Spectral dispersion of cloud droplet size distributions and radar threshold reflectivity for drizzle*

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From first principles, we find that the radar threshold reflectivity between nonprecipitating clouds and precipitating clouds is strongly related to not only the cloud droplet number concentration but also the spectral dispersion of cloud droplet size distributions. The further investigation indicates that the threshold value is an increasing function of spectral dispersion and cloud droplet number concentration. These results may improve our understanding of the cloud-precipitation interaction and the aerosol indirect effect.

Keywords: radar threshold reflectivity, spectral dispersion, cloud droplet number concentration

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1. Introduction

Distinguishing between nonprecipitating clouds and precipitating clouds is a critical issue for the use of remote sensing techniques. Many previous studies showed that the cloud droplet effective radius (r_e), cloud optical thickness (τ), and cloud radar reflectivity (Z) can be described as the basis of discrimination between nonprecipitating clouds and precipitating clouds or have the corresponding threshold values for drizzle.[1−3]

The radar technique is extensively applied to atmosphere physics.[4−7] The radar reflectivity is directly measured from millimeter-wavelength cloud radars, hence, observational investigations on radar threshold reflectivity for drizzle have attracted much more attention. A recent study indicates that the radar reflectivities of cloud and drizzle fields have very little overlap and are well separated by a threshold value around −15 dBZ in measurement of stratus cloud.[8] Chin *et al.*[9] also used this threshold −15 dBZ as an indication of drizzle occurrence in their microphysical retrieval of continental low-level stratiform clouds. The threshold −20 dBZ was used in their retrieval of continental stratus.[10] In Ref. [3], the study found that the threshold reflectivities for drizzle in warm marine stratus vary between −19 and −16 dBZ for three different cases. In the recent work,[11] −17 dBZ was used as the radar threshold reflectivity to distinguish between nonprecipitating clouds and precipitating clouds, and the influence of varying the threshold value between −20 dBZ and −15 dBZ on the results was examined.

In physics, the threshold value is related to the rapid growth of cloud droplets into rain drops, that is to say, cloud droplets at the threshold value grow into raindrops rapidly; the precipitation hardly forms below the value.[1] More recently, Liu *et al.*[12] presented a theory for the threshold reflectivity by relating it to the autoconversion process. Specifically, it is directly related to the number threshold function. They further showed that the dependence of threshold reflectivity on droplet number concentration suggests that the differences in empirically determined threshold reflectivity arise from the differences in droplet number concentration. In the present work, according to Liu *et al.*'s theory, we find that the radar threshold reflectivity between nonprecipitating and precipitating

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clouds is strongly related to not only the cloud droplet number concentration but also the spectral dispersion of cloud droplet size distribution. The further investigation indicates that the threshold value is an increasing function of cloud droplet number concentration and spectral dispersion.

The rest of the present paper is organized as follows. In Section 2, we review the threshold function related to the autoconversion process, and give the relationship between threshold function and spectral dispersion. In Section 3, the dependence of threshold reflectivity on droplet number concentration and spectral dispersion is derived. Concluding remarks and further discussion are given in Section 4.

2. Spectral dispersion and number threshold function

In most bulk schemes, cloud droplet size distribution is well described by the general Weibull droplet size distribution given by

$$n(r) = N_0 r^{q-1} \exp\left[-\left(\frac{r}{r_0}\right)^q\right], \tag{1}$$

where r is the radius of a cloud droplet, $n(r)$ is the total number concentration per unit volume of droplet radius r, N_0 is the intercept parameter, q is the shape parameter related to the spectral dispersion of the cloud droplet size distribution.

The spectral dispersion of the size distribution ε can be derived as

$$\begin{aligned}\varepsilon = \frac{\sigma}{\bar{r}} &= \sqrt{\frac{\int (r-\bar{r})^2 n(r)\mathrm{d}r}{\int n(r)\mathrm{d}r}}\frac{1}{\bar{r}} \\ &= \left[\frac{2q\Gamma(2/q)}{\Gamma^2(1/q)} - 1\right]^{1/2} \approx q^{-1}.\end{aligned} \tag{2}$$

According to the analytical Weibull distribution function (1), the formulas of the cloud droplet number concentration N, the cloud liquid water content L, and the radar reflectivity Z are given as

$$\begin{aligned}N &= \int_0^\infty n(r)\mathrm{d}r = \frac{N_0 r_0^q}{q}, \\ L &= \frac{4}{3}\pi\rho_\mathrm{w}\int_0^\infty n(r) r^3 \mathrm{d}r = \frac{4}{3}\pi\rho_\mathrm{w}\frac{N_0 r_0^q}{q} r_0^3 \Gamma\left(\frac{q+3}{q}\right) \\ &= \frac{4}{3}\pi\rho_\mathrm{w}\frac{N_0 r_0^q}{q} r_0^3 \Gamma(3\varepsilon+1), \\ Z &= 64\int_0^\infty n(r) r^6 \mathrm{d}r = 64\frac{N_0 r_0^q}{q} r_0^6 \Gamma\left(\frac{q+6}{q}\right) \\ &= 64\frac{N_0 r_0^q}{q} r_0^6 \Gamma(6\varepsilon+1).\end{aligned} \tag{3}$$

Consider the effect of truncating the cloud droplet size distribution on the autoconversion rate,[12,13] the threshold function for the bulk quantity Y is given as

$$T_Y = \left[\frac{\int_{r_c}^\infty r^6 n(r)\mathrm{d}r}{\int_0^\infty r^6 n(r)\mathrm{d}r}\right]\left[\frac{\int_{r_c}^\infty r^p n(r)\mathrm{d}r}{\int_0^\infty r^p n(r)\mathrm{d}r}\right], \tag{4}$$

where r_c is the critical radius beyond which the autoconversion process dominates, corresponds to the kinetic potential barrier of the droplet population and is a function of number concentration and mass content.[14]

According to Liu *et al.*'s theory,[12] we choose the number threshold function ($p = 0$) which is related to the threshold radar reflectivity as

$$\begin{aligned}T_N &= \gamma\left(\frac{6+q}{q}, x_\mathrm{cq}\right)\gamma(1, x_\mathrm{cq}), \\ x_\mathrm{cq} &= \Gamma^{q/3}\left(\frac{3+q}{q}\right) x_\mathrm{c}^{q/3}, \\ x_\mathrm{c} &= 9.7\times 10^{-17} N^{3/2} L^{-2},\end{aligned} \tag{5}$$

where $\Gamma(n,a) = \int_a^\infty x^{n-1}\mathrm{e}^{-x}\mathrm{d}x$ is the incomplete gamma function;

$$\gamma(n,a) = \int_a^\infty x^{n-1}\mathrm{e}^{-x}\mathrm{d}x \Big/ \int_0^\infty x^{n-1}\mathrm{e}^{-x}\mathrm{d}x$$

is the normalized incomplete gamma function; x_c is the ratio of the critical mass to mean mass. From Eq. (2), we have the following relations:

$$\begin{aligned}T_N &= \gamma(6\varepsilon+1, x_\mathrm{cq})\gamma(1, x_\mathrm{cq}), \\ x_\mathrm{cq} &= \Gamma^{1/3\varepsilon}(3\varepsilon+1) x_\mathrm{c}^{1/3\varepsilon}, \\ x_\mathrm{c} &= 9.7\times 10^{-17} N^{3/2} L^{-2}.\end{aligned} \tag{6}$$

According to the formulas of the cloud droplet number concentration N, the cloud liquid water content L, and the radar reflectivity Z in expression (3), x_c can be given as

$$x_\mathrm{c} = 9.7\times 10^{-17}\times 64\times\left(\frac{3}{4\pi\rho_\mathrm{w}}\right)^2 \Gamma(6\varepsilon+1)\Gamma^{-2}(3\varepsilon+1) N^{1/2} Z^{-1}. \tag{7}$$

Hence the number threshold function can be derived as

$$T_N = \gamma(6\varepsilon+1, \Gamma^{1/3\varepsilon}(3\varepsilon+1) x_\mathrm{c}^{1/3\varepsilon})\ \gamma(1, \Gamma^{1/3\varepsilon}(3\varepsilon+1) x_\mathrm{c}^{1/3\varepsilon}), \tag{8}$$

$$x_{\rm c} = 7.1 \times 10^{-16} \times \left[\frac{1}{2} \times \Gamma(6\varepsilon+1)\Gamma^{-2}(3\varepsilon+1)\right] N^{1/2} Z^{-1}. \tag{9}$$

It shows that the number threshold function T_N is dependent on droplet number concentration N, spectral dispersion ε and radar reflectivity Z. Figures 1 and 2 show that the dependences of the number threshold function calculated from Eqs. (8) and (9), on radar reflectivity for different values of N and ε, respectively. Figure 1 shows that the variations of number threshold function with radar reflectivity for different values of N (N = 100, 300, 500, 700, and 1000 cm^{-3}) at $\varepsilon = 0.33$ (or $q = 3$). Figure 2 shows

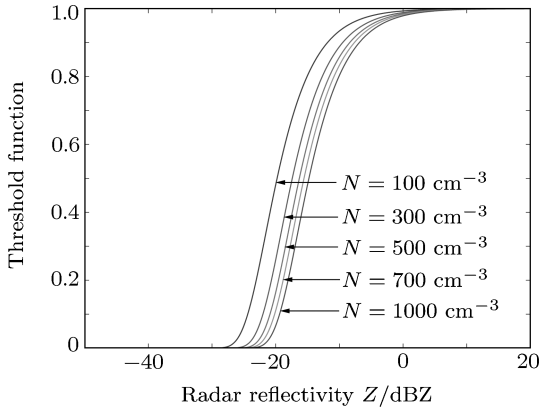


Fig. 1. Dependences of number threshold function on cloud radar reflectivity, calculated from the analytical threshold function for different values of cloud droplet number concentration N by setting $\varepsilon = 0.33$.

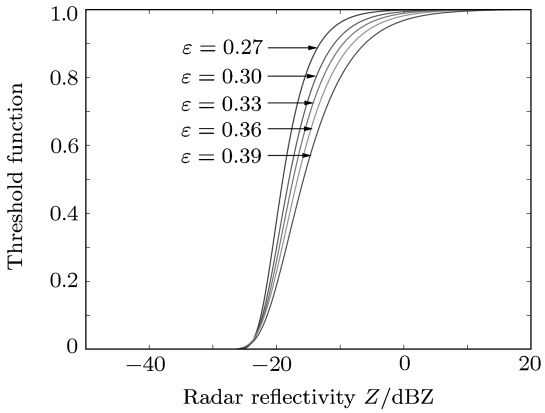


Fig. 2. Dependences of threshold reflectivity on cloud radar reflectivity for different values of spectral dispersion ε and cloud droplet number concentration $N = 300$ cm^{-3}.

that the variations of number threshold function with radar reflectivity for different values of ε (ε = 0.27, 0.30, 0.33, 0.36, and 0.39) at $N = 300$ cm^{-3}. The range of spectral dispersion almost keeps identical with the values measured by Levin,[15] and confirmed in many subsequent experiments (e.g., in Ref. [16]). From these two figures, it is evident that the change in threshold function with radar reflectivity is a phase transition, and it indicates the existence of threshold reflectivity. The phase-transition point shifts with the increase of N and ε, implying that the threshold value is not a constant, but it is dependent on both droplet number concentration N and spectral dispersion ε.

3. Radar threshold reflectivity

In this section, we will derive the radar threshold reflectivity by using the cloud droplet number concentration and the spectral dispersion from the number threshold function. According to Eqs. (8) and (9), the radar threshold reflectivity Z_c can be given as

$$\begin{aligned} Z_c(N,\varepsilon,T_N) &= 7.1\times10^{-16} \\ &\quad\times\left[\frac{1}{2}\Gamma(6\varepsilon+1)\Gamma^{-2}(3\varepsilon+1)\right] \\ &\quad\times N^{1/2}x_c^{-1}(\varepsilon,T_N). \end{aligned} \tag{10}$$

Next, we express the above formula in unit of dBZ for $Z_{\rm c}$, and then we have

$$\begin{aligned} Z_{\rm c}({\rm dBZ}) &= 10\log[10^{12}Z_{\rm c}({\rm mm}^6\cdot{\rm m}^{-3})] \\ &\approx -31+5\log N \\ &\quad+10\log[\frac{1}{2}\Gamma(6\varepsilon+1)\Gamma^{-2}(3\varepsilon+1)] \\ &\quad-10\log x_{\rm c}(\varepsilon,T_N). \end{aligned} \tag{11}$$

By setting $T_N = 0.9$,[12] we can obtain the following results for different values of ε (ε = 0.27, 0.30, 0.33, 0.36, and 0.39):

1) for $\varepsilon = 0.27$ and $x_{\rm c}(\varepsilon,T_N) \approx 0.17$,

$$Z_{\rm c}({\rm dBZ}) \approx -24+5\log N; \tag{12}$$

2) for $\varepsilon = 0.30$ and $x_{\rm c}(\varepsilon,T_N) \approx 0.13$,

$$Z_{\rm c}({\rm dBZ}) \approx -22+5\log N; \tag{13}$$

3) for $\varepsilon = 0.33$ and $x_{\rm c}(\varepsilon,T_N) \approx 0.11$,

$$Z_{\rm c}({\rm dBZ}) \approx -21+5\log N; \tag{14}$$

4) for $\varepsilon = 0.36$ and $x_{\rm c}(\varepsilon,T_N) \approx 0.08$,

$$Z_{\rm c}({\rm dBZ}) \approx -20+5\log N; \tag{15}$$

5) for $\varepsilon = 0.39$ and $x_{\rm c}(\varepsilon,T_N) \approx 0.07$,

$$Z_{\rm c}({\rm dBZ}) \approx -18+5\log N. \tag{16}$$

These five equations indicate that the radar threshold reflectivity is related to the droplet number concentration N at constant ε. Figure 3 shows that the threshold value is an increasing function of spectral dispersion and cloud droplet number concentration, that is to say, the radar threshold reflectivity has a larger value for larger spectral dispersion or higher cloud droplet number concentration. It is known that the spectral dispersion and the droplet number concentration can be enhanced by anthropogenic aerosols, hence the radar threshold reflectivity becomes larger in the more polluted background.

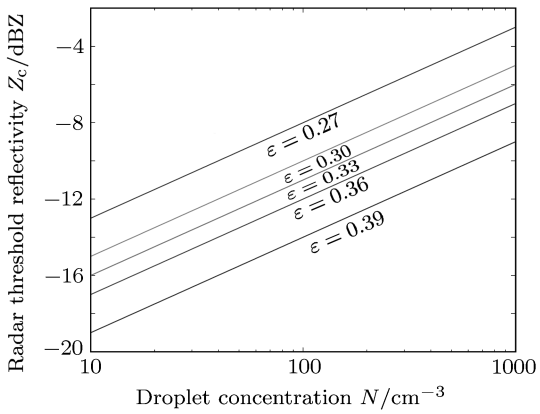


Fig. 3. Dependences of threshold reflectivity on cloud droplet number concentration for different values of spectral dispersion ε.

Note that the cloud droplet size distribution is described as the Weibull distribution function in the paper. For the gamma distribution function $n(r) = N_0 r^{\mu} \exp(-\lambda r)$ (with $\varepsilon = (\mu + 1)^{-1/2}$), the number threshold function ($p = 0$) is given as

$$\begin{aligned} T_N &= \gamma(\varepsilon^{-2} + 6, x_{\rm cq})\gamma(\varepsilon^{-2}, x_{\rm cq}), \\ x_{\rm cq} &= [\varepsilon^{-2}(\varepsilon^{-2} + 1)(\varepsilon^{-2} + 2)]^{1/3} x_{\rm c}^{1/3}, \\ x_{\rm c} &= 9.7 \times 10^{-17} N^{3/2} L^{-2}. \end{aligned} \tag{17}$$

Using the above method, we can also obtain the similar results: the threshold value is an increasing function of spectral dispersion and cloud droplet number concentration; only the sector about spectral dispersion is changed. Hence, the detailed derivation process is neglected.

4. Conclusion and discussion

Using the number threshold function, we derive the radar threshold reflectivity between nonprecipitating clouds and precipitating clouds which is strongly related to both the spectral dispersion of cloud droplet size distributions and the cloud droplet number concentration. The results further show that the radar threshold reflectivity value is an increasing function of spectral dispersion and cloud droplet number concentration. These results may help us to improve the understanding of the cloud-precipitation interaction and the aerosol indirect effect.

Since these analytical expressions for the radar threshold reflectivity are absolutely derived from theoretical studies, it will be very important to confirm the results in observational researches in the future.

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