New approximate solution for time-fractional coupled KdV equations by generalised differential transform method*

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In this paper, the generalised two-dimensional differential transform method (DTM) of solving the time-fractional coupled KdV equations is proposed. The fractional derivative is described in the Caputo sense. The presented method is a numerical method based on the generalised Taylor series expansion which constructs an analytical solution in the form of a polynomial. An illustrative example shows that the generalised two-dimensional DTM is effective for the coupled equations.

Keywords: fractional coupled KdV equations, Caputo fractional derivative, differential transform method, approximate analytic solution

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1. Introduction

In the last decade, the fractional differential equations have been widely used in various fields of physics and engineering. The analytical and the numerical approximations of such problems have attracted great attention and become a considerably interesting subject in mathematical physics. Some powerful methods including the Adomian decomposition method,[1,2] variational iteration method,[3,4] homotopy analysis method[5,6] and fractional difference method[7,8] have been developed to obtain exact and approximate analytic solutions. Among these solution techniques, the Adomian decomposition method and the variational iteration method are more clear and realistic methods of solving fractional differential equations, because they give the approximate solutions of the considered problems without performing any linearization or discretization.

In this paper, we consider the analytical solutions of the coupled KdV equations with time-fractional derivatives of the form[9]

$$\begin{cases} \dfrac{\partial^\alpha u}{\partial t^\alpha} = a\dfrac{\partial^3 u}{\partial x^3} + 6au\dfrac{\partial u}{\partial x} + 2bv\dfrac{\partial v}{\partial x}, & t > 0, \quad 0 < \alpha \leq 1, \\ \dfrac{\partial^\beta v}{\partial t^\beta} = -\dfrac{\partial^3 v}{\partial x^3} - 3u\dfrac{\partial v}{\partial x}, & t > 0, \quad 0 < \beta \leq 1, \end{cases} \tag{1}$$

where a and b are constants; α and β are parameters describing the order of the time-fractional derivatives of $u(x,t)$ and $v(x,t)$, respectively. The functions $u(x,t)$ and $v(x,t)$ are assumed to be the causal functions of time and space, i.e., vanishing for $t < 0$ and $x < 0$. The fractional derivatives are considered in the Caputo sense. In the case where $\alpha = \beta = 1$, Eq. (1) reduces to the classical coupled KdV equations.[10−13]

The differential transform method was proposed first by Zhou in 1986 to study electric circuits.[14] The differential transform is an iterative procedure based on the Taylor series expansion which constructs an analytic solution in the form of a polynomial. The method is well addressed in Refs. [15]–[23]. Recently, the generalised differential transform method of obtaining approximate analytic solutions of some linear and nonlinear differential equations of fractional order[24−26] is developed. The aim of the present

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paper is to directly extend the the generalised two-dimensional differential transform method (DTM) to the approximate solution of Eq. (1).

There are several definitions of a fractional derivative of order $\alpha > 0$. Here, we mention the basic definitions of the Caputo fractional-order integration and differentiation,[27,28] which are used in the following.

Definition 1 The fractional derivative of $f(x)$ in the Caputo sense is defined as

$$D^{\nu} f(x) = J_0^{m-\nu} D^m f(x) = \frac{1}{\Gamma(m-\nu)} \int_0^x (x-t)^{m-\nu-1} f^{(m)}(t) \mathrm{d}t,$$

for $m-1 < \nu < m$, and $m \in N, x > 0$. Here, J_0^{μ} is the Riemann–Liouville integral operator of order $\mu > 0$, defined by

$$J_0^{\mu} f(x) = \frac{1}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1} f(t) \mathrm{d}t, \quad \mu > 0.$$

Definition 2 For m to be the smallest integer that exceeds α, the Caputo time-fractional derivative of order $\alpha > 0$ is defined as

$$D^{\alpha} u(x,t) = \frac{\partial^{\alpha} u}{\partial t^{\alpha}} = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \int_0^t (t-\xi)^{m-\alpha-1} \dfrac{\partial^m u(x,\xi)}{\partial \xi^m} \mathrm{d}\xi, & \text{for } m-1 < \alpha < m, \\ \dfrac{\partial^m u(x,t)}{\partial t^m}, & \text{for } \alpha = m \in N. \end{cases}$$

The Caputo fractional derivative is considered here because it allows traditional initial and boundary conditions to be included in the formulation of the problem. For more information about the mathematical properties of fractional derivatives and integrals, one can consult the aforementioned references.

2. Generalised two-dimensional DTM

Consider a function of two variables, $u(x,y)$, and suppose that it can be represented as a product of two single variable functions, i.e., $u(x,y) = f(x)g(y)$. Based on the properties of generalised two-dimensional differential transform, the function $u(x,y)$ can be represented as

$$u(x,y) = \sum_{k=0}^{\infty} F_{\alpha}(k)(x-x_0)^{k\alpha} \sum_{h=0}^{\infty} G_{\beta}(h)(y-y_0)^{h\beta} = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U_{\alpha,\beta}(k,h)(x-x_0)^{k\alpha}(y-y_0)^{h\beta}, \tag{2}$$

where $0 < \alpha, \beta \leq 1$, and $U_{\alpha,\beta}(k,h) = F_{\alpha}(k)G_{\beta}(h)$ that is called the spectrum of $u(x,y)$. The generalised two-dimensional differential transform of the function $u(x,y)$ is given by

$$U_{\alpha,\beta}(k,h) = \frac{1}{\Gamma(\alpha k+1)\Gamma(\beta h+1)} [(D_{x_0}^{\alpha})^k (D_{y_0}^{\beta})^h u(x,y)]_{(x_0,y_0)}, \tag{3}$$

where $(D_{x_0}^{\alpha})^k = D_{x_0}^{\alpha} D_{x_0}^{\alpha} \ldots D_{x_0}^{\alpha}$, k is times. In the case where $\alpha = 1$ and $\beta = 1$, the generalised two-dimensional differential transform (3) reduces to a two-dimensional differential transform.

The fundamental theorems of the generalised two-dimensional differential transform are as follows.

Theorem 1 Suppose that $U_{\alpha,\beta}(k,h), V_{\alpha,\beta}(k,h)$ and $W_{\alpha,\beta}(k,h)$ are the differential transformations of the functions $u(x,y), v(x,y)$ and $w(x,y)$, respectively.

(a) If $u(x,y) = v(x,y) \pm w(x,y)$, then $U_{\alpha,\beta}(k,h) = V_{\alpha,\beta}(k,h) \pm W_{\alpha,\beta}(k,h)$.

(b) If $u(x,y) = av(x,y), a \in \mathbb{R}$, then $U_{\alpha,\beta}(k,h) = aV_{\alpha,\beta}(k,h)$.

(c) If $u(x,y) = v(x,y)w(x,y)$, then $U_{\alpha,\beta}(k,h) = \sum_{r=0}^{k} \sum_{s=0}^{h} V_{\alpha,\beta}(r,h-s)W_{\alpha,\beta}(k-r,s)$.

(d) If $u(x,y) = (x-x_0)^{n\alpha}(y-y_0)^{m\beta}$, then $U_{\alpha,\beta}(k,h) = \delta(k-n)\delta(h-m)$.

(e) If $u(x,y) = D_{x_0}^{\alpha} v(x,y), 0 < \alpha \leq 1$, then $U_{\alpha,\beta}(k,h) = \dfrac{\Gamma(\alpha(k+1)+1)}{\Gamma(\alpha k+1)} V_{\alpha,\beta}(k+1,h)$.

Theorem 2 If $u(x,y) = f(x)g(y)$, function $f(x) = x^{\lambda}h(x)$, where $\lambda > -1$ and $h(x)$ has the generalised Taylor series expansion $h(x) = \sum_{n=0}^{\infty} a_n (x-x_0)^{\alpha n}$, and

(i) $\beta < \lambda + 1$ and α arbitrary or

(ii) $\beta \geq \lambda + 1$, α arbitrary and $a_n = 0$ for $n = 0, 1, \ldots, m-1$, where $m-1 < \beta \leq m$,

then the generalised differential transform (3) becomes

$$U_{\alpha,\beta}(k,h) = \frac{1}{\Gamma(\alpha k+1)\Gamma(\beta h+1)} [D_{x_0}^{\alpha k} (D_{y_0}^{\beta})^h u(x,y)]_{(x_0,y_0)};$$

If $u(x,y)=D^{\gamma}_{x_0}v(x,y), m-1<\gamma\leq m$, and $v(x,y)=f(x)g(y)$, then

$$U_{\alpha,\beta}(k,h)=\frac{\Gamma(\alpha k+\gamma+1)}{\Gamma(\alpha k+1)}V_{\alpha,\beta}(k+\gamma/\alpha,h).$$

The details of some properties can be found in Ref. [29].

3. Application and result

Example Consider the following time-fractional coupled KdV equations:

$$\begin{cases}\dfrac{\partial^{\alpha}u}{\partial t^{\alpha}}=-\dfrac{\partial^{3}u}{\partial x^{3}}-6u\dfrac{\partial u}{\partial x}+3v\dfrac{\partial v}{\partial x}, & t>0, \quad 0<\alpha\leq 1,\\ \dfrac{\partial^{\beta}v}{\partial t^{\beta}}=-\dfrac{\partial^{3}v}{\partial x^{3}}-3u\dfrac{\partial v}{\partial x}, & t>0, \quad 0<\beta\leq 1,\end{cases} \tag{4}$$

subject to the initial conditions

$$u(x,0)=\frac{4c^{2}\mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^{2}}, \quad v(x,0)=\frac{4c^{2}\mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^{2}}. \tag{5}$$

The exact solutions of Eq. (4), for the special case where $\alpha=\beta=1$, given in Ref. [30], are

$$u(x,t)=v(x,t)=\frac{4c^{2}\mathrm{e}^{c(x-c^{2}t)}}{(1+\mathrm{e}^{c(x-c^{2}t)})^{2}}, \tag{6}$$

where c is an arbitrary constant.

Suppose that the solutions $u(x,t)$ and $v(x,t)$ can be represented as the products of single-valued functions, apply the generalised two-dimensional differential transform to both sides of Eq. (4), and use the related theorems, then we will have

$$\begin{aligned}\frac{\Gamma(\alpha(h+1)+1)}{\Gamma(\alpha h+1)}U_{1,\alpha}(k,h+1)\ =\ &-(k+1)(k+2)(k+3)U_{1,\alpha}(k+3,h)\\ &-6\sum_{r=0}^{k}\sum_{s=0}^{h}(k+1-r)U_{1,\alpha}(r,h-s)U_{1,\alpha}(k+1-r,s)\\ &+3\sum_{r=0}^{k}\sum_{s=0}^{h}(k+1-r)V_{1,\beta}(r,h-s)V_{1,\beta}(k+1-r,s),\end{aligned} \tag{7}$$

and

$$\begin{aligned}\frac{\Gamma(\beta(h+1)+1)}{\Gamma(\beta h+1)}V_{1,\beta}(k,h+1)\ =\ &-(k+1)(k+2)(k+3)V_{1,\beta}(k+3,h)\\ &-3\sum_{r=0}^{k}\sum_{s=0}^{h}(k+1-r)U_{1,\alpha}(r,h-s)V_{1,\beta}(k+1-r,s).\end{aligned} \tag{8}$$

The generalised two-dimensional differential transforms of the initial conditions can be obtained as follows:

$$\begin{aligned}&U_{1,\alpha}(k,0)=V_{1,\beta}(k,0)=0, \quad \text{if} \quad k=1,3,5,\ldots,\\ &U_{1,\alpha}(0,0)=V_{1,\beta}(0,0)=c^{2}, \quad U_{1,\alpha}(2,0)=V_{1,\beta}(2,0)=-c^{4}/4, \quad U_{1,\alpha}(4,0)=V_{1,\beta}(4,0)=c^{6}/24,\ldots.\end{aligned}$$

Utilizing the recurrence relations (7), (8) and the transformed initial conditions, we can obtain all the $U_{1,\alpha}(k,h)$ and $V_{1,\beta}(k,h)$ with help of MATHEMATICA. Moreover, substituting all $U_{1,\alpha}(k,h)$ and $V_{1,\beta}(k,h)$ into Eq. (2) yields the series form solutions of Eq. (4) as

$$\begin{aligned}u(x,t)&=u_{1}(x,t)+u_{2}(x,t)+u_{3}(x,t)+u_{4}(x,t)+\ldots,\\ v(x,t)&=v_{1}(x,t)+v_{2}(x,t)+v_{3}(x,t)+v_{4}(x,t)+\ldots,\end{aligned}$$

where

$$
\begin{aligned}
u_1(x,t) &= c^2 - \frac{c^4}{4}x^2 + \frac{c^6}{24}x^4 - \frac{17c^8}{2880}x^6 + \frac{31c^{10}}{40320}x^8 + \dots,\\
u_2(x,t) &= \left\{ \frac{c^6}{2}x - \frac{c^8}{6}x^3 + \frac{17c^{10}}{480}x^5 - \frac{31c^{12}}{5040}x^7 + \frac{691c^{14}}{725760}x^9 + \dots \right\} \frac{t^\alpha}{\Gamma(1+\alpha)},\\
u_3(x,t) &= \left\{ \left(\frac{3c^8}{2} - \frac{21c^{10}}{8}x^2 + \frac{47c^{12}}{32}x^4 - \frac{251c^{14}}{480}x^6 + \frac{553c^{16}}{3840}x^8 + \dots \right) \frac{\Gamma(1+\alpha)}{\Gamma(1+\beta)} \right.\\
&\quad \left. + \left(-2c^8 + \frac{25c^{10}}{8}x^2 - \frac{79c^{12}}{48}x^4 + \frac{163c^{14}}{288} - \frac{769c^{16}}{5040}x^8 + \dots \right) \right\} \frac{t^{2\alpha}}{\Gamma(1+2\alpha)},\\
u_4(x,t) &= \left\{ \left(\frac{3c^{12}}{4}x - c^{14}x^3 + \frac{91c^{16}}{160}x^5 - \frac{181c^{18}}{840}x^7 + \frac{61787c^{20}}{967680}x^9 + \dots \right) \frac{\Gamma(1+2\alpha)}{\Gamma^2(1+\beta)} \right.\\
&\quad + \left(\frac{3c^{12}}{4}x + \frac{41c^{14}}{4}x^3 - \frac{967c^{16}}{80}x^5 + \frac{96959c^{18}}{13440}x^7 - \frac{2918563c^{20}}{967680}x^9 + \dots \right) \frac{\Gamma(1+\alpha)}{\Gamma(1+\beta)}\\
&\quad + \left(-4c^{12}x - \frac{23c^{14}}{3}x^3 + \frac{329c^{16}}{30}x^5 - \frac{276413c^{18}}{40320}x^7 + \frac{2120047c^{20}}{725760}x^9 + \dots \right)\\
&\quad + \left(-\frac{3c^{12}}{2}x + 2c^{14}x^3 - \frac{91c^{16}}{80}x^5 + \frac{181c^{18}}{420}x^7 - \frac{61787c^{20}}{483840}x^9 + \dots \right) \frac{\Gamma(1+2\alpha)}{\Gamma^2(1+\alpha)}\\
&\quad + \left(-\frac{3c^{12}}{4}x + \frac{35c^{14}}{8}x^3 - \frac{169c^{16}}{40}x^5 + \frac{389c^{18}}{168}x^7 - \frac{127511c^{20}}{138240}x^9 + \dots \right) \frac{\Gamma(1+2\alpha)}{\Gamma(1+2\beta)}\\
&\quad \left. + \left(\frac{9c^{12}}{2}x - \frac{33c^{14}}{8}x^3 + \frac{249c^{16}}{40}x^5 - \frac{3401c^{18}}{1120}x^7 + \dots \right) \frac{\Gamma(1+2\alpha)\Gamma(1+\beta)}{\Gamma(1+\alpha)\Gamma(1+2\beta)} \right\} \frac{t^{3\alpha}}{\Gamma(1+3\alpha)};
\end{aligned}
$$

and

$$
\begin{aligned}
v_1(x,t) &= c^2 - \frac{c^4}{4}x^2 + \frac{c^6}{24}x^4 - \frac{17c^8}{2880}x^6 + \frac{31c^{10}}{40320}x^8 + \dots,\\
v_2(x,t) &= \left\{ \frac{c^6}{2}x - \frac{c^8}{6}x^3 + \frac{17c^{10}}{480}x^5 - \frac{31c^{12}}{5040}x^7 + \frac{691c^{14}}{725760}x^9 + \dots \right\} \frac{t^\beta}{\Gamma(1+\beta)},\\
v_3(x,t) &= \left\{ \left(-\frac{c^8}{2} - \frac{c^{10}}{4}x^2 + \frac{31c^{12}}{96}x^4 - \frac{211c^{14}}{1440}x^6 + \frac{3653c^{16}}{80640}x^8 + \dots \right) \right.\\
&\quad \left. + \left(\frac{3c^{10}}{4}x^2 - \frac{c^{12}}{2}x^4 + \frac{91c^{14}}{480}x^6 - \frac{181c^{16}}{3360}x^8 + \dots \right) \Gamma(1+\beta) \right\} \frac{t^{2\beta}}{\Gamma(1+2\beta)},\\
v_4(x,t) &= \left(\frac{15c^{12}}{2}x - \frac{125c^{14}}{8}x^3 + 13c^{16}x^5 - \frac{18203c^{18}}{2688}x^7 + \frac{255125c^{20}}{96768}x^9 + \dots \right) \frac{\Gamma(1+\beta)}{\Gamma(1+\alpha)}\\
&\quad + \left(-\frac{25c^{12}}{4}x + \frac{40c^{14}}{3}x^3 - \frac{1109c^{16}}{96}x^5 + \frac{49949c^{18}}{8064}x^7 - \frac{1428271c^{20}}{580608}x^9 + \dots \right)\\
&\quad + \left(-\frac{3c^{12}}{4}x + c^{14}x^3 - \frac{91c^{16}}{180}x^5 + \frac{181c^{18}}{840}x^7 - \frac{61787c^{20}}{967680}x^9 + \dots \right) \frac{\Gamma(1+2\beta)}{\Gamma(1+\alpha)\Gamma(1+\beta)}\\
&\quad + \left(-3c^{12}x + \frac{91c^{14}}{16}x^3 - \frac{679c^{16}}{160}x^5 + \frac{54857c^{18}}{26880}x^7 - \frac{61787c^{20}}{967680}x^9 + \dots \right) \frac{\Gamma(1+2\beta)}{\Gamma(1+2\alpha)}\\
&\quad \left. + \left(\frac{9c^{12}}{4}x - \frac{75c^{14}}{16}x^3 + \frac{147c^{16}}{40}x^5 - \frac{3271c^{18}}{1792}x^7 + \dots \right) \frac{\Gamma(1+\alpha)\Gamma(1+2\beta)}{\Gamma(1+\beta)\Gamma(1+2\alpha)} \right\} \frac{t^{3\beta}}{\Gamma(1+3\beta)}.
\end{aligned}
$$

The closed forms of $u_1(x,t)$, $u_2(x,t)$, $u_3(x,t)$ and $u_4(x,t)$ are

$$
\begin{aligned}
u_1(x,t) &= \frac{4c^2 \mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^2},\\
u_2(x,t) &= \left\{ \frac{8\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^3} - \frac{4\mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^2} \right\} \frac{c^5 t^\alpha}{\Gamma(1+\alpha)},\\
u_3(x,t) &= \left\{ \left[\frac{480\mathrm{e}^{3cx}}{(1+\mathrm{e}^{cx})^6} - \frac{96\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^4} \right] \frac{\Gamma(1+\alpha)}{\Gamma(1+\beta)} + \left[\frac{4\mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^2} + \frac{72\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^4} - \frac{480\mathrm{e}^{3cx}}{(1+\mathrm{e}^{cx})^6} \right] \right\} \frac{c^8 t^{2\alpha}}{\Gamma(1+2\alpha)},
\end{aligned}
$$

$$
\begin{aligned}
u_4(x,t) = & \left\{ \left[\frac{360\mathrm{e}^{4cx}}{(1+\mathrm{e}^{cx})^2} - \frac{120\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^5} + \frac{168\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^7} - \frac{408\mathrm{e}^{5cx}}{(1+\mathrm{e}^{cx})^7} \right] \frac{\Gamma(1+2\alpha)}{\Gamma^2(1+\beta)} \right. \\
& - \frac{96\mathrm{e}^{2cx}(-8+99\mathrm{e}^{cx}-253\mathrm{e}^{2cx}+253\mathrm{e}^{3cx}-99\mathrm{e}^{4cx}+8\mathrm{e}^{5cx})}{(1+\mathrm{e}^{cx})^9} \frac{\Gamma(1+\alpha)}{\Gamma(1+\beta)} \\
& + \frac{4\mathrm{e}^{cx}(-1-185\mathrm{e}^{cx}+2475\mathrm{e}^{2cx}-5981\mathrm{e}^{3cx}+5981\mathrm{e}^{4cx}-2475\mathrm{e}^{5cx}+185\mathrm{e}^{6cx}+\mathrm{e}^{7cx})}{(1+\mathrm{e}^{cx})^9} \\
& + \frac{96\mathrm{e}^{2cx}(-1+5\mathrm{e}^{cx}-5\mathrm{e}^{2cx}+\mathrm{e}^{3cx})}{(1+\mathrm{e}^{cx})^7} \frac{\Gamma(1+2\alpha)}{\Gamma^2(1+\alpha)} \\
& - \frac{96\mathrm{e}^{2cx}(-1+24\mathrm{e}^{cx}-71\mathrm{e}^{2cx}+71\mathrm{e}^{3cx}-24\mathrm{e}^{4cx}+\mathrm{e}^{5cx})}{(1+\mathrm{e}^{cx})^9} \frac{\Gamma(1+2\alpha)}{\Gamma(1+2\beta)} \\
& \left. - \frac{576\mathrm{e}^{3cx}(-3+13\mathrm{e}^{cx}-13\mathrm{e}^{2cx}+3\mathrm{e}^{3cx})}{(1+\mathrm{e}^{cx})^9} \frac{\Gamma(1+2\alpha)\Gamma(1+\beta)}{\Gamma(1+\alpha)\Gamma(1+2\beta)} \right\} \frac{c^{11}t^{3\alpha}}{\Gamma(1+3\alpha)}.
\end{aligned}
$$

The closed forms of $v_1(x,t)$, $v_2(x,t)$, $v_3(x,t)$ and $v_4(x,t)$ are

$$
\begin{aligned}
v_1(x,t) &= \frac{4c^2\mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^2}, \\
v_2(x,t) &= \left\{ \frac{8\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^3} - \frac{4\mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^2} \right\} \frac{c^5t^\beta}{\Gamma(1+\beta)}, \\
v_3(x,t) &= \left\{ \left[\frac{192\mathrm{e}^{3cx}}{(1+\mathrm{e}^{cx})^6} - \frac{72\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^2} + \frac{4\mathrm{e}^{cx}}{(1+\mathrm{e}^{cx})^2} \right] + \left[\frac{48\mathrm{e}^{2cx}}{(1+\mathrm{e}^{cx})^4} - \frac{192\mathrm{e}^{3cx}}{(1+\mathrm{e}^{cx})^6} \right] \Gamma(1+\beta) \right\} \frac{c^8t^{2\beta}}{\Gamma(1+2\beta)}, \\
v_4(x,t) &= \left\{ \frac{384\mathrm{e}^{2cx}(-1+15\mathrm{e}^{cx}-50\mathrm{e}^{2cx}+50\mathrm{e}^{3cx}-15\mathrm{e}^{4cx}+\mathrm{e}^{5cx})}{(1+\mathrm{e}^{cx})^9} \frac{\Gamma(1+\beta)}{\Gamma(1+\alpha)} \right. \\
& + \frac{4\mathrm{e}^{cx}(-1+139\mathrm{e}^{cx}-1521\mathrm{e}^{2cx}+4675\mathrm{e}^{3cx}-4675\mathrm{e}^{4cx}+1521\mathrm{e}^{5cx}-139\mathrm{e}^{6cx}+\mathrm{e}^{7cx})}{(1+\mathrm{e}^{cx})^9} \\
& + \frac{48\mathrm{e}^{2cx}(-1+5\mathrm{e}^{cx}-5\mathrm{e}^{2cx}+\mathrm{e}^{3cx})}{(1+\mathrm{e}^{cx})^7} \frac{\Gamma(1+2\beta)}{\Gamma(1+\alpha)\Gamma(1+\beta)} \\
& + \frac{48\mathrm{e}^{cx}(-1-21\mathrm{e}^{cx}+100\mathrm{e}^{2cx}-100\mathrm{e}^{3cx}+21\mathrm{e}^{4cx}+\mathrm{e}^{5cx})}{(1+\mathrm{e}^{cx})^9} \frac{\Gamma(1+2\beta)}{\Gamma(1+2\alpha)} \\
& \left. - \frac{1152\mathrm{e}^{3cx}(-1+4\mathrm{e}^{cx}-4\mathrm{e}^{2cx}+\mathrm{e}^{3cx})}{(1+\mathrm{e}^{cx})^9} \frac{\Gamma(1+\alpha)\Gamma(1+2\beta)}{\Gamma(1+\beta)\Gamma(1+2\alpha)} \right\} \frac{c^{11}t^{3\beta}}{\Gamma(1+3\beta)}.
\end{aligned}
$$

The approximate solutions of Eq. (4) in the finite series form are given by

$$u(x,t) = u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t), \tag{9}$$

$$v(x,t) = v_1(x,t) + v_2(x,t) + v_3(x,t) + v_4(x,t). \tag{10}$$

The exact solution (6) and the approximate solutions (9) and (10), for the special case where $\alpha = \beta = 1$, are shown in Fig. 1. It can be seen from Fig. 1 that the solutions obtained by the presented method are nearly identical with the exact solution.

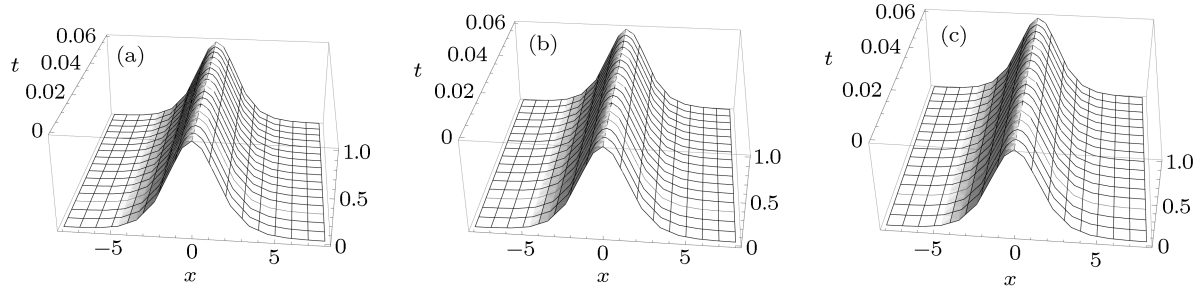


Fig. 1. Solutions for Eq. (4): (a) exact solution $u(x,t)$ or $v(x,t)$, (b) approximate solution (9), (c) approximate solution (10), when $\alpha = \beta = c = 1$.

Figure 2(a) shows the exact solutions when $\alpha = 1/3$ and $\beta = 1/5$. Figures 2(b) and 2(c) display the approximate solutions $u(x, y)$ and $v(x, y)$ when $\alpha = 1/3$ and $\beta = 1/5$. Figure 3(a) indicates the exact solutions when $\alpha = 0.005$ and $\beta = 0.002$. Figures 3(b) and 3(c) exhibit the approximate solutions $u(x, y)$ and $v(x, y)$ when $\alpha = 0.005$ and $\beta = 0.002$. From Figs. 3(b) and 3(c), it is easy to conclude that as the time-fractional derivative parameters α and β decrease, the solutions bifurcate into two waves.

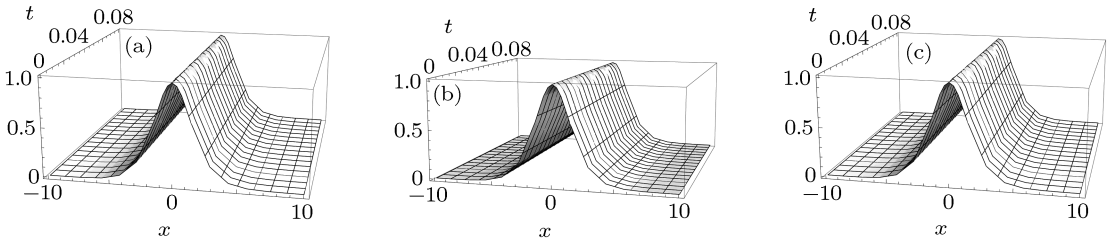


Fig. 2. Solutions for Eq. (4): (a) exact solution $u(x, t)$ or $v(x, t)$, (b) approximate solution (9), (c) approximate solution (10), when $\alpha = 1/3, \beta = 1/5$ and $c = 1$.

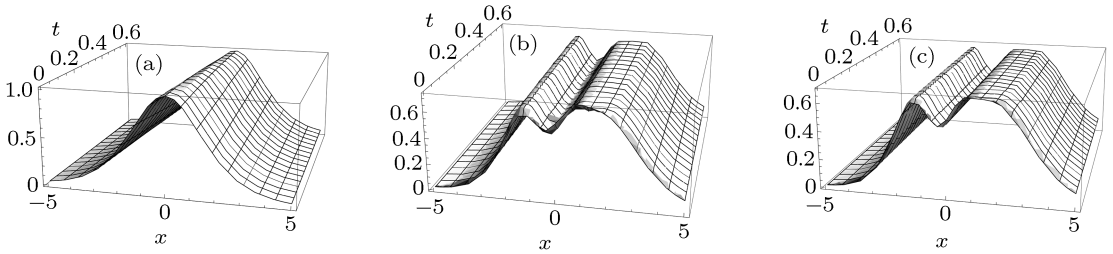


Fig. 3. Solutions for Eq. (4): (a) exact solution $u(x, t)$ or $v(x, t)$, (b) approximate solution (9), (c) approximate solution (10), when $\alpha = 0.005, \beta = 0.002$ and $c = 1$.

4. Conclusions

In the paper, we extend the generalised two-dimensional DTM to the solving of nonlinear fractional coupled partial differential equations. The time-fractional KdV equations with initial conditions are chosen to illustrate the proposed method. As a result, based on symbolic computation system MATHEMATICA, some approximate solutions for closed forms of fractional coupled KdV equations are obtained in a high accuracy. The obtained results demonstrate the reliability of the algorithm and its wider applicability to nonlinear fractional coupled partial differential equations.

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