# Glassy phase and thermodynamics for random field Ising model on spherical lattice in magnetic field

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Phase diagram and thermodynamic parameters of the random field Ising model (RFIM) on spherical lattice are studied by using mean field theory. This lattice is placed in an external magnetic field ($B$). The random field ($h_i$) is assumed to be Gaussian distributed with zero mean and a variance $\langle h_{\rm i}^2 \rangle = H_{\rm RF}^2$. The free energy ($F$), the magnetization ($M$) and the order parameter ($q$) are calculated. The ferromagnetic (FM) spin-glass (SG) phase transition is clearly observed. The critical temperature ($T_{\rm C}$) is computed under a critical intensity of random field $H_{\rm RF} = \sqrt{2/\pi}J$. The phase transition from FM to paramagnetic (PM) occurs at $T_{\rm C} = J/k$ in the absence of magnetic field. The critical temperature decreases as $H_{\rm RF}$ increases in the phase boundary of FM-to-SG. The magnetic susceptibility ($\chi$) shows a sharp cusp at $T_{\rm C}$ and the specific heat ($C$) has a singularity in small random field. The internal energy ($U$) has a similar behaviour to that obtained from the Monte Carlo simulation.

**Keywords:** random field, mean field theory, magnetization, specific heat and phase diagram

**PACC:** 7510N, 7540B, 7620

## 1. Introduction

The random field Ising model (RFIM)[1] is one of the simplest models used to describe the frustration introduced by disorder in interacting many-body systems. Despite its simplicity of RFIM, its behaviour has proved to be the source of much controversy.[1] We are lacking in reliable theoretical methods which can be used for such systems. Still, efforts to elucidate the basic properties of the RFIM continue to attract considerable attention, primarily because of its direct relevance to a number of important physical problems. These include not only the behaviours of diluted magnets in external magnetic fields,[2] but also several aspects of electronic transport in disordered insulators[3] and systems near the metal-insulator transition.[4−6] In addition, the non-equilibrium behaviour of the RFIM has been used to model the physics of hysteresis and avalanche behaviour and the origin of self-organized criticality.[7,8] The non-disordered models with infinitesimal random fields have been studied[9] in order to investigate self-generated glassy behaviour[10−15] observed in the systems such as supercooled liquids,[16] or even underdoped cuprates.[17,18]

The simplest effect of turning on a weak random field is the resulting depression of the critical temperature of the uniform ordering. While for sufficiently strong randomness the ordered phase completely disappears. Very early on, perturbative renormalization group (RG) results by Parisi and Sourlas (PS)[19] suggested the existence of a "dimensional reduction" by which the random problem belongs to the same universality class as a clean one in two dimensions. Unfortunately, the beautiful result of PS was found to be in conflict not only with the heuristic argument of Imry and Ma,[20] but also with the exact results of Imbrie[21] and thus deemed to be incorrect. More recently, the origin of these discrepancies was traced[22] to the implicit assumption of PS that outside the ordered phase, a single thermodynamically stable phase exists. This assumption may not be warranted if another phase transition, presumably of glassy character, would precede any uniform ordering, which would lead to the breakdown of dimensional reduction. Of course, such a glassy phase is not found in the naïve Bragg–Williams mean field theory (BWMFT), so that more sophisticated theoretical schemes have to be used in order to identify the corresponding instability of the high-temperature paramagnetic phase. Such a theory was formulated by Mezard and Young[22] who utilized the self-consistent screening (SCS) approach

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of Bray,[23] and identified the glassy phase by carrying out a replica-symmetry-breaking stability analysis.

Similar results were obtained by numerically solving the mean-field equations for a fixed realization of disorder by Lancaster *et al.*,[24] confirming the existence of the glassy phase. Mezard and Monasson,[25] and de Dominicis *et al.*[26] presented the arguments that the glass phase should persist even at weak disorder, and everywhere precede the uniform ordering, in agreement with the breakdown of the dimensional reduction. Pastor *et al.*[27] presented the simplest possible approach that is capable of providing a description of glassy phase. The recent theoretical efforts used sophisticated numerical simulation approaches to study the behaviour of the order parameter under magnetic field and the temperature for the RFIM.[28−30] The phase diagram has been investigated by Monte–Carlo simulation.[31,32] The one-dimensional disordered Ising systems are considered to be one of the most important physical problems.[33,34] The thermal behaviour of the square Ising model studied with exchange ($J$) and dipole ($g$) interactions is well understood for values of $J/g$ far from the phase boundaries between striped configurations of different values of width $h$. A variety of phase transitions were found by Monte–Carlo simulations, where the first order and continuous phase transitions were found depending on the value of the ratio $J/g$, but no intermediate phase was found between the low temperature ordered striped phase and the high temperature paramagnetic one. Phase diagram of a square Ising model with exchange and dipole interactions was obtained by using Monte–Carlo simulations.[35] The statistical properties of peculiar responses are studied in glassy systems on the mesoscopic scale based on a class of mean-field spin-glass models which exhibit one step replica symmetry breaking.[36]

Experimentally, the Ising models in random fields (RF) are a good representation of a large number of impure materials. For the RFIM, Belanger[37] and Nattermann[38] discussed experiments and surveyed the theory for only the nearest neighbour interactions in the absence of an external magnetic field ($B$). They pointed out that the experiments gave clear evidence of a finite temperature transition in three dimensions and discussed the observed critical behaviour noting the difficulties arising from irreversible effects below the critical point. The recent experiments on the diluted anti-ferromagnets under high random fields provide an experimental characterization of the equilibrium critical behaviour.[39] The theoretical study of infinite one-dimensional RFI systems was done by Andelman,[40] and Vilfan and Stefan.[41]

In this work, we present the thermodynamic properties of the RFIM on spherical lattice in the presence of an external magnetic field by using the replica trick.[42,43] Our aim is to investigate the phase diagram, the critical temperature and the field dependence of the thermodynamic parameters, including free energy, magnetization, order parameter, magnetic susceptibility, internal energy and specific heat. Gaussian distribution of the local random field is considered with zero mean and variance $H_{\mathrm{RF}}^2$. The remainder of the present paper is organized as follows. In Section 2 we describe the details of the model and use the replica trick to obtain the free-energy ($F$). We calculate in Section 3 magnetization ($M$) and order parameter ($q$) self-consistently. In Section 4, we analyse our results for RFIM on spherical lattice in the presence of an external magnetic field. We discuss and compare our results with other numerical simulations obtained before. Finally, in Section 5 we draw some conclusions from the present study.

## 2. Model

We consider the Hamiltonian of RFIM as follows:

$$\mathscr{H} = -J\sum_{ij} S_i S_j - \sum_i H_i S_i - B\sum_i S_i, \qquad (1)$$

where $J$ is uniform ferromagnetic interaction among the nearest neighbours spins, rescaled with the coordination number $z$ in order to obtain a finite result in the $z \to \infty$ limit (spherical lattice); $S_i$ and $S_j$ are the corresponding Ising spin variables respectively and $S_i, S_j = \pm 1$; $B$ is the external magnetic field applied to our system. The random magnetic field variables $\{H_i\}$ are Gaussian distributed, with zero mean and variance $\langle h_i^2 \rangle = H_{\mathrm{RF}}^2$.

Thermodynamic properties of the model are obtained from the following ensemble averaged free energy:

$$F(\beta) = -\frac{1}{\beta N} \lim_{N\to\infty} \ln Z(\beta). \qquad (2)$$

In Eq. (2) the partition function $Z = \sum_{\{S_i\}} \exp(-\beta\mathscr{H}(S_i))$, where $\sum_{\{S_i\}}$ is the summation over all states of configurations and $\beta = 1/kT$ with $k$ being Boltzmann constant and $T$ the absolute temperature.

It is very difficult to compute $[\ln Z]_{\rm av}$ directly, because of the summation over the disorder. In order to overcome this difficulty, we use the so-called replica trick and one can formally average over disorder. This trick uses the simple mathematical identity given as

$$\ln Z = \lim_{n\to 0}\left(\frac{Z^n-1}{n}\right). \tag{3}$$

As usual, we aim to calculate $Z^n$ for positive integer $n$ by replicating the system $n$ times. Introducing $n$ similar replicas of the original system, we will compute the partition functions of these $n$ replicas i.e. $Z^n$. Then we do the analytic continuation $n \to 0$ in Eq. (3). The effective Hamiltonian no longer contains any disorderness. In this case, the resulting partition function takes the form,

$$Z^n = {\rm Tr}_n \exp\left(\frac{\beta J}{z}\sum_{ij}\sum_{\alpha} S_i^\alpha S_j^\alpha + \frac{1}{2}(\beta H_{\rm RF})^2 \sum_{ij}\sum_{\alpha\beta} S_i^\alpha S_i^\beta S_j^\alpha S_j^\beta + B\sum_i\sum_\alpha S_i^\alpha\right), \tag{4}$$

where $\alpha$ and $\beta$ label $n$ dummy replicas (indexed by $\alpha = 1, \ldots, n$; $n \to 0$). The trace is now over $n$ replicas at a single site and extends over all states of a single replicated spin $S^\alpha$.

The relative magnitudes of $H_{\rm RF}$ and $J$ determine whether ferrromagnet (FM) or spin-glass (SG) ordering occurs at low temperature. Higher cumulants will scale as higher inverse powers of number of spin than the first two ones. Thus these do not affect the thermodynamic properties of an infinite-range order. Using the following two identities:

$$\sum_\alpha\sum_{(ij)} S_i^\alpha S_j^\alpha = \frac{1}{2}\sum_\alpha\left(\sum_i S_i^\alpha\right)^2 - \frac{Nn}{2},$$

$$\sum_{\alpha\beta}\sum_{(ij)} S_i^\alpha S_i^\beta S_j^\alpha S_j^\beta = \frac{1}{2}\sum_{\alpha\beta}\left(\sum_i S_i^\alpha S_i^\beta\right)^2 + \frac{N^2n}{2} - \frac{Nn^2}{2},$$

then, dropping the terms which vanish in the thermodynamic limit, we find that, the resulting free energy in thermodynamic limit can be expressed as

$$\begin{aligned} F = -kT\lim_{N\to\infty}\lim_{n\to 0}(Nn)^{-1}\Bigg\{ & {\rm Tr}_n \exp\left(\frac{\beta J}{2z}\sum_\alpha\left(\sum_i S_i^\alpha\right)^2\right. \\ & \left. + \frac{1}{4}(\beta H_{\rm RF})^2\sum_{(\alpha\beta)}\left(\sum_i S_i^\alpha S_i^\beta\right)^2 + \beta B\sum_\alpha\sum_i S_i^\alpha\right) - 1\Bigg\}, \end{aligned} \tag{5}$$

where $(\alpha\beta)$ refers to the combination of $\alpha$ and $\beta$ with $\alpha \neq \beta$. Note that the exchange terms in the exponent are in the form of $\lambda\left(\sum_i O_i\right)^2$, where $O_i$ is the local intensive operator, which leads to physical thermodynamic consequence only if $\lambda \propto N^{-1}$. By applying the following Hubbard–Stratononvitch identity:

$$\exp(\lambda a^2) = \left(\frac{N}{2\pi}\right)^{1/2}\int {\rm d}x \exp\left(-\frac{Nx^2}{2} + (2\lambda N)^{1/2} ax\right) \tag{6}$$

to the squared terms in Eq. (5), we find that

$$\begin{aligned} F = & -kT\lim_{N\to\infty}\lim_{n\to 0}(Nn)^{-1} \\ & \times\left[\int\prod_\alpha\left(\frac{N}{2\pi}\right)^{1/2}{\rm d}x^\alpha\right]\left[\int\prod_{(\alpha\,\beta)}\left(\frac{N}{2\pi}\right)^{1/2}{\rm d}y^{(\alpha\beta)}\right]\exp\left\{-N\sum_\alpha\frac{1}{2}(x^\alpha)^2 + \sum_{(\alpha\beta)}\frac{1}{2}\left(y^{\alpha\beta}\right)^2\right. \\ & \left.-\ln {\rm Tr}_n\exp\left(\beta B\sum_\alpha S^\alpha + (\beta J)\sum_\alpha x^\alpha S^\alpha + \frac{1}{2}(\beta H_{\rm RF})^2\sum_{(\alpha\beta)} y^{(\alpha\beta)} S^\alpha S^\beta\right)\right\} - 1. \end{aligned} \tag{7}$$

Using the single-spin property,

$${\rm Tr}_n\exp\left(g\sum_i S_i^\alpha\right) = \exp\left[N\ln {\rm Tr}_n \exp g\left(S^\alpha\right)\right] \tag{8}$$

and assuming that both the limit $n \to 0$ and the thermodynamic limit $N \to \infty$ can be interchanged we can rewrite Eq. (7) in the form

$$F = -kT \lim_{N\to\infty} \lim_{n\to 0} (Nn)^{-1} \left\{ \exp\left[\frac{1}{4}\beta^2 H_{\mathrm{RF}}^2 Nn\right] \left[\int \prod_{\alpha} \left(\frac{N}{2\pi}\right)^{1/2} \mathrm{d}x^{\alpha}\right] \times \left[\int \prod_{(\alpha\beta)} \left(\frac{N}{2\pi}\right)^{1/2} \mathrm{d}y^{(\alpha\beta)}\right] \exp\left(-NG\right) - 1 \right\} \tag{9}$$

with

$$G = \frac{1}{2}\sum_{\alpha} (x^{\alpha})^2 + \frac{1}{2}\sum_{(\alpha\beta)} \left(y^{\alpha\beta}\right)^2 - \ln \mathrm{Tr}_n \exp\left(\beta B \sum_{\alpha} S^{\alpha} + (\beta J)\sum_{\alpha} x^{\alpha} S^{\alpha} + \frac{1}{2}\left(\beta H_{\mathrm{RF}}\right)^2 \sum_{(\alpha\beta)} y^{(\alpha\beta)} S^{\alpha} S^{\beta}\right), \tag{10}$$

where the trace extends over all states of a single replicated spin $S^{\alpha}$. For $N \to \infty$, the integral (9) with Eq. (10) can be performed by the steepest descents method (The integral being dominated by the region of maximum integrand). Since the replicas are indistinguishable, we consider only the extreme of the exponential for which all values of the $y^{(\alpha\beta)}$ are equal, as well as $x^{\alpha}$. We denote their values by $y_n$ and $x_n$. We obtain that $M = M_n = x_0^{\alpha}$ and $q = q_n = y_0^{\alpha\beta}$, where $M = \langle S^{\alpha} \rangle$ and $q = \left\langle S^{\alpha} S^{\beta} \right\rangle_{\alpha\neq\beta}$.

One of our goals is to use the saddle-point condition to evaluate the integral $I$, where

$$I = \int \exp\left\{-NG\left(x_0^{\alpha}, y_0^{\alpha\beta}\right)\right\} \left[\prod_{\alpha} \left(\frac{N}{2\pi}\right)^{1/2} \mathrm{d}x^{\alpha}\right] \left[\prod_{(\alpha\beta)} \left(\frac{N}{2\pi}\right)^{1/2} \mathrm{d}y^{\alpha\beta}\right]. \tag{11}$$

If one takes care of the correction to the leading result one finds that

$$I \propto (\det \Lambda_n)^{1/2} \exp\{-NG(x_0^{\alpha}, y_0^{\alpha\beta})\}[1 + O(N^{-1}],$$

where $\Lambda_n$ is a matrix of $\left[\frac{1}{2}n\,(n+1)\right] \times \left[\frac{1}{2}n\,(n+1)\right]$ whose elements are simple functions of $M$ and $q$ of order unity. Both the square root of the determinant and the $O\left(N^{-1}\right)$ correction involve the square root of the products of eigen-values of the Hessian: a negative eigen-value would imply an imaginary result. It is therefore absolutely necessary that the eigenvalues of the Hessian can be non-negative; if they are zero, one should consider that the higher order terms and the corrections to the leading terms in Eq. (11) can then have a different behaviour, e.g. $O\left(N^{1/2}\right)$. By substituting

$$\sum_{(\alpha\beta)} y_{\max}^{(\alpha\beta)} S^{\alpha} S^{\beta} = \frac{1}{2}\left[\left(\sum_{\alpha} S^{\alpha}\right)^2 - n\right], \tag{12}$$

where $y_{\max}^{(\alpha\beta)} = 1$ for ferromagnetic state, into Eq. (11), and using Eqs. (6) and (10), one can obtain

$$I = \left(\det \Lambda_n\right)^{-1/2} \exp\left[-N\left(\frac{1}{2}\beta J n m_n^2 + \frac{1}{2}\left(\beta H_{\mathrm{RF}}\right)^2 \left[n\,(n-1)\,q_n^2 + n\right] - \ln \int \mathrm{d}z\, \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \left(2\cosh Z_n\right)^n\right)\right] \times \left[1 + O\left(N^{-1}\right)\right], \tag{13}$$

where $Z_n = \beta\left(B + JM + H_{\mathrm{RF}} x\right)$. By substituting Eq. (13) into Eq. (9), the final form of the free energy ($F$) is given as

$$F = -kT\left[\frac{1}{2}\beta J m^2 + \frac{\left(\beta H_{\mathrm{RF}}\right)^2}{4}(1-2q)^2 - \frac{1}{\sqrt{2\pi}} \int \exp\left(-\frac{x^2}{2}\right) \ln\left(2\cosh Z_n\right) \mathrm{d}x\right]. \tag{14}$$

## 3. Thermodynamic parameters

Magnetization ($M$), order parameter ($q$), susceptibility ($\chi$), internal energy ($U$) and specific heat ($C$) are calculated each as a function of temperature ($T$) and random field ($H_{\rm RF}$).

By differentiating Eq. (14) with respect to $B$, we find the magnetization ($M$) in the following form:

$$M(T) = \frac{1}{\sqrt{2\pi}} \int \exp\left(-\frac{x^2}{2}\right) \times \tanh\left(\beta B + \beta J M + \beta H_{\rm RF} x\right) {\rm d}x. \quad (15)$$

It is a self-consistent equation which can be solved with the saddle-point conditions. The mean-square of frozen moment ($q$) per site is defined as $q = \left\langle S^{\alpha} S^{\beta} \right\rangle_{\alpha \neq \beta}$. This leads to

$$q\,(T) = \frac{1}{\sqrt{2\pi}} \int \exp\left(-\frac{x^2}{2}\right) \times \tanh^2\left(\beta B + \beta J M + \beta H_{\rm RF} x\right) {\rm d}x. \quad (16)$$

We solve Eqs. (15) and (16) numerically by using Newton–Raphson method (NRM). This contrasts with the behaviour of $M(T)$ in a uniform Ising magnet, for which all temperature derivatives vanish at $T = 0$ since the excitations from the ferromagnetic ground state require a finite energy.

We can find that the zero-temperature magnetization is

$$M(0) = {\rm erf}\left[\frac{1}{\sqrt{2}}\left(\frac{J}{H_{\rm RF}} M(0) + \frac{B}{H_{\rm RF}}\right)\right]. \quad (17)$$

Inserting $B = 0$ into Eq. (15), a straightforward generalization of the well–known Bragg–Williams's condition is obtained. The critical temperature where the magnetization vanishes is easily computed by differentiating the right-hand side of Eq. (15) with respect to $M$ through letting both $M$ and $B$ be zeros, and given as the following equation:

$$\frac{kT}{J} = \frac{1}{\sqrt{2\pi}} \int \exp\left(\frac{-x^2}{2}\right) \times {\rm sech}^2\left(\frac{H_{\rm RF} x / J}{kT/J}\right) {\rm d}x. \quad (18)$$

From this equation, we can plot the phase diagram. By differentiating the term on the right-hand side of Eq. (18) with respect to temperature at $T = T_{\rm C} = 0$, the zero solution occurs when $H_{\rm RF}/J \geq \sqrt{2/\pi}$ which means that the system becomes SG for all values of $T$.

Differentiating Eq. (15) with respect to the magnetic field ($B$) and taking the limit as $B \to 0$, the susceptibility ($\chi$) is given as

$$\chi(T) = \frac{1-q}{kT - J(1-q)}. \quad (19)$$

If we let $q = 0$ in the above equation, we can obtain the Curie–Weiss law. By substituting Eq. (18) into Eq. (19), we find that $\chi \to \infty$ at $T = T_{\rm C}$.

We use the adaptive Simpson quadrate method to compute the integral in Eq. (14) within MATLAB package. We are able to obtain numerical results for the internal energy ($U$) by differentiating Eq. (14) with respect to $\beta$ with applying the central difference method. The specific heat ($C$) can be calculated also by differentiating Eq. (14) with respect to temperature ($T$).

## 4. Results and discussion

The thermodynamic parameters obtained above are calculated where the Gaussian distribution of the local random field is considered. In the absence of external magnetic field ($B$), we find that the free energy ($F$) is constant at low temperature and then linearly decreases with temperature increasing whatever the random field is, which is shown in Fig. 1.

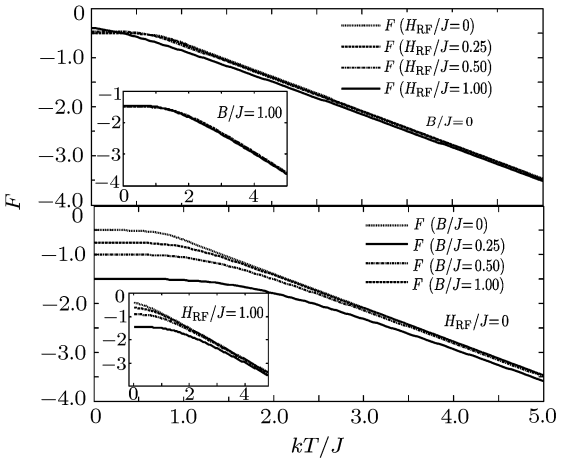



**Fig. 1.** Curves for free energy ($F$) versus $kT/J$. In the upper panel, at different strengths of random field ($H_{\rm RF}/J$) in the presence of external magnetic field ($B/J$), the main part is for the case of $B/J = 0.0$ and the inset is for $B/J = 1.0$. In the lower panel, at different values of external magnetic field ($B/J$), the main part is for the case of $H_{\rm RF}/J = 0.0$ and the inset is for $H_{\rm RF}/J = 1.0$.

In the presence of $B$, $F$ has the similar behaviour (see the inset in the upper panel of Fig. 1). In addition, we find that the free energy equals the ground

state internal energy ($U(0)$) at $T = 0$. This result accords with that obtained by Binder and Young.[44] In the absence of random field ($H_{\rm RF}$), it is constant at low temperature and decreases with the value of $B$ increasing. In the presence of $H_{\rm RF}$, $F$ has a similar behaviour (see the inset in the lower panel of Fig. 1).

$M(T)$ and $q^{1/2}(T)$ obtained by numerical solution of Eqs. (15) and (16) are shown in Fig. 2 for different values of random magnetic field $H_{\rm RF}$ and external magnetic field $B$. The frozen moment $q^{1/2}\,(T)$ is always greater than $M(T)$ at the same temperature (for $H_{\rm RF} \neq 0$). The magnetization decreases as random field increases and tends to zero as $H_{\rm RF}/J \geq \sqrt{2/\pi}$ for all temperatures. Also, we find that $M$ and $q^{1/2}$ decrease as temperature increases. This accords with the result obtained by Rosas and Coutinho.[45]

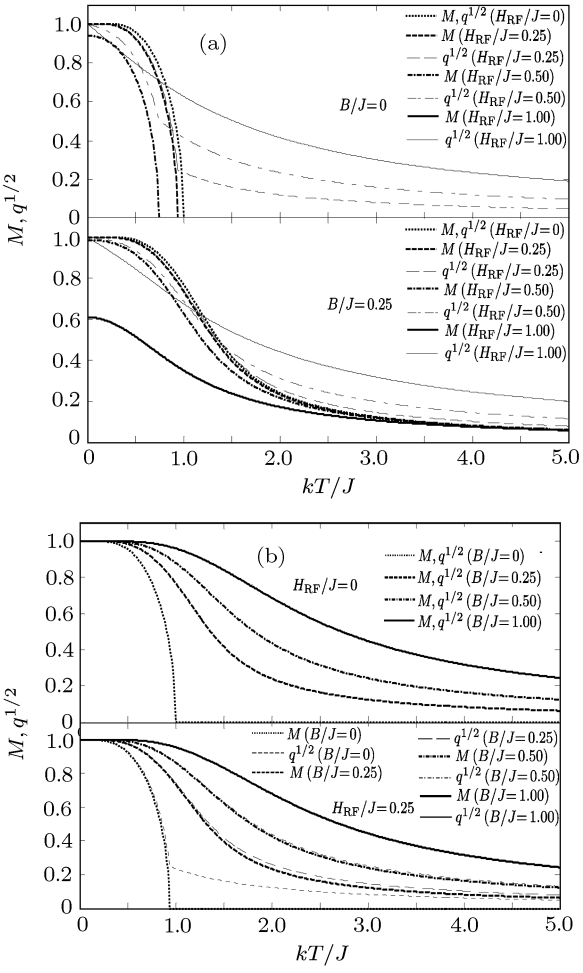



**Fig. 2.** Curves for magnetization ($M$) and the order parameter ($q^{1/2}$ ) versus $kT/J$ at different values of random field ($H_{\rm RF}/J$) with a Gaussian distribution (GD) for the RFIM in two cases of external field $B/J = 0$ and 0.25 (a), and at different values of magnetic field ($B/J$) in $H_{\rm RF}/J$ =0 and 0.25 (b).

They found that whatever the probability distribution is the magnetization decreases fast for high fields (below the critical field) as temperature increases, indicating that the stronger the random field is the greater the probability of appearance of correlated clusters of reversed spins with respect to the magnetization will be. In Fig. 2(a), $M$ decreases faster than $q^{1/2}$ and vanishes at $H_{\rm RF} = 1$ since the integrals in Eqs. (15) and (16) depend on the value of $H_{\rm RF}$. This means that there is a phase transition from FM to SG at strong random field ($H_{\rm RF} = 1$). In Fig. 2(b), we find that $M$ and $q^{1/2}$ increase with external magnetic field ($B$) increasing. Also, we find that $M = q^{1/2}$ as $H_{\rm RF} = 0$. This result is in agreement with those obtained in Refs. [42] and [43]. Because they found that a finite external field removes sharp phase transitions by allowing $M$ and $q^{1/2}$ to be nonzero at all temperatures. The separation between $M$- and $q^{1/2}$-curves is increasing with the value of $H_{\rm RF}$ increasing as shown in the lower panel of Fig. 2(a) while it decreases by increasing $B$ as indicated in the lower panel of Fig. 2(b). The FM region (i.e. at $B$=0) decreases gradually by increasing $H_{\rm RF}$ as shown in the upper panel of Fig. 2(a) and vanishes at $H_{\rm RF} = 1$. Finally, we can conclude that $M = q^{1/2} \cong 1$, at low values for both $H_{\rm RF}/J$ and $kT/J$.

The ground-state magnetization ($M(T = 0)$) increases as $B$ increases and vanishes as $H_{\rm RF}/J \geq \sqrt{2/\pi}$, which is shown in Fig. 3.

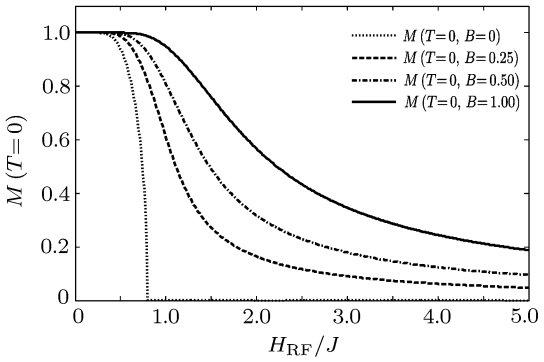



**Fig. 3.** Ground-state magnetizations versus $H_{\rm RF}/J$ at different values of magnetic field.

We observe that $M(T = 0)$ is equal to one at low $H_{\rm RF}/J$ and decreases as the random field increases.

From the phase diagram in Fig. 4 it follows that (i) the phase transition from FM to PM occurs in the absence of external magnetic field ($B$) and at $T_{\rm C} = J/k$ only; (ii) the $T_{\rm C}$ decreases as $H_{\rm RF}$ increases in the boundary of phase transition from FM to SG; (iii) our RF system behaves as SG as $H_{\rm RF}/J \geq \sqrt{2/\pi}$,

whatever the $T$ is.

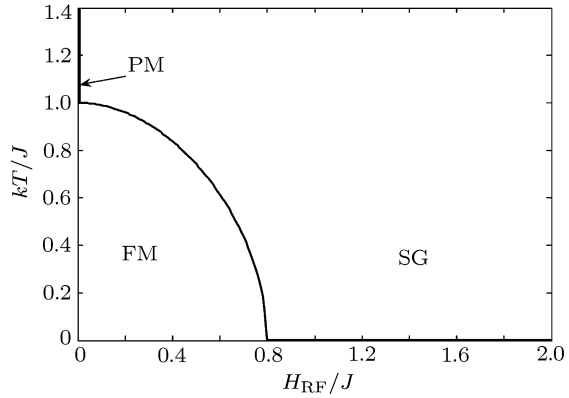



**Fig. 4.** Phase diagram of RFIM at $B = 0$.

In the absence of external magnetic field, we find that the susceptibility ($\chi$) has a sharp cusp at small random fields as shown in Fig. 5(a).

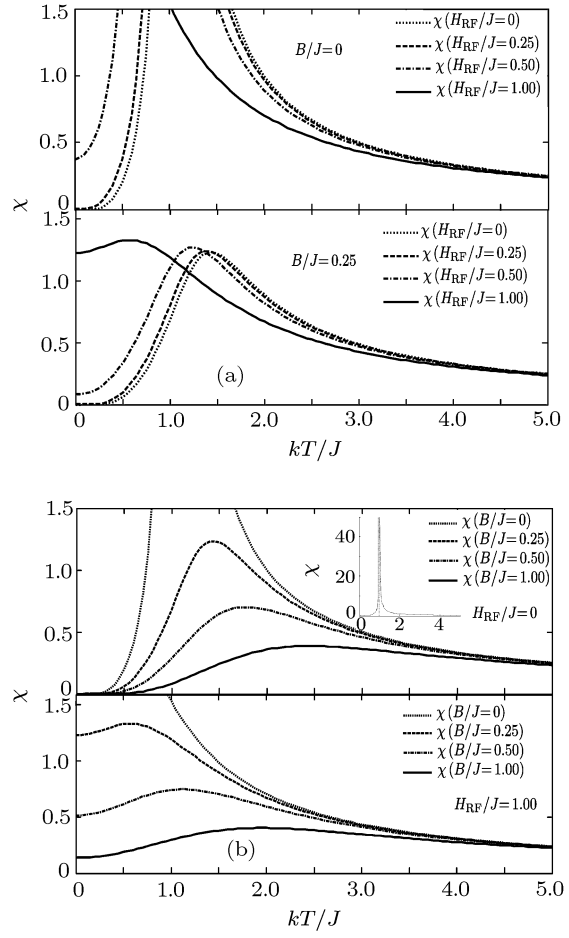



**Fig. 5.** Curves for susceptibility ($\chi$) versus $kT/J$ at different values of $H_{\rm RF}/J$ in $B/J = 0$ and 0.25 (a) and at different values of $B/J$ for $H_{\rm RF}/J$ =0 and 1 (b).

On increasing the random field it decreases with temperature increasing and tends to zero at high temperature. In the presence of external magnetic field, $\chi$ shows a rounded peak which increases and moves toward high temperatures as shown in Fig. 5(a).

In the absence of random field, $\chi$ shows a rounded peak whatever the value of external magnetic field is. The rounded peak moves to right with the value of $B$ increasing. The maximum of the peak lowers with the value of $B$ increasing. In the presence of random fields, the susceptibility shows a similar behaviour as shown in Fig. 5(b). This result accords with that obtained by Ismail *et al.*[43] They found that the real part and the imaginary part of the dynamic susceptibility display maxima at $T = T_{\rm C}$. These maxima can be described by the Arrhenius law.

We note that the internal energy ($U$) is almost constant at low temperature. With the further increase of temperature, $U$ increases as $T$ increases and it always negative and vanishes at $T_{\rm C}$ when $H_{\rm RF} = 0$ as shown in Fig. 6.

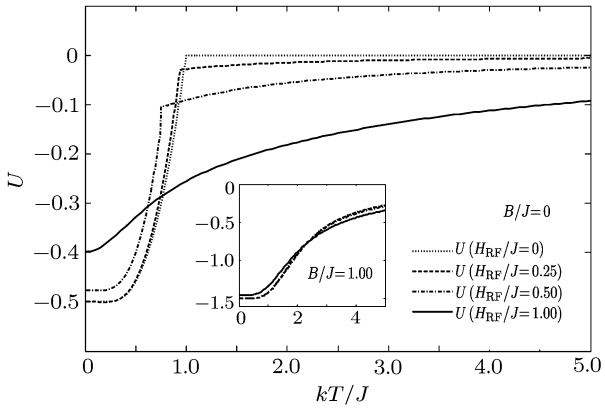



**Fig. 6.** Curves for internal energy ($U(T)$) versus $kT/J$ at different values of $H_{\rm RF}/J$ with GD for the FIRM at $B/J = 0$ and 1.

In the absence of $B$, $U$ decreases on increasing $H_{\rm RF}$. Then it tends to zero by increasing the temperature above $T_{\rm C}$. In the presence of $B$, $U$ increases slowly at low temperatures then it increases rapidly as $T$ increases, whatever $H_{\rm RF}$ is (see the inset of Fig. 6). This result is in agreement with that obtained in Ref. [42]. The internal energy at zero temperature comes out to be $U(0) = -\sqrt{2/\pi}J \cong -0.798J$, while numerical simulation gives rise to $U(0) \cong -0.79J$, which is in quantitative agreement with the Monte Carlo simulation.[42]

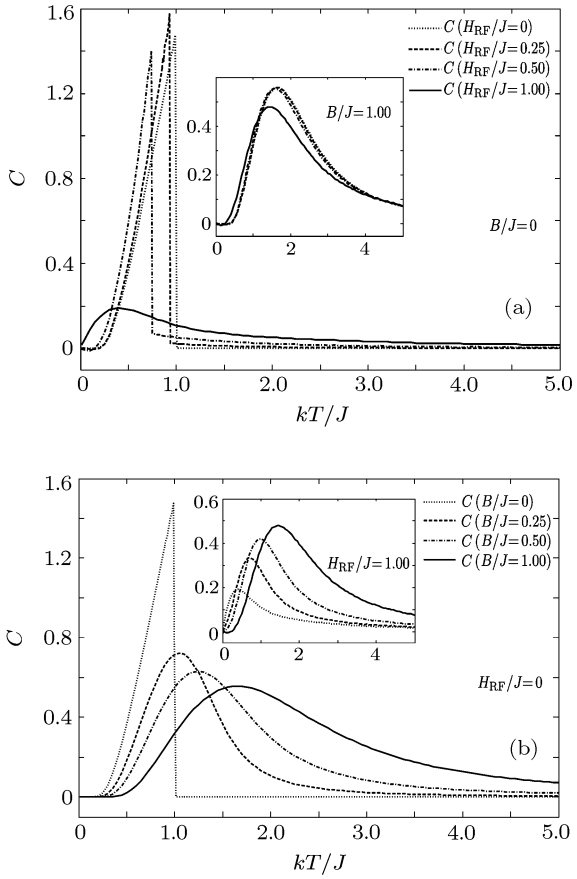



**Fig. 7.** Curves for specific heat ($C(T)$) versus $kT/J$ with GD for the RFIM at different values of $H_{\rm RF}/J$ for $B/J = 0$ and 1 (a) and at different values of $B/J$ for $H_{\rm RF}/J = 0$ and 1 (b).

Finally, in the absence of an external magnetic field, the specific heat ($C$) has a singularity at a critical temperature under small random fields (see Fig. 7(a)). This means that there is a phase transition from FM to SG at a critical temperature ($T_{\rm C}$). On increasing random field it has a rounded peak as shown in Fig. 7(a). This result is in agreement with what obtained in Ref. [42]. This behaviour should be contrasted with the situation for a pure infinite-ranged ferromagnet for which $C$ vanishes above $T_{\rm C}$. It is shown also that the value of $T_{\rm C}$ decreases on increasing random field. While in the presence of $B$, $C$ shows a rounded peak which decreases on increasing $H_{\rm RF}$ (see the inset of Fig. 7(a)). On the other hand, in the absence of random field (i.e., $H_{\rm RF} = 0$), $C$ shows a rounded peak under $B$. The peaks decrease on increasing $B$ and shifted toward high $T$ as shown in Fig. 7(b).

But in the presence of random field, $C$ shows a rounded peak which moves toward high $T$ on increasing $B$ (see the inset of Fig. 7(b)). This behaviour is similar to that in Ref. [45]. They observed that the specific heat for the delta-bimodal model displays a "plateau" absent in the Gaussian model. All our results are in agreement with those obtained by Ismail and Salem,[33,34] who showed that the variation of $C$ as a function of $T$ for several values of frequency ($\omega$), and $C$ has a maximum. The maxima have very rounded peaks and moves towards $T = 0$ on increasing $\omega$.

## 5. Conclusions

We can conclude that in the present spherical-lattice Ising model with random field, the critical temperature is $T_{\rm C} = J/k$ at a critical strength $H_{\rm RF} = \sqrt{2/\pi}J$ of the random field. The critical temperature is decreasing on increasing $H_{\rm RF}$. The random field system behaves like the conventional SG system on $H_{\rm RF} \geq \sqrt{2/\pi}J$.

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