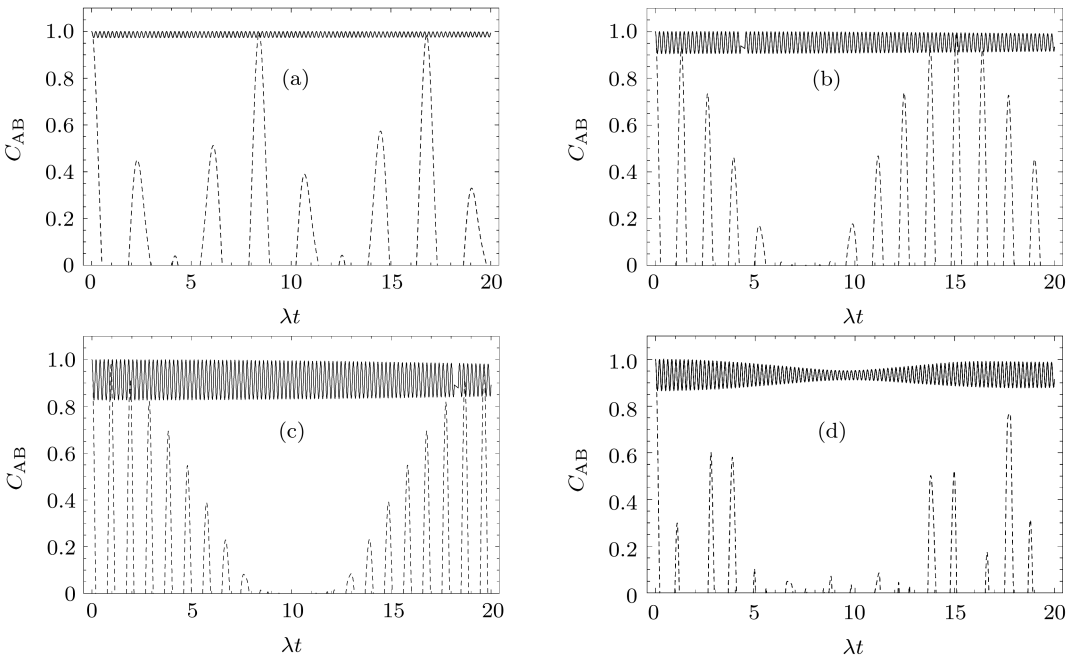


Fig. 3. Concurrences as a function of λt with the initial atomic state $|\Phi\rangle_{\rm AB} = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{\rm AB}$ for $\delta = \lambda$ (dash line), $\delta = 30\lambda$ (solid line) and (a) $n_a = n_b = 1$, (b) $n_a = n_b = 3$, (c) $n_a = n_b = 5$ and (d) $n_a = 5, n_b = 10$ with $\alpha = \pi/4$.

Now, we discuss the effect of the degree of entanglement of initial state on the death time. From Fig. 4 we find that the length of death time is dependent on the degree of entanglement of the initial atomic state, and the smaller the initial degree of entanglement, the longer the sudden death time is. This property is different from the result in Ref. [9].

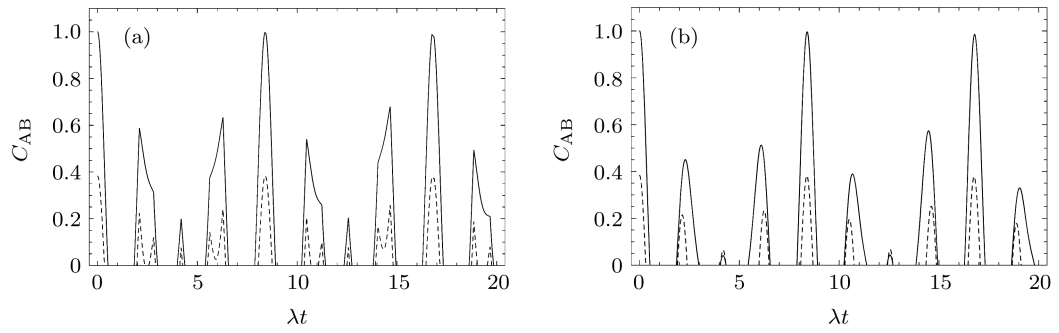


Fig. 4. Concurrences as a function of λt with the initial atomic state (a) $|\Psi\rangle_{\rm AB} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}$ and (b) $|\Phi\rangle_{\rm AB} = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{\rm AB}$ for $\delta = \lambda$, $n_a = n_b = 1$ and $\alpha = \pi/4$ (solid line), $\alpha = \pi/16$ (dash line).

Next, we make a comparison between concurrence and negativity of their evaluations on the degree of entanglement. By comparing concurrence with negativity of their evaluations on degree of entanglement as shown in Fig. 5, we find that for initial atomic state $|\Psi\rangle_{\rm AB} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}$, there is a small difference between concurrence and negativity, whereas the evaluations of concurrence and negativity are equivalent (the dash line and the solid line become one line in Fig. 5(b)) for initial atomic state $|\Phi\rangle_{\rm AB} = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{\rm AB}$.

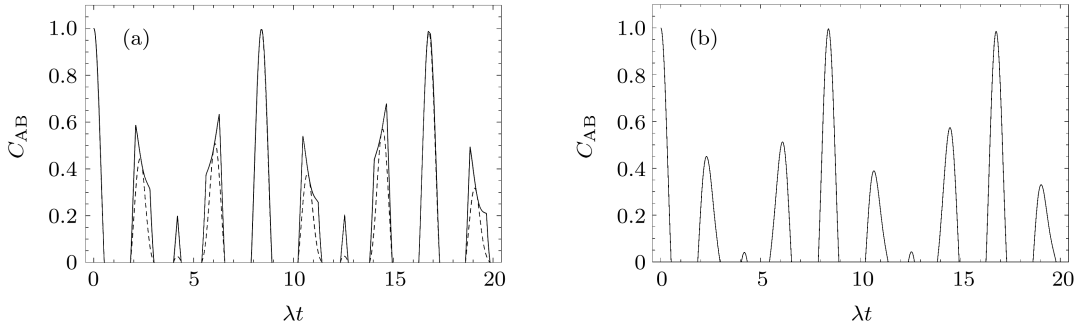


Fig. 5. Concurrences (solid line) and negativities (dash line) as a function of λt with the initial atomic state (a) $|\Psi\rangle_{\rm AB} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}$ and (b) $|\Phi\rangle_{\rm AB} = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{\rm AB}$ for $\delta = \lambda$, $n_a = n_b = 1$ and $\alpha = \pi/4$.

4. Discussion and conclusion

We have described the entanglement dynamics of two distant two-level atoms interacting off-resonantly with two Fock-state cavity fields, respectively. We have found that the evolution of entanglement has interesting sudden death and sudden birth phenomena. With the increase of photon numbers in the two cavities, the alternate frequency of sudden death and sudden birth becomes fast and the amplitude of concurrence oscillates regularly with oscillation frequency becoming slow when the two cavity fields have the same photon numbers and are in small detuning, while, the sudden death and sudden birth disappear and the average concurrence is largely increased when the cavities are in large detuning. That is to say, we can enlarge the detuning of the cavities to improve the degree of entanglement. Whereas, when the two cavity fields have different photon numbers the maximum of concurrence declines. This character shows that we can keep the two cavities having the same photon numbers to obtain high concurrence. Furthermore, different from the result in Ref. [9], the length of death time is dependent on the degree of entanglement of the initial state. We have compared concurrence with negativity of their evaluations about degree of entanglement and find that there is a small difference between concurrence and negativity for initial atomic state $|\Psi\rangle_{\rm AB} = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{\rm AB}$, which has not been explained convincingly due to the fact that concurrence and negativity are not observables and cannot been represented by Hermitian operators. But for initial atomic state $|\Phi\rangle_{\rm AB} = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{\rm AB}$, the evaluations of concurrence and negativity are equivalent.

References

[1] Bennet C H and DiVincenzo D P 2000 *Nature* **404** 247

[2] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge, England: Cambridge University)

[3] Ekert A 1991 *Phys. Rev. Lett.* **67** 661

[4] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[5] Steane A 1996 *Phys. Rev. Lett.* **77** 793

[6] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404

[7] Yu T and Eberly J H 2006 *Opt. Commun.* **264** 393

[8] Yönac M, Yu T and Eberly J H 2006 *J. Phys.* B: *At. Mol. Opt. Phys.* **39** S621

[9] Li Z J, Li J Q, Jin Y H and Nie Y H 2007 *J. Phys.* B: *At. Mol. Opt. Phys.* **40** 3401

[10] Yönac M, Yu T and Eberly J H 2007 *J. Phys.* B: *At. Mol. Opt. Phys.* **40** S45

[11] Abdel-Atya M 2008 *Eur. Phys. J.* D **46** 537

[12] Mazzola L, Maniscalco S, Piilo J, Suominen K A and Garraway 2009 *Phys. Rev.* A **79** 042302

[13] Chen L, Shao X Q and Zhang S 2009 *Chin. Phys.* B **18** 888

[14] Deng X J, Fang M F and Kang G D 2009 *Chin. Phys.* B **18** 4100

[15] Chen L, Shao X Q and Zhang S 2009 *Chin. Phys.* B **18** 4676

[16] Zhang Y J, Man Z X and Xia Y J 2009 *J. Phys.* B: *At. Mol. Opt. Phys.* **42** 095503

[17] Thilagam A 2009 *J. Phys.* A: *Math. Theor.* **42** 335301

[18] Zhang G F and Pu J J 2010 *Acta Phys. Sin.* **59** 1462 (in Chinese)

[19] Shan C J, Chen T, Huang Y X, Li H, Liu J B and Liu T K 2010 *Acta Phys. Sin.* (to be published) (in Chinese)

[20] Almeida M P, De Melo F, Hor-Meyll M, Salles A, Walborn S P, Souto Ribeiro P H and Davidovich 2007 *Science* **316** 579

[21] Laurat J, Choi S, Deng H, Chou C W and Kimble H J 2007 *Phys. Rev. Lett.* **99** 180504

[22] Jiménez Farías O, Lombard Latune C, Walborn S P, Davidovich L and Souto Ribéiro P H 2009 *Science* **324** 1414

[23] Verstraete F, Audenaert K, Dehaene and De Moor B 2001 *J. Phys.* A: *Math. Gen.* **34** 10327