Non-equilibrium statistical theory about microscopic fatigue cracks of metal in magnetic field

Liu Zhao-Long(刘兆龙)†, Hu Hai-Yun(胡海云),
Fan Tian-You(范天佑), and Xing Xiu-San(邢修三)

Department of Physics, Beijing Institute of Technology, Beijing 100081*, China*

(Received 5 February 2010; revised manuscript received 4 March 2010)

This paper develops the non-equilibrium statistical fatigue damage theory to study the statistical behaviour of micro-crack for metals in magnetic field. The one-dimensional homogeneous crack system is chosen for study. To investigate the effect caused by magnetic field on the statistical distribution of micro-crack in the system, the theoretical analysis on microcrack evolution equation, the average length of micro-crack, density distribution function of micro-crack and fatigue fracture probability have been performed. The derived results relate the changes of some quantities, such as average length, density distribution function and fatigue fracture probability, to the applied magnetic field, the magnetic and mechanical properties of metals. It gives a theoretical explanation on the change of fatigue damage due to magnetic fields observed by experiments, and presents an analytic approach on studying the fatigue damage of metal in magnetic field.

Keywords: fatigue damage, micro-crack distribution, non-equilibrium statistical theory, magnetic field

PACC: 8140N, 6220M

1. Introduction

It is well known that cracks are responsible for the material failure. In 1920, British engineer Alan Griffith established a mathematical description on the macroscopic behaviour of cracks.[1] His work gave rise to the establishment of fracture mechanics. With a deepening understanding of cracks, not only engineers but also a growing number of physicists have been involved in the study of cracks due to lots of interesting discoveries in the mature area.[2−5] It is realized that growth of cracks is a rich source of complex physical phenomena. Combined experimental and theoretical researches on crack physics are developing in order to understand how cracks grow.

One of the challenging areas in fracture research is to investigate the effect of magnetic field on fatigue damages, which are irreversible processes for materials subjected to alternative stresses. Lü Bao-Tong *et al.*[6] reported that the magnetic field could repair fatigue damage under some conditions. Bhat *et al.*,[7] Fahmy *et al.*,[8] and Ayhan Çelik *et al.*[9] observed the changes in fatigue life of steels in magnetic field, respectively. All these results derived from experiments show that the magnetic field makes some impacts on fatigue damage. Studying the mechanism of the change in fatigue damage processes caused by magnetic fields is a new area waiting to be explored.

The changes in fatigue damage for metals in magnetic field show that the cracking behaviours must be influenced by magnetic field. In order to understand the fatigue damage of material under the combined action of the cyclic stresses and magnetic field, the evolution and distribution of micro-crack in materials need to be considered. In this paper, we investigate the distribution of micro-cracks in magnetic field using non-equilibrium statistical theory.

Fatigue in magnetic field is a very complicated subject, which encompasses many disciplines involving physics, chemistry, fracture mechanics, material testing and structural analysis. There are many problems not understood in the area. To simplify the study we discuss the one-dimensional crack systems only.

2. Micro-crack evolution equation

It is the evolution of micro-crack including the nucleation and propagation that determines the nature of a fatigue fracture process. To investigate the change in fracture damage caused by magnetic field, we need to have the microcrack evolution equation in magnetic field.

†Corresponding author. E-mail: zlliu@bit.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

The microscopic composition, defects and phase structures in real metals are inhomogeneous, so the metallic structure may be regarded as an average structure background superimposed by stochastic fluctuation due to microscopic compositions, defects and inhomogeneous phase structures. The average structure is deterministic, whereas the inhomogeneous fluctuation is stochastic. The fatigue fracture process may be regarded as a stochastic process due to the fluctuation of microscopic structure, so the law governing the evolution of microcrack is stochastic, but not deterministic only. From this point of view, we use non-equilibrium statistical theory to study the fracture damage in magnetic fields.

Consider a metal in magnetic field $\boldsymbol{H}$ under the action of cyclic stress with amplitude $\sigma_{\rm a}$ and period T_N. There are a large number of micro-cracks nucleating, growing continuously in the metal under the action of cyclic stress in magnetic field. For convenience, we discuss one-dimensional micro-cracks of the same shape, and assume that their space orientation distribution in the metal is homogenous. Let N denote the cyclic number of applied stress, and $c(N)$ denote the length of a micro-crack for N cycles. The derivative of $c(N)$ with respect to the cyclic number is $\dot{c}(N) = {\rm d}c/{\rm d}N$, where $\dot{c}(N)$ is the growth rate of micro-crack with length $c(N)$ per cycle for N cycles. The growth rate obeys generalized Langevin equation:[10]

$$\dot{c}(N) = K(c,N) + cf(N), \tag{1}$$

where $K(c,N)$ called transportation growth rate of the microcrack is the deterministic part of growth rate of the microcrack; $f(N)$ termed the fluctuation function is the stochastic part of growth rate. The function $f(N)$ has a Guassian distribution.

$$\begin{cases} \langle f(N)\rangle = 0, \\ \langle f(N)f(N')\rangle = D\delta(N-N'), \end{cases} \tag{2}$$

where D is the fluctuation growth coefficient.

For the one-dimensional crack system, let c_0 be the length of nucleus of a micro-crack and assume $M(c,N)$ to be the density distribution function of the cracks. The $M(c,N){\rm d}c$ is the number of micro-cracks per unit volume evolving in the length between c and c+dc for N cycles. The density distribution function $M(c,N)$ satisfies the micro-crack evolution equation[10,11]

$$\begin{aligned} \frac{\partial M(c,N)}{\partial N} &= -\frac{\partial}{\partial c}\left[\left(K(c,N)+\frac{Dc}{2}\right)M(c,N)\right] \\ &\quad + \frac{D}{2}\frac{\partial^2}{\partial c^2}\left[c^2 M(c,N)\right] \\ &\quad + q(N)\delta(c-c_0), \end{aligned} \tag{3}$$

where $q(N)$ is the number of nuclei per unit volume per cycle for N cycles and $\delta(c-c_0)$ is the Dirac function. Equation (3) is the micro-crack evolution equation describing the main stage of fatigue fracture. The physical meaning of Eq. (3) is that the change in $M(c,N)$ with respect to cyclic number consists of three parts. The first part is transport growth (the first term on the right-hand side). The second part is fluctuation growth (the second term on the right-hand side). The third part is the nucleation (the third term of the right-hand side). The initial and boundary conditions for Eq. (3) are

$$\begin{cases} M(c, N=0) = 0, \\ M(c\to\infty, N) = 0, \end{cases} \tag{4}$$

where $K(c,\ N)$ and D in Eq. (3) are given by the equations:[10,12]

$$K(c,N) = Ac, \tag{5}$$

$$A = \frac{(\sigma_{\rm a}-2\varsigma)\sigma_{\rm a}^{1/\beta} - 2\dot{\lambda}_H \dot{H}_N N_0 L^2 \sigma_0^{1/\beta}}{2\left(\gamma+\lambda\right)N_0 L^2 \sigma_0^{1/\beta}}, \tag{6}$$

$$D = 2\eta A, \tag{7}$$

$$\varsigma = \frac{\mu_0}{2}\int_0^M \boldsymbol{H}\cdot{\rm d}\boldsymbol{M}, \quad \dot{H}_N = \frac{{\rm d}H}{{\rm d}N} = \frac{{\rm d}H}{{\rm d}t}T_N,$$

where σ_0 is the cyclic strength coefficient, β is cyclic hardening exponent, γ is the surface energy, $\boldsymbol{M}$ is magnetization intensity of metal, N_0 is the mean mobile dislocation sources per unit volume, L is the length of the slip plane, n is the number of dislocations squeezed in the microcrack wedge, λ is the magnetic coefficient given by

$$\lambda = \frac{\mu_0 nb}{2}\int_0^M \boldsymbol{H}\cdot{\rm d}\boldsymbol{M}. \tag{8}$$

The magnetic coefficient depends on the magnetic properties and the chemical composition of metal. μ_0 is the permeability of free space and b is the magnitude of Burger's vector. The quantities N_0, L, γ, and σ_0 are structurally sensitive, they fluctuates about their averages due to the fluctuation of inhomogeneous microstructure about an average background in metals. We need to consider the relative deviations of these quantities. The quantity η is used to describe the total relative deviation of these quantities due to inhomogeneous fluctuation of microcrack structure in magnetic field. The nucleation rate of fatigue micro-cracks per unit volume for N cycles is given by the non-equilibrium statistical theory of fatigue damage[10,11]

$$q(N) = AM_0{\rm e}^{-AN}, \tag{9}$$

where M_0 is the average number of latent nuclei per unit volume.

Solving the microcrack evolution equation under the boundary and initial conditions for $M(c,N)$, the change in $M(c,N)$ caused by magnetic field can be revealed.

3. Average length of micro-crack in magnetic field

To understand the effect of magnetic field on the distribution of micro-cracks for metals, the average length of micro-crack in magnetic field needs to be investigated. Substituting Eq. (5) into Eq. (1) and integrating on both sides of the equation, obtain

$$c = c_0 \exp(AN) \exp\left(\int_0^N f(N')\mathrm{d}N' \right).$$

Using Eq. (2), we derive the average length $\bar{c}$of the micro-crack in magnetic field for the$N$ cycles as

$$\bar{c} = c_0 \exp(AN) \left\langle \exp\left(\int_0^N f(N')\mathrm{d}N' \right) \right\rangle = c_0 \exp\left(AN + \frac{DN}{2} \right).$$

Substituting Eqs. (6) and (7) into the above equation, we obtain the average length of micro-crack in magnetic field as

$$\bar{c} = c_0 \exp\left[\frac{(\sigma_\mathrm{a} - 2\varsigma)\sigma_\mathrm{a}^{1/\beta} - 2\dot{\lambda}_H \dot{H}_N N_0 L^2 \sigma_0^{1/\beta}}{2(\gamma+\lambda) N_0 L^2 \sigma_0^{1/\beta}} (1+\eta) N \right]. \tag{10}$$

For static field, $\dot{H}_N = 0$, we have the average length of micro-crack in static field

$$\bar{c}_\mathrm{s} = c_0 \exp\left[\frac{(\sigma_\mathrm{a} - 2\varsigma)\sigma_\mathrm{a}^{1/\beta}}{2(\gamma+\lambda) N_0 L^2 \sigma_0^{1/\beta}} (1+\eta) N \right]. \tag{11}$$

It can be seen from Eqs. (10) and (11) that the average length of micro-crack is influenced by magnetic field. For simplicity, we discuss the specific case of fatigue fracture in static magnetic field. To see how the average length of micro-crack varies, we plot $\bar{c}_\mathrm{s}$ as a function of λ and N from Eq. (11), as shown in Fig. 1. For the calculation, respective parameters are assumed as follows: $\sigma_\mathrm{a} = 40$ kg/mm^2, $\sigma_0 = 80$ kg/mm^2, $L = 10^{-2}$ mm, $\gamma = 3$ kg/mm, $\beta = 0.1$, $\eta = 0.3$, $N_0 = 6 \times 10^6$ mm^{-3}.

The curves in Fig. 1 show that the average length $\bar{c}_\mathrm{s}$ varies with the magnetic coefficient λ, besides increases with the number of cycles. If λ is positive (i.e. ferromagnetic materials), the average length decreases with λ, the presence of magnetic field makes the average length of micro-crack less than that without magnetic field. If λ is negative, the average length $\bar{c}_\mathrm{s}$ increases with the absolute value of λ, the presence of magnetic field makes the average length of micro-crack greater than that without magnetic field. So the average length of micro-crack can increase or decrease by applied magnetic field, depending on λ.

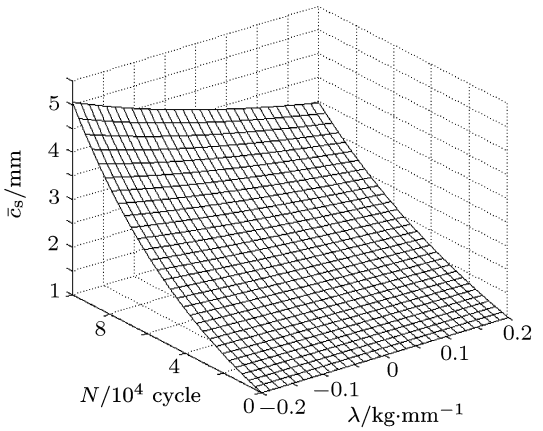


Fig. 1. The average length of micro crack in static magnetic field varies as a function of cycles N and magnetic coefficient λ.

4. Micro-crack density distribution function $M(c,N)$

To obtain the micro-crack density distribution function for metal, we substitute Eqs. (5) and (9) into

Eq. (3), and obtain the solution satisfying Eq. (4)

$$M(c,N) = AM_0 \int_0^N \frac{\exp(-AN')}{\sqrt{2\pi D(N-N')c^2}} \times \exp\left\{-\frac{\left[\ln\frac{c}{c_0} - A(N-N')\right]^2}{2D(N-N')}\right\} \mathrm{d}N', \tag{12}$$

Equation (12) is the micro-crack density distribution function in magnetic field, it shows that $M(c,N)$ can be influenced by magnetic field in addition to other known quantities. The affection of magnetic field on metal is closely related to λ. To be specific, let us look at materials of positive λ in static magnetic fields. According to Eq. (12), (6) and (7), we plot $M(c,N)$ as a function of the length of micro-crack at various magnetic fields, as shown in Fig. 2, where $N_0 = 8\times10^5$ mm^{-3}, $M_0 = 5\times10^6$ mm^{-3}, the other parameters are the same as those in Fig. 1. It can be seen from the graph that the influence of magnetic field on the density distribution function is effective mainly for small cracks. For the parameters used, $M(c,N)$ increases with magnetic field basically when the length of crack is very small (the initial part of the curves). With the increase of length of micro-crack (the final part of the curves), the influence of magnetic field on the density distribution function tends to be negligible.

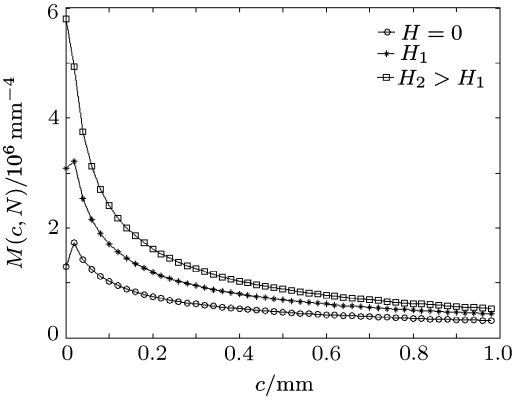


Fig. 2. Graph of $M(c, N)$ versus c for material of positive λ at various magnetic fields.

5. Fatigue fracture probability due to a single micro-crack

From the density distribution function, the probability distribution of micro-crack length $P(c,N)$ may be derived. The $P(c,N)$ is the probability of finding the micro-crack with length between c and $c+\mathrm{d}c$ among all micro-cracks per unit volume of metals under the action of cyclic stress for N cycles. It can be calculated via

$$P(c,N)\mathrm{d}c = \frac{M(c,N)\mathrm{d}c}{\int_{c_0}^{\infty} M(c,N)\mathrm{d}c} = \frac{A\mathrm{d}c}{(1-\mathrm{e}^{-AN})}\int_0^N \frac{\exp(-AN')}{\sqrt{2\pi D(N-N')c^2}} \times \exp\left\{-\frac{\left[\ln\frac{c}{c_0} - A(N-N')\right]^2}{2D(N-N')}\right\}\mathrm{d}N'. \tag{13}$$

Obviously, $P(c,N)$ satisfies the normalization condition

$$\int_{c_0}^{\infty} P(c,N)\mathrm{d}c = 1.$$

The integral expression of Eq. (13) is not convenient for further calculation, so we use its approximate formula to replace Eq. (13) in the following discussions[10]

$$P(c,N)\mathrm{d}c \approx \frac{1}{\sqrt{2\pi DNc^2}} \times \exp\left\{-\frac{\left[\ln\frac{c}{c_0} - AN\right]^2}{2DN}\right\}\mathrm{d}c. \tag{14}$$

Let the probability corresponding stress amplitude between σ_a and $\sigma_\mathrm{a}+\mathrm{d}\sigma_\mathrm{a}$ to cause micro-crack be $P(\sigma_\mathrm{a},N)\mathrm{d}\sigma_\mathrm{a}$. According to probability theory, we have

$$P(\sigma_\mathrm{a},N)\mathrm{d}\sigma_\mathrm{a} = P(c,N)|\mathrm{d}c|. \tag{15}$$

When the length of a micro-crack is greater than the critical length c_k, it will have probability to propagate. The critical length of micro-crack in magnetic field is given by[12]

$$c_\mathrm{k} = \frac{(\gamma+\lambda)E}{\pi(1-\nu^2)\sigma_\mathrm{a}^2}, \tag{16}$$

where E is Young's modulus and ν is Poisson's ratio. Substituting Eqs. (14) and (16) into Eq. (15), we obtain

$$P(\sigma_{\rm a},N)\,{\rm d}\sigma_{\rm a} = \sqrt{\frac{2}{\pi DN\sigma_{\rm a}^2}} \times \exp\left\{-\left[\frac{\ln\dfrac{(\gamma+\lambda)\,E}{\pi\,(1-\nu^2)\,c_0\sigma_{\rm a}^2} - AN}{\sqrt{2DN}}\right]^2\right\}{\rm d}\sigma_{\rm a}$$
$$= \frac{1}{\sqrt{\pi}}\exp(-X^2)\frac{\partial X}{\partial\sigma_{\rm a}}{\rm d}\sigma_{\rm a}, \tag{17}$$

where

$$X = \frac{\ln\left(\dfrac{c_{\rm k}}{c_0}\right) - AN}{\sqrt{2DN}} = \frac{\ln\left[\dfrac{(\gamma+\lambda)\,E}{\pi\,(1-\nu^2)\,c_0\sigma_{\rm a}^2}\right] - AN}{\sqrt{2DN}}.$$

The $P(\sigma_{\rm a},N){\rm d}\sigma_{\rm a}$ expressed by Eq. (17) is the probability of finding a micro-crack propagating between the stress amplitude $\sigma_{\rm a}$ and $\sigma_{\rm a}+{\rm d}\sigma_{\rm a}$ to cause metal to fracture from all the micro-cracks per unit volume of metal under the combined action of cyclic stress and magnetic field for N cycles.

Suppose that the stress amplitude does not change. Let the probability of finding a micro-crack to cause metal to fracture between the cyclic number N and $N+{\rm d}N$ be $P_n(\sigma_{\rm a},N){\rm d}N$. We have

$$P_n(\sigma_{\rm a},N){\rm d}N = P(X)\left|\frac{\partial X}{\partial N}\right|{\rm d}N.$$

Substituting Eq. (17) into this relation, we obtain

$$P_n(\sigma_{\rm a},N){\rm d}N = \frac{1}{\sqrt{\pi}}\exp(-X^2){\rm d}X. \tag{18}$$

We plot the probability $P_n(\sigma_{\rm a},N)$ as a function of cycles N and magnetic coefficient λ for material in static field, as shown in Fig. 3, where $N_0 = 5\times10^4$ mm^{-3}, the other parameters are the same as those in Fig. 2.

The graph shows that the probability is affected by magnetic coefficient λ. Hence the probability varies with magnetic field. To investigate how the probability changes with magnetic field, we consider the case for the material of positive λ in static magnetic field and plot its $P_n(\sigma_{\rm a},N)$ as a function of N at various magnetic fields, as shown in Fig. 4, the parameters used are the same as those in Fig. 3. Comparing the curves plotted at different static fields with the curve without magnetic field, it can be found that the applied magnetic field could make the probability $P_n(\sigma_{\rm a},N)$ increase or decrease even for the same material under the action of the same cyclic stress. The change in $P_n(\sigma_{\rm a},N)$ due to magnetic field has something to do with the number of cycles of applied stress. This means that the change relates to the time also. The initial parts of the curves show that the probability $P_n(\sigma_{\rm a},N)$ is less than that without magnetic field, and $P_n(\sigma_{\rm a},N)$ decreases with magnetic field. On contrary, after the action of magnetic field and cyclic stress for some time, the probability $P_n(\sigma_{\rm a},N)$ is greater than that without magnetic field, and $P_n(\sigma_{\rm a},N)$ increases with magnetic field.

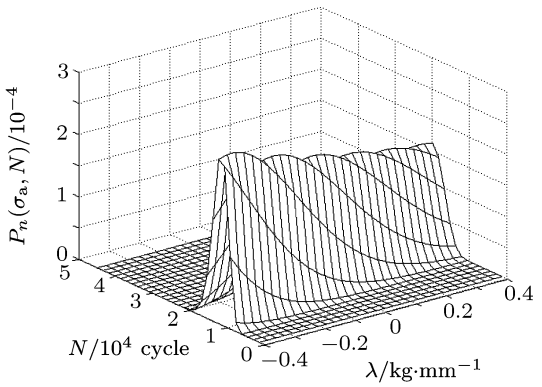


Fig. 3. Graph of $P_n(\sigma_{\rm a},N)$ versus N and λ.

Although the quantity of fracture probability for a material is hard to test experimentally, some experiments support the result derived from our theory on fatigue damage. Çelik *et al.*[9] investigated the effect of magnetic field on fatigue life of AISI 4140 steel. They found that fatigue life is improved when magnetic field was applied especially to the specimens at the stage of fatigue crack initiation. The magnetic field was detrimental on fatigue life if it was applied through the whole process of the test. Bhat *et al.*[7] found the decreasing in fatigue life of mild steel subjected to sustained magnetic field with increasing magnetic field. Based on our theory, it is understandable that the fatigue life of metal in magnetic field has something to do with the time stayed in magnetic field, because $P_n(\sigma_{\rm a},N)$ can be less or greater than that without magnetic field for different time durations as shown in Fig. 4.

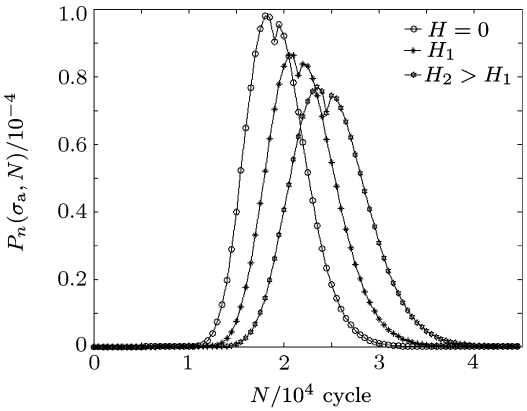


Fig. 4. Graph of $P_n(\sigma_a, N)$ for material of positive λ versus N at various magnetic fields.

6. Conclusions

Fatigue damage in magnetic field is a very complicated problem. It is more difficult to study than the fatigue damage under the only action of cyclic stress. In this paper, we present a non-equilibrium statistical theory on micro-crack distribution for metal undergoing fatigue fracture process in magnetic field. The results derived from our work indicate that the distribution of micro-crack depends on the applied magnetic field as well as on the magnetic and mechanical properties of material, and that the magnetic coefficient is an important quantity in describing the effect caused by magnetic field on fatigue fracture damage.

References

[1] Griffith A A 1920 *Phil. Trans. Roy. Soc. London* A **221** 163

[2] Abraham F F 2003 *Adv. Phys.* **52** 727

[3] Buehler M J, Abraham F F and Gao H 2003 *Nature* **426** 141

[4] Spatschek R, Hartmann M, Brener E, Müller-Krumbhaar H and Kassner K 2006 *Phys. Rev. Lett.* **96** 015502

[5] Cao L X and Wang C Y 2007 *Acta Phys. Sin.* **56** 413 (in Chinese)

[6] Lü B T, Qiao S R and Sun X Y 1999 *Scripta Mater.* **40** 767

[7] Bhat I K, Muju M K and Mazumdar P K 1993 *Int. J. Fatigue* **15** 193

[8] Fahmy Y, Hare T, Tooke R and Conrad H 1998 *Scripta Mater.* **38** 1355

[9] Çelik A, Yetim A, Alsaran A and Karakan M 2005 *Materials and Design* **26** 700

[10] Xing X S 1987 *Engng. Fracture Mech.* **26** 393

[11] Xing X S 1989 *Engng. Fracture Mech.* **32** 935

[12] Liu Z L, Hu H Y, Fan T Y and Xing X S 2009 *Chin. Phys.* B **18** 1283