Entanglement of two atoms interacting with a dissipative coherent cavity field without rotating wave approximation*

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(Received 1 August 2009; revised manuscript received 7 June 2010)

Considering two identical two-level atoms interacting with a single-model dissipative coherent cavity field without rotating wave approximation, we explore the entanglement dynamics of the two atoms prepared in different states using concurrence. Interestingly, our results show that the entanglement between the two atoms that initially disentangled will come up to a large constant rapidly, and then keeps steady in the following time or always has its maximum when prepared in some special Bell states. The model considered in this study is a good candidate for quantum information processing especially for quantum computation as steady high-degree atomic entanglement resource obtained in dissipative cavity.

Keywords: atomic entanglement, Tavis–Cummings model, rotating wave approximation, characteristic function

PACC: 0365, 4250

1. Introduction

Quantum entanglement plays a key role in quantum information and quantum computation,[1] quantum teleportation,[2] quantum dense code,[3] quantum cryptography[4] and so on. Therefore it is important to study entanglement dynamics in different scenarios in order to obtain entangled resource to be implemented in these quantum information processing. Most experimental works of quantum entanglement have been implemented with photons. Although individual polarisation states of photons are easily controlled and their quantum coherence can be preserved over many kilometers in an optical fibre,[5,6] photons cannot be stored for a long time, and the manipulation of collective entangled states demonstrates considerable difficulties even if photons are confined in the same cavity. The creation of long lived entangled pairs with atoms, on the other hand, is a relatively recent pursuit, which may provide reliable quantum information storage. Thus the issue of creating and controlling two-atom entanglement has attracted much attention.[7−12] Noted that Tavis–Cummings (TC) model is introduced to describe dynamics especially atomic entanglement dynamics of systems consisting multi atoms coupled with quantised electromagnetic field, and many complicated systems which can be reduced to this model, of course, have been widely studied in many papers[13−22] under rotating wave approximation (RWA). But with the experimental progress, it has been revealed that for some of systems such as trapped ions,[23] cooper-pair boxes,[24,25] flux qubits,[26] the coupling can be very large, and the RWA breaks down, so that the counter-rotating terms must be considered to describe the dynamics correctly. Recently, cavity systems with strong couplings have been discussed.[27] Peng and Li have numerically analysed the effect of the virtual-photon field on phase fluctuation and the periodic collapse revivals of atomic population in the Jaynes–Cummings (JC) mode.[28−30] However, so far, no paper has reported the effect of the virtual-photon field on the atomic entanglement in the TC model. We do need to study the TC model without RWA especially when the coupling is very large. In this study, we investigate the system consisting two atoms coupled with a dissipative coherent cavity field without RWA. We explore the entanglement dynamics of the two atoms numerically after obtaining the precisely analytic solution of the density matrix. Our results show that the atomic en-

*Project supported by the National Natural Science Foundation of China (Grant No. 10374025), the Education Ministry of Hunan Province, China (Grant No. 06A038) and the Natural Science Foundation of Hunan Province, China (Grant No. 07JJ3013).
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tanglement is enhanced due to the effect of the virtual-photon field if the two atoms are initially in some special states, and the high-degree entangled state of the two atoms remains valid after a long time. Compared with the cases in Refs. [13], [16], [19] and [31], the atomic entanglement resource we obtained in these special cases is superior. The model is a good candidate in quantum information processing especially in quantum computing as steady high-degree entanglement essential resource.

2. Precisely analytic solution of the two-atom TC model without RWA

The model presented in this paper is a two identical two level atoms coupled with a dissipative coherent cavity field. The cavity model is assumed to be resonant with atomic transition frequency. Considering cavity decay, the time dependent amplitude of the cavity field is assumed to be $\alpha(t) = (2\mathrm{i}g/k)(1 - \mathrm{e}^{-kt})$. Without rotating wave approximation, the Hamiltonian of the whole system in the interaction picture can be written as

$$\hat{H}_{\mathrm{I}} = \hbar g \sum_{i=1}^{2} (a^{\dagger} + a)(\sigma_i^{+} + \sigma_i^{-}), \tag{1}$$

where g is the coupling constant (taken equal for both atoms), $a^{\dagger}(a)$ is the creation (annihilation) operator for the field model, $\sigma_i^{+} = |e\rangle_i\langle g|(\sigma_i = |g\rangle_i\langle e|)$ the atomic raising (lowering) operator. Based on $\{|+\rangle_i, |-\rangle_i\}$ rotated basis connected to the standard basis $\{|g\rangle_i, |e\rangle_i\}$ via $|\pm\rangle_j = (|g\rangle_i \pm |e\rangle_i)/\sqrt{2}$, the Hamiltonian (1) becomes

$$\hat{H}'_{\mathrm{I}} = \hbar g \sum_{i=1}^{2} (a^{\dagger} + a)(|+\rangle_i\langle +| - |-\rangle_i\langle -|). \tag{2}$$

Considering the dissipative dynamics of the cavity field,[32,33] the density operator $\hat{\rho}$ of the whole system can be solved through the following master equation

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{\mathrm{i}}{\hbar}[\hat{H}'_{\mathrm{I}}, \hat{\rho}] + \hat{L}_{\mathrm{f}}\hat{\rho}, \tag{3}$$

where $\hat{L}_{\mathrm{f}}\hat{\rho} = (k/2)[2a\hat{\rho}a^{\dagger} - a^{\dagger}a\hat{\rho} - \hat{\rho}a^{\dagger}a]$, and k is the cavity decay rates. The density operator of the whole system can be expressed in decomposition form: $\hat{\rho}(t) = \sum_{n,m=1}^{4}\langle\varphi_n|\hat{\rho}(t)|\varphi_m\rangle|\varphi_n\rangle\langle\varphi_m| = \sum_{n,m=1}^{4}\hat{\rho}_{nm}(t)|\varphi_n\rangle\langle\varphi_m|$, where $|\varphi_m\rangle_{m=1}^{4} = \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ is the rotated basis of the atomic Hilbert space, and $\hat{\rho}(t)$ satisfies the normalised condition $\mathrm{Tr}[\hat{\rho}(t)] = 1$. Thus Eq. (3) is equal to the following set of uncoupled equations of the field operator.

$$\begin{aligned}
\frac{\partial\hat{\rho}_{11}}{\partial t} &= -2\mathrm{i}g[a^{\dagger} + a, \hat{\rho}_{11}] + \hat{L}_{\mathrm{f}}\hat{\rho}_{11},\\
\frac{\partial\hat{\rho}_{12}}{\partial t} &= -2\mathrm{i}g(a^{\dagger} + a)\hat{\rho}_{12} + \hat{L}_{\mathrm{f}}\hat{\rho}_{12},\\
\frac{\partial\hat{\rho}_{13}}{\partial t} &= -2\mathrm{i}g(a^{\dagger} + a)\hat{\rho}_{13} + \hat{L}_{\mathrm{f}}\hat{\rho}_{13},\\
\frac{\partial\hat{\rho}_{14}}{\partial t} &= -2\mathrm{i}g\{a^{\dagger} + a, \hat{\rho}_{14}\} + \hat{L}_{\mathrm{f}}\hat{\rho}_{14},\\
\frac{\partial\hat{\rho}_{22,23,33}}{\partial t} &= \hat{L}_{\mathrm{f}}\hat{\rho}_{22,23,33},\\
\frac{\partial\hat{\rho}_{24}}{\partial t} &= -2\mathrm{i}g\hat{\rho}_{24}(a^{\dagger} + a) + \hat{L}_{\mathrm{f}}\hat{\rho}_{24},\\
\frac{\partial\hat{\rho}_{34}}{\partial t} &= -2\mathrm{i}g\hat{\rho}_{34}(a^{\dagger} + a) + \hat{L}_{\mathrm{f}}\hat{\rho}_{34},\\
\frac{\partial\hat{\rho}_{44}}{\partial t} &= 2\mathrm{i}g[a^{\dagger} + a, \hat{\rho}_{44}] + \hat{L}_{\mathrm{f}}\hat{\rho}_{44},
\end{aligned} \tag{4}$$

where the brackes $[\,,]$ and $\{\,,\}$ denote the commutator and anti-commutator symbols respectively. Noted that $\hat{\rho}_{mn}(t) = \hat{\rho}_{nm}(t)^{\dagger}$, the density matrix of the field system can be obtained solving Eq. (4). As only creation and annihilation operators are contained in Eq. (4), and characteristic functions[34,35] enable the expectation value of any operator function consisting a and $a^{\dagger}$ to be calculated, so we can solve Eq. (4) via characteristic function $\chi(\xi, t) = \mathrm{Tr}_{\mathrm{f}}[\hat{\rho}\hat{D}(\xi)]\exp(|\xi|/2)$ which contains all information needed to reconstruct the density matrix of the system, where $\hat{D}(\xi)$ is the well known Glauber displacement operator. $\hat{D}(\xi) = \exp(\xi a^{\dagger} - \xi^* a)$, and ξ is a complex number. Through simple calculation we can transform Eq. (4) into the following forms

$$\begin{aligned}
\dot{\chi}_{11} &= \left[2\mathrm{i}g(\xi + \xi^*) - k\Big(\xi\frac{\partial}{\partial\xi} + \xi^*\frac{\partial}{\partial\xi^*} + |\xi|^2\Big)\right]\chi_{11},\\
\dot{\chi}_{12} &= \left\{-2\mathrm{i}g\left[\frac{\partial}{\partial\xi} - \frac{\partial}{\partial\xi^*} - \frac{1}{2}(\xi + \xi^*)\right] + k\left(\xi\frac{\partial}{\partial\xi} - \xi^*\frac{\partial}{\partial\xi^*}\right)\right\}\chi_{12},\\
\dot{\chi}_{13} &= \left\{-2\mathrm{i}g\left[\frac{\partial}{\partial\xi} - \frac{\partial}{\partial\xi^*} - \frac{1}{2}(\xi + \xi^*)\right] + k\left(\xi\frac{\partial}{\partial\xi} - \xi^*\frac{\partial}{\partial\xi^*}\right)\right\}\chi_{13},
\end{aligned}$$

$$\begin{aligned}
\dot{\chi}_{14} &= \left[-4\mathrm{i}g\left(\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\xi^*}\right)-k\left(\xi\frac{\partial}{\partial\xi}+\xi^*\frac{\partial}{\partial\xi^*}\right)\right]\chi_{14},\\
\dot{\chi}_{22,23,33} &= -k\left(\xi\frac{\partial}{\partial\xi}+\xi^*\frac{\partial}{\partial\xi^*}+|\xi|^2\right)\chi_{22,23,33},\\
\dot{\chi}_{24} &= \left\{-2\mathrm{i}g\left[\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\xi^*}+\frac{1}{2}(\xi+\xi^*)\right]-k\left(\xi\frac{\partial}{\partial\xi}-\xi^*\frac{\partial}{\partial\xi^*}\right)\right\}\chi_{24},\\
\dot{\chi}_{34} &= \left\{-2\mathrm{i}g\left[\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\xi^*}+\frac{1}{2}(\xi+\xi^*)\right]-k\left(\xi\frac{\partial}{\partial\xi}-\xi^*\frac{\partial}{\partial\xi^*}\right)\right\}\chi_{34},\\
\dot{\chi}_{44} &= \left[-2\mathrm{i}g(\xi+\xi^*)-k\left(\xi\frac{\partial}{\partial\xi}+\xi^*\frac{\partial}{\partial\xi^*}\right)\right]\chi_{44}. \qquad (5)
\end{aligned}$$

The initial state of the whole system can be written as

$$\begin{aligned}
\hat{\rho}(0) &= \hat{\rho}_{\mathrm{f}}(0)\otimes\hat{\rho}_a(0)\\
&= |\alpha(0)\rangle\langle\alpha(0)|\otimes|\Psi(0)\rangle\langle\Psi(0)|, \qquad (6)
\end{aligned}$$

where $|\Psi(0)\rangle=\sum_{m=1}^{4}\lambda_m|\varphi_m\rangle$ is the initiall state of the atom-system. Thus the initial characteristic function of the cavity field is

$$\chi_{m,n}(\xi,0)=\Sigma_{m,n=1}^{4}\exp[\xi\alpha^*(0)-\xi^*\alpha(0)]\lambda_m\lambda_n^*. \quad (7)$$

By representing $\partial/\partial\xi=(1/2)(\partial/\partial x-\mathrm{i}\partial/\partial y)$, and $\partial/\partial\xi^*=(1/2)(\partial/\partial x+\mathrm{i}\partial/\partial y)$, Eq. (5) turns to be differential equations in which the real symbols x and y are the only parameters. Then we obtain

$$\begin{aligned}
\chi_{11} &= \frac{1}{2}|\lambda_1+\lambda_3|^2\exp[-\alpha^*(t)\xi+\alpha(t)\xi^*],\\
\chi_{12} &= -\frac{1}{2}(\lambda_1+\lambda_3)(\lambda_2-\lambda_4)^*F(t)\exp[\alpha(t)\xi^*],\\
\chi_{13} &= -\frac{1}{2}(\lambda_1+\lambda_3)(\lambda_2+\lambda_4)^*F(t)\exp[\alpha(t)\xi^*],\\
\chi_{14} &= \frac{1}{2}(\lambda_1+\lambda_3)(\lambda_1-\lambda_3)^*F^4(t)\\
&\quad\times\exp[\alpha(t)\xi^*+\alpha^*(t)\chi],\\
\chi_{22} &= \frac{1}{2}|\lambda_2-\lambda_4|^2,\\
\chi_{23} &= \frac{1}{2}(\lambda_2-\lambda_4)(\lambda_2+\lambda_4)^*,\\
\chi_{24} &= -\frac{1}{2}(\lambda_2-\lambda_4)(\lambda_1-\lambda_3)^*F(t)\exp[\alpha^*(t)\xi],\\
\chi_{33} &= \frac{1}{2}|\lambda_2+\lambda_4|^2,\\
\chi_{34} &= -\frac{1}{2}(\lambda_2+\lambda_4)(\lambda_1-\lambda_3)^*F(t)\exp[\alpha^*(t)\xi],\\
\chi_{44} &= \frac{1}{2}|\lambda_1-\lambda_3|^2\exp[\alpha^*(t)\xi+\alpha(t)\xi^*], \qquad (8)
\end{aligned}$$

where the function $F(t)$ is $\exp[-4g^2/kt+(4g^2/k^2)(1-\mathrm{e}^{-kt})]$. Consequently, the corresponding density matrixelements of the field system reconstructed from Eq. (8) are given by

$$\begin{aligned}
\hat{\rho}_{11} &= \frac{1}{2}|\lambda_1+\lambda_3|^2|-\alpha(t)\rangle\langle-\alpha(t)|,\\
\hat{\rho}_{12} &= \frac{1}{2}(\lambda_1+\lambda_3)(\lambda_2-\lambda_4)^*F(t)|-\alpha(t)\rangle\langle 0|,\\
\hat{\rho}_{13} &= \frac{1}{2}(\lambda_1+\lambda_3)(\lambda_2+\lambda_4)^*\\
&\quad\times\frac{F(t)}{\exp\left(-\dfrac{|\alpha(t)|^2}{2}\right)}|-\alpha(t)\rangle\langle 0|,\\
\hat{\rho}_{14} &= \frac{1}{2}(\lambda_1+\lambda_3)(\lambda_1-\lambda_3)^*\\
&\quad\times\frac{F^4(t)}{\exp(-2|\alpha(t)|^2)}|-\alpha(t)\rangle\langle\alpha(t)|,\\
\hat{\rho}_{22} &= \frac{1}{2}|\lambda_2+\lambda_4|^2|0\rangle\langle 0|,\\
\hat{\rho}_{23} &= \frac{1}{2}(\lambda_2-\lambda_4)(\lambda_2+\lambda_4)^*|0\rangle\langle 0|,\\
\hat{\rho}_{24} &= -\frac{1}{2}(\lambda_2-\lambda_4)(\lambda_1-\lambda_3)^*\\
&\quad\times\frac{F(t)}{\exp\left(-\dfrac{|\alpha(t)|^2}{2}\right)}|0\rangle\langle\alpha(t)|,\\
\hat{\rho}_{33} &= \frac{1}{2}|\lambda_2+\lambda_4|^2|0\rangle\langle 0|,\\
\hat{\rho}_{34} &= -\frac{1}{2}(\lambda_2+\lambda_4)(\lambda_1-\lambda_3)^*\\
&\quad\times\frac{F(t)}{\exp\left(-\dfrac{|\alpha(t)|^2}{2}\right)}|0\rangle\langle\alpha(t)|,\\
\hat{\rho}_{44} &= \frac{1}{2}|\lambda_1-\lambda_3|^2|-\alpha(t)\rangle\langle-\alpha(t)|. \qquad (9)
\end{aligned}$$

For instance, to reconstruct $\hat{\rho}_{11}$, i.e., $|-\alpha(t)\rangle\langle-\alpha(t)|\hat{D}(\xi)=|-\alpha(t)\rangle\langle-\alpha(t)|\exp[-\alpha^*(t)\xi+\alpha(t)\xi^*-\exp(|\xi|^2/2)]$ indicates that only $\hat{\rho}_{11}=(1/2)|\lambda_1+\lambda_2|^2|-\alpha(t)\rangle\langle-\alpha(t)|$ makes the equation $\mathrm{Tr}_{\mathrm{f}}[(1/2)|\lambda_1+\lambda_2|^2|-\alpha(t)\rangle\langle-\alpha(t)|\hat{D}(\xi)]\exp(|\xi|^2/2)=\chi_{11}$ tenable. Similarly, other elements of the density matrix can be reconstructed.

3. Dynamics of the atomic entanglement

In this section, we investigate the entanglement dynamics of the two atoms. Assuming the initial state of the atom-system as

$$|\Psi(0)\rangle = \sum_{m=1}^{4} \lambda_m |\varphi_m\rangle, \tag{10}$$

where $\{|\varphi_m\rangle\}_{m=1,2,3,4} = \{\phi^+, \phi^-, \Psi^+, \Psi^-\}$ is the Bell basis with $\phi^{\pm} = (|ee\rangle \pm |gg\rangle)/\sqrt{2}$, $\Psi^{\pm} = (|eg\rangle \pm |ge\rangle)/\sqrt{2}$, and the coefficients λ_m are normalised. The atomic density matrix based on the Bell basis $\{\Phi^+, \Phi^-, \Psi^+, \Psi^-\}$, which is obtained from Eq. (9) via a unitary transformation after tracing over the coherent field variables is given by

$$\hat{\boldsymbol{\rho}}_a(t) = \begin{pmatrix} \dfrac{|1+F^4||\lambda_1|^2 + |1-F^4||\lambda_3|^2}{2} & -\mathrm{i}F\lambda_1\lambda_2^* & -\mathrm{i}\dfrac{|1+F^4||\lambda_1\lambda_3^* + |1-F^4||\lambda_1^*\lambda_3}{2} & F\lambda_1\lambda_4^* \\ \mathrm{i}F\lambda_1\lambda_2^* & |\lambda_2|^2 & F\lambda_2\lambda_3^* & \mathrm{i}\lambda_2\lambda_4^* \\ \mathrm{i}\dfrac{|1+F^4||\lambda_1\lambda_3^* + |1-F^4||\lambda_1^*\lambda_3}{2} & F\lambda_2^*\lambda_3 & \dfrac{|1-F^4||\lambda_1|^2 + |1+F^4||\lambda_3|^2}{2} & \mathrm{i}F\lambda_3\lambda_4^* \\ F\lambda_1^*\lambda_4 & -\mathrm{i}\lambda_2^*\lambda_4 & -\mathrm{i}F\lambda_3^*\lambda_4 & |\lambda_4|^2 \end{pmatrix}. \tag{11}$$

Then we explore the atomic entanglement-dynamics using concurrence $C(t)$ defined as

$$C(t) = \max(0, \sqrt{e_1} - \sqrt{e_2} - \sqrt{e_3} - \sqrt{e_4}), \tag{12}$$

where e_i are the eigenvalues of the matrix of $R(t)$ taken in decreasing order.

$$R(t) = \hat{\rho}_a(t)(\sigma_y \otimes \sigma_y)\hat{\rho}_a^*(t)(\sigma_y \otimes \sigma_y). \tag{13}$$

First, we consider a simple condition that the two atoms are prepared in the maximally entangled state Φ^- with $\lambda_2 = 1$ ($\lambda_1 = \lambda_3 = \lambda_4 = 0$). In this case, only one term of the density matrix $|\lambda_2|^2|\Phi^-\rangle\langle\Phi^-|$ left, and of course the concurrence $C(t)$ keeps the maximum ($C(t) = 1$) and independent of time. This result is also obtained for the case that two atoms are prepared in the maximally entangled state Ψ^-. Second, if the two atoms are prepared in the Bell state Φ^+ ($\lambda_1 = 1$, $\lambda_2 = \lambda_3 = \lambda_4 = 0$), the concurrence mainly determined by function $F(t)$ decreases to zero gradually which is numerically showed in Fig. 1. The similar result is obtained for the condition that the two atoms are initially in the Bell state Ψ^+. Third, if the coefficients $\lambda_1 = 1/\sqrt{2}$, $\lambda_2 = -1/\sqrt{2}$, or in other words if the two atoms are initially in the ground states, atomic entanglement will come up to a constant quickly, and then keeps steady despite the dissipative behaviour of the coherent cavity field, and the numeric result is plotted in Fig. 2. The reason is possible that one ground-state atom will make a transition when absorb a photon from the cavity, and will simultaneously emit a virtual photon which is absorbed by the other atom. Then the second atom makes a transition from $|g\rangle$ to $|e\rangle$ by emitting one photon which is absorbed by the first one. This means that the two atoms will be related via virtual-photon changing coperation (induced coperation of virtual-photon field). Under the effect of virtual-photon field, energy shift of the two atoms will soon occur, and then there will be one-photon detune between the cavity and the atomic system, so there is no energy changing between them, and of course the atomic dynamics is stable despite the cavity decay in the following time. On the other hand, we give a intuitionistic explanation via the wave function corresponding to the initial atomic $|gg\rangle$ state. Initially, when dimensionless time kt is very small thus $(1 - \mathrm{e}^{-kt})$ is equivalent to kt, from Eq. (10), we can obtain the state of the whole system based on rotated basis

$$|\Psi(t)\rangle \propto |--\rangle \otimes |\alpha(t)\rangle + (|+-\rangle + |-+\rangle) \otimes |0\rangle + |++\rangle \otimes |-\alpha(t)\rangle, \tag{14}$$

which is pure with zero-concurrence. Then with

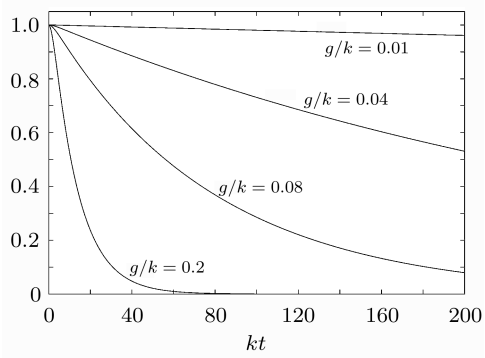


Fig. 1. The time evolution of atom–atom concurrence as a function of dimensionless time kt for different values of g/k: 0.01, 0.04, 0.08, 0.2, atoms prepared in the Bell state Φ^+.

time evolution, the state of the whole system become mixed, and the stabilize ($kt >> 1$, $e^{-kt} \simeq 0$) density matrix of the atomic system is

$$\begin{aligned}&\hat{\rho}_a(t)\\ &= \frac{1}{4}(|++\rangle\langle++|+|--\rangle\langle--|+|+-\rangle\langle+-|\\ &\quad +|+-\rangle\langle-+|+|-+\rangle\langle+-|\\ &\quad +|-+\rangle\langle-+|),\end{aligned} \tag{15}$$

with a value of the concurrence, 0.712, which is identical with the result in Fig. 2. And this entanglement dynamics is the same with the condition ($\lambda_3 = 1/\sqrt{2}$, $\lambda_4 = -1/\sqrt{2}$, $\lambda_1 = \lambda_2 = 0$). It shows that although the virtual-photon field plays a noise role in the JC model,[28−30] it assists the generation of entanglement in the TC model in some special cases. But, how would it be if the two atoms are prepared in the superposition of the Bell state Φ^- and Ψ^+. We can predict it from Eq. (11) that the atomic entanglement will decrease to zero gradually as the concurrence of Ψ^+ is mainly ruled by the decoherent function $F(t)$, while the concurrence of Φ^- does not evolve as numerically proved in Fig. 3.

The analogical result can be obtained if other conditions are considered. We noted that there is an interesting outcome: when the atomic system is initially in superposition of different Bell states, relative phase between these Bell states does not affect the entanglement dynamics. It is observed intuitionisticly by comparing Figs. 4(a) and 4(b) with Figs. 2 and 3. From numeric results, we noted that the larger the coupling constant g is, the stronger the effect of the virtual-photon field will be, and the most wonderful case is that the two atoms are initially in the Bell state Φ^- or Ψ^-. It is not necessary to preliminarily entangle two atoms because the steady and high-degree entanglement of atoms disentangled initially could be obtained with the assistance of the virtual-photon field.

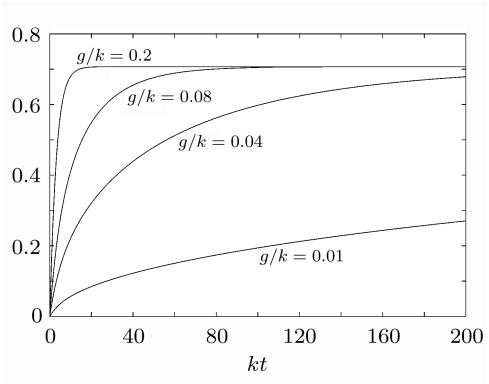


Fig. 2. Entanglement-dynamics of the atoms prepared in the ground state vs. dimensionless time kt for different coupling constants g/k: 0.01, 0.04, 0.08, 0.2.

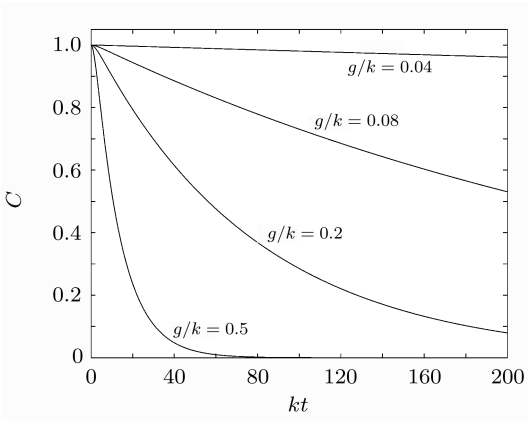


Fig. 3. The time evolution of atom–atom concurrence as a function of dimensionless time kt for different values of g/k: 0.04, 0.08, 0.2, 0.5. Atoms prepared in the state $\frac{1}{\sqrt{(2)}}(\Phi^- + \Psi^+)$.

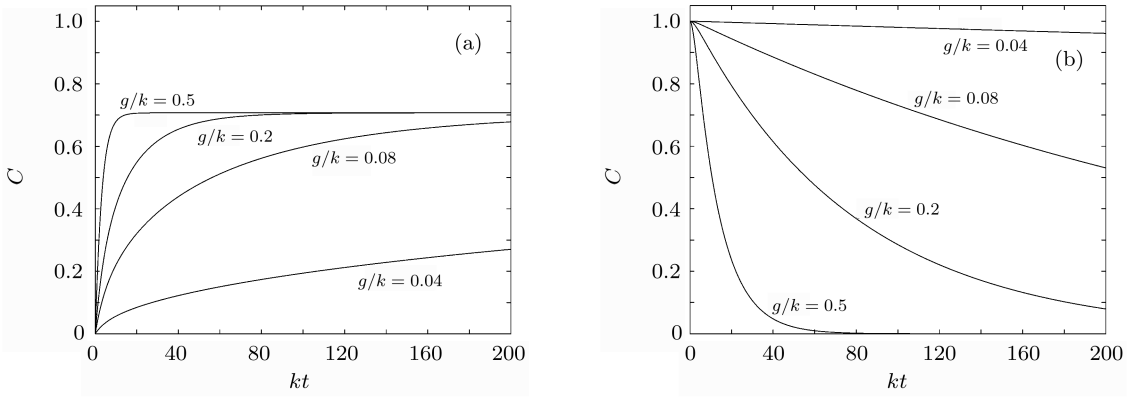


Fig. 4. Entanglement-dynamics of the atoms prepared in the partially entangled state (a) $|\psi(0)\rangle = 1/\sqrt{2}\Phi^+ + 1/\sqrt{2}\,e^{i\theta}\Phi^-$, (b) $|\psi(0)\rangle = 1/\sqrt{2}\Psi^+ + 1/\sqrt{2}\,e^{i\theta}\Phi^-$ where $\theta \in (0, \pi)$ vs. dimensionless time kt for different coupling constants g/k: 0.04, 0.08, 0.2, 0.5.

4. Summary

We investigate the entanglement dynamics of two atoms interacting with a dissipative coherent cavity field without RWA, and it is necessary especially when the effect of the virtual-photon field is evident. Some interesting results are obtained by analytical and numerical describing. We find that if the two atoms are initially in some disentangled states ($|gg\rangle$,$|ee\rangle$), concurrence will come up to a constant from zero quickly, then keeps steady due to the effect of virtual-photon field (counter-rotating terms), and the larger the coupling constant is the faster the entanglement-enhancing process is. But if the two atoms are initially in the maximally entangled Bell state (Φ^- or Ψ^-), the entanglement always deposited in the cavity without any change. on the other hand, if the two atoms are initially in the superposition of some Bell states discussed in the third section, concurrence will be decreasing with the evolution of the dissipative coherent field, and the larger the coupling constant is the faster the decreasing process is, which is contrary to the case that the two atoms are initially in the ground or excited state. Different with the result obtained in the JC model by Peng and Li in Refs. [28]–[30], we proved that the effect of virtual-photon field is not always the noise in a system, it may play an assistant role in the generation of entanglement in some special cases in the TC model. And this is coincident with the ideal that noise or dissipation can assist the generation of entanglement.[36−39] The system we considered is a good candidate to realise some quantum information processing with high probability, especially in quantum computing where the noise of qubit (such as decoherence and cavity dissipative) stored in the hardware makes it so hard to implement a quantum computer.

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