Effect of number density on velocity distributions in a driven quasi-two-dimensional granular gas*

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The motion of mono-disperse spherical steel particles in a vibration driven quasi-two-dimensional (2D) square cell is studied. The cell is horizontally vibrated to eliminate the effect of gravity compaction. The velocity distributions at different particle number densities are studied and found to obey the form $\exp[-\beta(|v_y|/\sigma_y)^\alpha]$, in which v_y and σ_y are velocity and its variance in the transverse direction, and α and β are fitting parameters. The value of α is found to decrease with the number density of particles increasing. To investigate the effect of the bottom plate, the molecular dynamics simulation without considering any bottom friction is performed. The accordance between the simulation result and the experimental result shows that the influence of bottom plate friction force on the high energy tail of the velocity distribution can be neglected.

Keywords: granular matter, velocity distribution

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1. Introduction

Classical equilibrium statistical mechanics has reached a rather mature phase, but the understanding of the system far away from thermodynamic equilibrium is still lacking. Because of its apparent simplicity, granular system, collection of macroscopic particles, is a good example of non-equilibrium statistical system. Granular material behaves differently compared with any other familiar form of matter—solid, liquid, or gas.[1−5]

One of the important features of the granular system is the dissipative nature of interactions between grains. Without energy supply, the kinetic energy of granular system is lost during collisions. Therefore, to achieve their steady state and avoid inelastic collapse, energy should be continuously supplied to them from some external sources. In experiments, energy is supplied to the granular system through vibration or shearing. In some simulations[6,7] and in most analytic theories, the external driving has been considered as heating the particles uniformly throughout the container, with all the particles in the gas driven independently by a white-noise source. However, in experimental study, energy is usually supplied to the granular gas by vibrating the walls of the container. So the gas will develop gradients in density and kinetic energy.[8,9] Even in the case of uniform heating, granular systems are found to be inhomogeneous.[6]

One of the most basic properties of ordinary gas lies in the velocity distribution, which obeys Maxwell–Boltzmann distribution. Whether the granular dissipative gas obeys this velocity distribution, has been an important topic for researchers. But the significantly different behaviour in velocity distribution of granular gas from classical one, has been observed and the characteristic velocity distribution in the form of

$$P(v) = C\exp[-\beta(|v|/\sigma)^\alpha] \tag{1}$$

is obtained so far, where in resemblance to the equilibrium gas, the term $\sigma = \langle v^2\rangle$ is often called the granular temperature. In experiment, Rouyer and Menon[10] suggested that a universal exponent i.e. $\alpha = 1.55$ should occur over a wide range of experimental parameters.

In a setup, where the particles on a horizontal plate were vibrated along the vertical direction, Olafsen and Urbach[11,12] found a crossover from exponential to Gaussian as the amplitude of vibration was

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increased. Later on with the help of high speed digital imaging, experiments[13] showed the deviation in the tail of velocity distribution from Gaussian distribution. Blair and Kudrolli[14] used an inclined plane in their experiment and observed velocity correlation over large distance comparable to system size. But in all of these experiments, there was compaction of particles due to gravity. Our goal is to explore what the effect will be on the velocity distributions in dilute and dense regimes, if we eliminate the compaction of particles due to gravity.

2. Experiment

In this experiment a square cell of aluminum is used which has an internal area of 40×40 mm^2 and a depth of 2.35±0.01 mm. Spherical stainless steel particles, each of which has a diameter of 2.0±0.01 mm, are brought into play. The cell is equipped with a fixed glass base and a removable glass lid. An electromagnetic oscillator is used to vibrate the cell. The whole arrangement is mounted on a table having fine pitch screws, which can be used to level the table. The boundaries perpendicular to the direction of vibration are the main source of energy inputs. These spherical particles are viewed from the top using a high speed DALSA-1 Camera with 955 frames/second and a spatial resolution of 256×256 pixels^2. The cell is vibrated sinusoidally with a fixed amplitude of 2 mm and fixed frequency $\nu = 44.33\pm0.2$ Hz. The peak acceleration is $A\omega^2$, where A is the amplitude of the vibrations and $\omega = 2\pi\nu$ is the angular frequency of vibrator. The schematic diagram of the experimental setup is shown in Fig. 1.

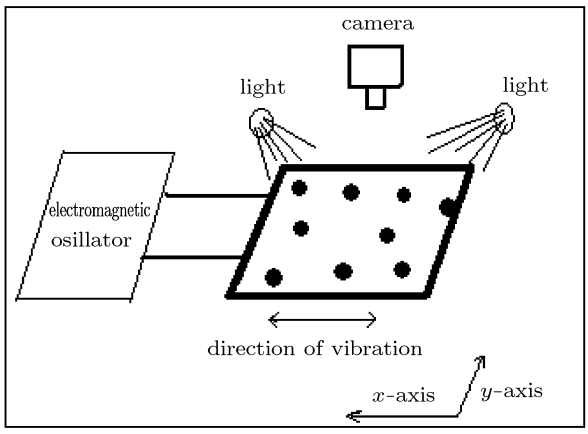


Fig. 1. Schematic diagram of the experimental setup.

The surface of the cell is leveled with the help of adjustable fine pitch screws, within ±0.004°, to avoid clustering of particles. As the internal length of the square cell is 40 mm and the diameter of each particle is 2 mm, there are 20 particles in each layer. The number of particles in this experiment ranges from 40 to 200. The mean free path of the system is less than 4 times as large a particle diameter,[15] and this means the granular system in our experiment is not in the range of the Knudsen regime.

3. Results and discussion

When the cell is vibrated, there are two types of collisions i.e., particle–particle and particle–boundary collisions. The particle–boundary collisions act as a source of energy inputs into the system, while during particle–particle collisions, the particles dissipate energy. As the number of particles is increased, clustering starts. The reason is that the particles are compressed in the centre of the cell by the pressure of those particles, which are moving in, after striking the boundaries. The particles in the dense regions collide more frequently as compared with the particles in the dilute region. Therefore, there is strong dissipation of energy in dense region and the particles in there cool faster as compared with in dilute region. As a result, a stable dense fluid is produced.[16]

The positional ordering of the particles can be studied by examining the structural configuration of the granular system. The structural configuration has been inspected by measuring the radial distribution function $g(r)$ as referred to by Reis *et al.*,[17] where r is the distance between the centres of particles, i.e.

$$g(r) = A(r)\left\langle \sum_i \sum_{i\neq j} \delta(r - r_{ij}) \right\rangle, \qquad (2)$$

where r_{ij} is the separation between the i-th and j-th particles and $A(r)$ is a normalization constant. The curve with black squares shows a radial distribution function for particles in up to eight layers. There is a single peak at $r/d = 1$ on the curve of $g(r)$ and it decays quickly at large distances as expected in the case of disordered gas. The curve with gray circles shows a radial distribution function corresponding to ten layers of particles as shown in Fig. 2. There are two peaks at $r/d = 1$ and $r/d = 2$, exhibiting the liquid-like behaviour. The neighbouring spheres show the correlation of their positions. The hard sphere dense

fluids (liquid-like), commonly show such behaviours.

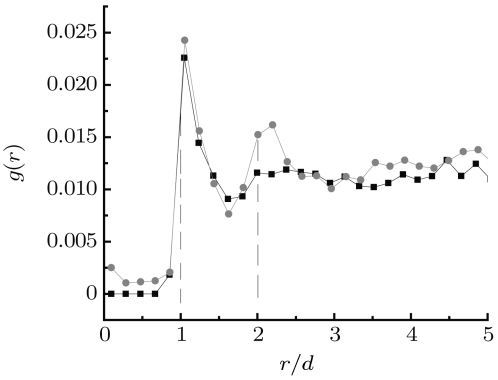


Fig. 2. Radial distribution function $g(r)$, where black squares are for cells of 2 to 8 layers of particles, gray circles for cell of 10 layers of particles; the vertical dashed lines correspond to the expected positions of the peaks in each of the phases.

The difference between the radial distribution functions with different numbers of layers implies the change of the granular system configuration with the number of particles, and it is interesting to know how the velocity distributions vary when the system transits from the gas-like state to the liquid-like state. In order to eliminate the dependence of the velocity distribution on the spatial location, a rectangular area $4d \times 20d$ of the cell along the direction of vibration was selected to obtain the result as shown in Fig. 3. Within this region the number density varies by 7% of the total variation in density. Since the change in density in this area is small, the velocity distributions are considered almost to be the same throughout this area.

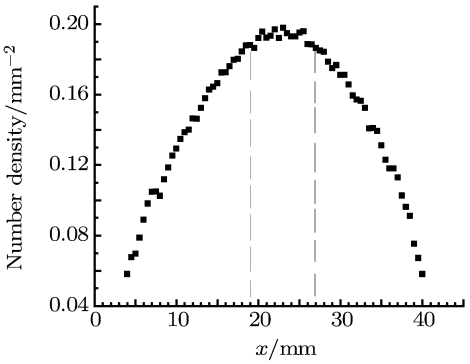


Fig. 3. Variation of density along x axis for the cell of ten layers, where vertical lines show the area chosen for the calculations.

Along the transverse-driven direction the velocity distribution is found to obey the probability distribution function $P(v_y)$, given by Eq. (1). By increasing the number of layers, the values of exponent α, rather a constant as described by Rouyer and Menon,[10] are found to decrease to 1.35 and 1.2 for systems of layer numbers 6 and 10, respectively. The corresponding curves are shown in Fig. 4. It is indicated that on increasing number density the velocity distribution curves deviate from Gaussian distribution curves.

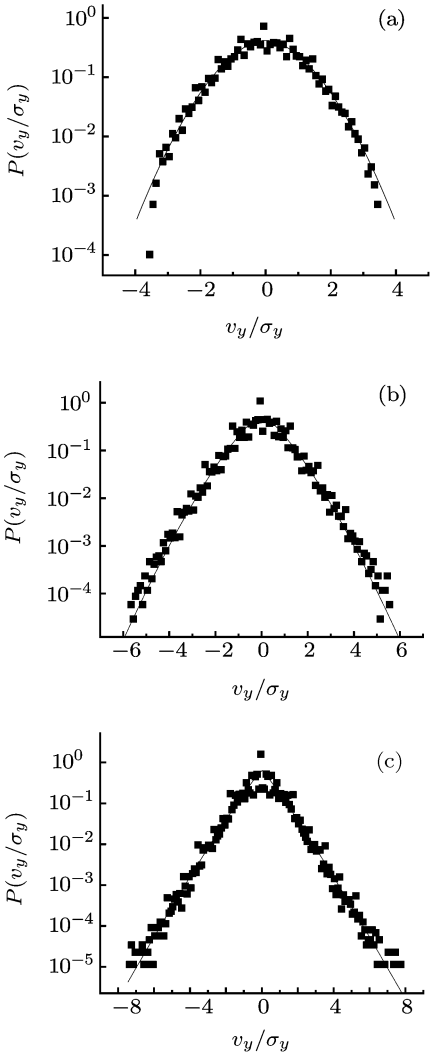


Fig. 4. Velocity distribution curves, obtained from experimental data, with fitting function $P(v_y) = C\exp[-\beta(|v_y|/\sigma_y)^\alpha]$, (a) $\alpha = 1.8$ for two layers, (b) $\alpha = 1.35$ for six layers, and (c) $\alpha = 1.2$ for ten layers.

The kurtosis or flatness of distribution $P(v_y)$ is calculated and then used to confirm the deviation of velocity distribution $P(v_y)$ from Gaussian distribution. The kurtosis or flatness of distribution curves is given by

$$F_y = \langle v_y \rangle^4 / \langle v_y^2 \rangle^2. \tag{3}$$

If the distribution is true Gaussian, the value of F_y should be equal to 3.0.[11,12] The value of flatness of curve is found to be $F_y = 3.11$ for two layers, illustrating that the distribution is nearly Gaussian and

this value increases to $F_y = 4.26$ for ten layers. This shows that by increasing the number density the flatness of the curve increases and the curve deviates from Gaussian distribution.

To understand the effect of the number density on velocity distribution, the concept brought by Vanzon *et al.*[16] is utilized. They found theoretically that the form of the observed velocity distribution is governed primarily by the restitution coefficient e and $q = N_{\rm H}/N_{\rm C}$, the ratio between the average number of heating events $N_{\rm H}$ and the average number of collisions $N_{\rm C}$ experienced by a particle in unit time. The velocity distribution is Gaussian distribution when heating dominates dissipation for $q \gg 1$; when the value of q decreases the velocity distribution deviates from Gaussian distribution. In our experiment, the energy is transferred in the transverse direction through the collisions of the slow moving particles with the fast moving particles which have gained the energy just from the collisions with the boundaries, so we consider one collision between driven boundary with particle as one heating event. When the number of particles increases, the average number of heating events experienced by one particle should decrease and the average number of collisions $N_{\rm C}$ increases, so the ratio q decreases and leads the velocity distribution to deviate from Gaussian distribution.

It should be mentioned here that the analysis above is rather rough, and the expression of q in the analysis is available only when the granular gas is not very dilute. Recalling the experiment performed under micro-gravity environment,[15] the velocity distribution follows an exponential decay when the granular system is in the Knudsen regime rather than Gaussian distribution which can be deduced if we follow the analysis above. One possible explanation for this is that when the granular system is dilute enough we cannot take the collision between driven wall and particle as the heating event because the particle might not collide with any other particle between two collisions with the driven boundary. Also, the collision between the particle and the parallel boundary, which dissipates the transverse kinetic energy of particles, becomes significant, and this should contribute to the average number of collision $N_{\rm C}$. These mean that the value of q might not become very large when the system is in the Knudsen regime; and more careful investigation on the definition of q in the experiment should be carried on.

4. Bottom plate friction

The container is vibrated horizontally to eliminate the effect of gravity but, here, the bottom plate friction cannot be neglected. The energy dissipation increases and the collision dynamics may become complex when two particles collide on a surface.[18] In order to investigate the influence of frictional force offered by the bottom plate, molecular dynamics simulation without any bottom friction and gravity is performed. Particles are considered to be perfect rigid spheres with the same radius and same mass. The coefficients of restitution in normal and tangential directions are kept to be 0.9 and the coefficient of friction is 0.3. Granular particles are placed in a 2D container with a dimension of $20d \times 20d$. The driven velocity is kept the same, i.e., 90π diameter/second. After running for a few seconds, the system reaches a steady state. The velocities of granular particles in the transverse direction, for an area of $4d \times 20d$ located in the centre of the container, are obtained. The results obtained from this simulation are plotted in Fig. 5. For two layers the value of exponent is found to be 2.0, shown in Fig. 5(a) which indicates a Gaussian distribution. The values of exponent decrease to 1.45 and 1.25 for six and ten layers respectively as shown in Figs. 5(b) and 5(c). These results demonstrate that by increasing the number density the corresponding curve deviates from Gaussian distribution. These simulation results also support the idea that the value of the exponent should not be constant with density varying.

Although the numerical value of α is a little different from the experimental results, however, the behaviour is unchanged when the value of α varies with the increase of the particle number density. A comparison between the simulation and the experimental results obviously shows that the shapes of the high energy tails are the same. But the peak for the low velocity particles in the experimental results is sharp and it is smoother for the simulation results. One possible explanation is that the friction force by the bottom plate is more likely to trap the slow moving particles. The frictional force affects both the low and the high velocity particles, but the energy dissipated by the bottom friction for the high energy particles can be ignored. Recalling the explanation for the large energy dissipation by the bottom surface, Painter and Behringer[18] proposed that particles experience a sliding process for a substantial time following collisions.

But the high velocity particles are expected to collide with other particles more frequently, which means that the time interval between two collisions is much shorter for the high velocity particles. Hence, these particles do not have enough time to let the sliding effect dissipate more energy and the bottom plate frictional force has little influence on the high energy part of the velocity distribution.

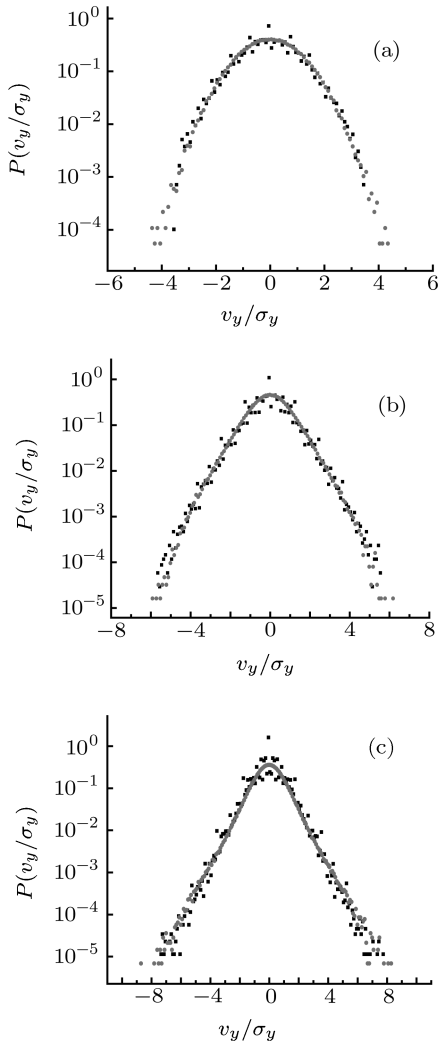


Fig. 5. Velocity distribution curves with fitting function $P(v_y) = C\exp[-\beta(|v_y|/\sigma_y)^\alpha]$, obtained from the simulation of 2, 6 and 10 layers respectively where black rectangles denote experimental results, gray circles represent simulation results, (a) $\alpha = 2.0$ for two layers, (b) $\alpha = 1.45$ for six layers, and (c) $\alpha = 1.25$ for ten layers.

5. Conclusion

In this paper, to exclude the compaction effect of the gravity field and the direct effect of the vibrating boundary, we investigated the transverse velocity distribution in the central part of a horizontally vibrated granular system, and the velocity distribution could be represented by the form of $\exp[-\beta(|v_y|/\sigma_y)^\alpha]$ in which the value of α describing the shape of the velocity distribution depends on the total number of particles. When there are 2 layers of particles, we obtain a Gaussian-like velocity distribution. Increasing the total number of particles, the state of the system changes from the gas-like state to the liquid-like state by checking the structure of the radial distribution function, meanwhile, the velocity distribution deviates from Gaussian distribution. Using the concept brought by Vanzon, we explain the change of the velocity distribution with the number of particles. The ratio q between the average number of heating events and the average number of collisions experienced by a particle in unit time can be expressed by the ratio between the average number of the particle–boundary (driven) collisions and that of particle–particle collisions in unit time. The value of q decreases with the increase of the particle number, and the velocity distribution should more and more deviate from Gaussian distribution.

A 2D molecular dynamics simulation without considering the friction force by the bottom plate shows the same change in the velocity distribution with the increase of particle number. A comparison between the simulation results and the experiment results shows that the high energy tails of velocity distributions accord well with each other, but the experimental results have a sharper peak at the low velocity part. This implies that the bottom friction affects the low velocity part of the distribution but has little influence on the high energy tail, which determines the main feature of the velocity distribution.

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