Effect of transparency on Josephson junction between s+g-wave superconductors

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In this paper, a dc Josephson junction between borocarbide superconductors has been studied theoretically. The s+g-wave pairing symmetry which is observed in rare earth complex of borocarbides has a huge anisotropy and is an interesting form of unconventional superconductivity. We calculate the Josephson current in a superconductor–insulator–superconductor (SIS) Josephson junction with s+g-wave superconducting pairing symmetry. In our planar junction *c*-axis is parallel to an interface with finite transparency but *ab*-planes of two tetragonal superconductors are misorientated by angle α. We obtain that the Josephson current is strongly dependent on mis-orientation between the left and the right *ab*-planes. An insulator sandwiched between two superconductors which acts as a potential barrier is demonstrated by a transparency coefficient. The effects of the potential barrier and the mis-orientation on the current are studied analytically and numerically. Occurrence of 0–π transition in this s+g-wave junction is investigated in this paper. A comparison between d-wave Josephson junction and s+g-wave one is also made in the present paper.

Keywords: Josephson junction, unconventional superconductor, borocarbides

PACC: 7340G, 7430F, 7450, 7470H

1. Introduction

The s+g-wave pairing symmetry as a new form of unconventional superconductivity has been considered in borocarbide complexes, such as YNi_2B_2C, YPd_2B_2C and $LuNi_2B_2C$.[1−8] These superconductors have relatively high critical temperatures T_c = 10–20 K and also have energy gap $\Delta(0) = 2.2$ meV.[9] As an interesting property, both of superconductivity and magnetic order coexist in some of these compounds at low temperatures.[3] This opens up the possibilities for the interplay between superconducting and magnetic ordering. Also the fourfold symmetry originating from the anisotropy of the superconducting gap structure has been observed in Ref. [10]. On the other hand, the Josephson junction between unconventional superconductivities is one of the most interesting approaches to the investigating of the pairing symmetry of unconventional superconductivity. Also, a Josephson junction between two crystals of borocarbides has been investigated theoretically in Ref. [11]. Two borocarbide crystals have a mis-orientation angle α and their common interface is ideally transparent (without reflection for electrons and pairs). The effect of mis-orientation on the current phase diagram was studied in Ref. [11]. In the mentioned paper, it was obtained that current phase relation for this type of Josephson junction is totally different from the conventional s-wave one.[12] Motivation of the present paper is to generalize the structure described in Ref. [11] to the case of finite and arbitrary transparency of interface between superconductors and also to generalize the case in Ref. [13] to the s+g-wave Josephson junction. While the ideally transparent system is a scientific case but the finite transparent system has more experimental applications. In this paper we generalize structure described in Ref. [11] from the ideally transparent junction superconductor–superconductor to the real case of superconductor–insulator–superconductor. In the present paper, current–phase diagrams for the junction between s+g-wave superconductors with finite transparency interface and the mis-orientation between crystals are plotted and investigated and the effects of transparency and mis-orientation on the current are also studied. Also, the possibility of the π state in weak link structures is one of the most interesting effects in the field of quantum transport in superconducting devices.[14,15] The π state as a phase shift of π in the ground state of a junction energy can be demonstrated by the sign reversal of critical

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Josephson supercurrent. The π junction can be produced by conventional Josephson junction including ferromagnet or by unconventional superconducting junction. For example, the π junction was observed in superconductor–ferromagnet–superconductor junction experimentally in Refs. [16] and [17]. In addition, the possibility of the π junction in the ballistic systems was studied theoretically in Refs. [18]–[21]. Owing to the discovery of unconventional superconductivity, the π transition has become a more interesting case of study. For instance, the π shift of the superconducting phase between the a- and the b-crystallographic directions of thin film of a high T_c superconductor was investigated in Ref. [22]. In addition, the π junction in spin-triplet Josephson junction was investigated in Ref. [23]. Here, we investigate the possibility of 0–π transition by changing the mis-orientation between borocarbide crystals and compare it with the case of d-wave Josephson junction.

The rest of this paper is organized as follows. In Section 2 we describe our configuration to be investigated. For a non-self-consistent model of the order parameter, the quasiclassical Eilenberger equations[24] are solved and suitable Green functions are obtained analytically. In Section 3 the obtained formulas for the Green functions are used for calculating the spontaneous and Josephson current densities in the interface. An analysis of numerical results is made. In Section 4 the analytical investigation of the system near the critical temperature is carried out. In Section 5 possibility of 0–π transition is investigated and from this point of view an s+g-wave Josephson junction is compared with d-wave Josephson junction. The paper will be finished with some conclusions in Section 6.

2. Formalism and basic equations

In this paper, we solve the nonlocal Eilenberger equations[24] imposed with Zaitsev boundary conditions[25,26] and obtain the corresponding Green functions exactly and analytically. These boundary conditions have been given in Ref. [27], and they have been refined by the author of this paper in Ref. [28]. Then, using the Green functions, we compute the spontaneous current and the Josephson current. The system which is investigated in this paper, consists of two borocarbide superconductors, between which an insulated interface is sandwiched. The interface has finite transparency for quasiparticles, and two superconductors have a mis-orientation with each other. Quasiclassical theory of superconductivity is based on the nonlocal Eilenberger equations (Keldysh technique) in thermal equilibrium for the quasiclassical matrix propagator. In the case of a clean (ballistic) singlet anisotropic paired superconductor, the equations for the energy ξ-integrated Green functions reduce to the following 2×2 matrix form:[24]

$$\begin{aligned}&\boldsymbol{v}_{\rm F}\cdot\nabla\hat{G}_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})\\&+[\omega\hat{\sigma}_3+\hat{\Delta}(\boldsymbol{v}_{\rm F},\boldsymbol{r}),\hat{G}_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})]=0,\end{aligned}\tag{1}$$

where

$$\widehat{\Delta}=\begin{pmatrix}0&\Delta\\\Delta^+&0\end{pmatrix},\quad \widehat{G}_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})=\begin{pmatrix}g_\omega&f_\omega\\f_\omega^+&-g_\omega\end{pmatrix}.\tag{2}$$

Here, Δ is the superconducting order parameter, $\hat{\sigma}_3$ is the third Pauli matrix, $\hat{G}_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})$ is the matrix Green function which depends on the electron velocity in the Fermi surface $\boldsymbol{v}_{\rm F}$, the coordinate $\boldsymbol{r}$ and the Matsubara frequency $\omega=(2n+1)\pi T$, with n and T being an integer number and temperature respectively. The normalization condition also needs to be satisfied, $g_\omega=\sqrt{1-f_\omega f_\omega^+}$, with f_ω^+ being time-reversal counterpart of f_ω. In general, Δ depends on the direction of $\boldsymbol{v}_{\rm F}$ and is determined by the self-consistency equation

$$\begin{aligned}&\Delta(\boldsymbol{v}_{\rm F},\boldsymbol{r})\\&=2\pi N(0)T\sum_{\omega>0}\langle V(\boldsymbol{v}_{\rm F},\boldsymbol{v}'_{\rm F})f_\omega(\boldsymbol{v}'_{\rm F},\boldsymbol{r})\rangle_{\boldsymbol{v}'_{\rm F}},\end{aligned}\tag{3}$$

where $V(\boldsymbol{v}_{\rm F},\boldsymbol{v}'_{\rm F})$ is the interaction potential. The solution of the matrix equation (1) together with self-consistency equation (3) and normalization condition determines the current density $\boldsymbol{j}(\boldsymbol{r})$ in the system as

$$\boldsymbol{j}(\boldsymbol{r})=-4\pi{\rm i}eN(0)T\sum_\omega\langle\boldsymbol{v}_{\rm F}g_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})\rangle_{\boldsymbol{v}_{\rm F}}.\tag{4}$$

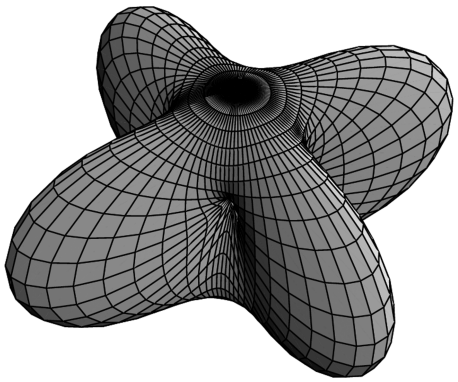


Fig. 1. The s+g-wave order parameter, $|\Delta(\hat{k})|$ of borocarbide superconductors. The crystallographic ab-plane coincides with xy-plane in the Cartesian system of coordinates. Here ab-plane of the order parameter is the plane of Lu atoms in borocarbide single crystal $LuNi_2B_2C$.[9]

In three-dimensional (3D) case, $N(0)$ is the 3D density of states at the Fermi level of energy and

$$\langle \cdots \rangle = \frac{1}{4\pi} \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \mathrm{d}(\cos\theta) \cdots$$

is the averaging over directions of 3D vector $\boldsymbol{v}_{\mathrm{F}}$. A symmetric configuration is considered, and so the equations are changed in to a two-dimensional (2D) problem. Thus the spatial distributions of $\Delta(\boldsymbol{r})$ and $\boldsymbol{j}(\boldsymbol{r})$ depend only on the coordinate in the ab-plane, and Eilenberger equations (1) reduce into a simpler problem. In the present paper, the transparency of the interface is assumed to be finite $0 \leq D \leq 1$. Also, equations (1) for Green function $\widehat{G}_\omega(\boldsymbol{v}_{\mathrm{F}}, \boldsymbol{r})$ have to be imposed by the suitable boundary conditions.[25,27,28] These boundary conditions couple the transmitted and the reflected trajectories.[27] So, because of finite reflection at the interface, it is impossible to impose the continuity of the Green functions along a trajectory that crosses over the interface as done in an ideal transparent system.[11] Instead, the Zaitsev boundary conditions[27−29] should be exerted as follows:

$$\widehat{d}^{\,l} = \widehat{d}^{\,r} \equiv \widehat{d}, \tag{5}$$

$$\left(\frac{D}{2-D}\right)\left[\left(1+\frac{\widehat{d}}{2}\right)\widehat{s}^{\,r},\ \widehat{s}^{\,l}\right] = \widehat{d}\,(\widehat{s}^{\,l})^2, \tag{6}$$

where

$$\widehat{s}^{\,r} = \widehat{G}^{\,r}_{\omega}(\boldsymbol{v}_{\mathrm{F}}, x=0) + \widehat{G}^{\,r}_{\omega}(\boldsymbol{v}'_{\mathrm{F}}, x=0), \tag{7}$$

and

$$\widehat{d}^{\,r} = \widehat{G}^{\,r}_{\omega}(\boldsymbol{v}_{\mathrm{F}}, x=0) - \widehat{G}^{\,r}_{\omega}(\boldsymbol{v}'_{\mathrm{F}}, x=0) \tag{8}$$

with $\boldsymbol{v}'_{\mathrm{F}}$ being the reflection of $\boldsymbol{v}_{\mathrm{F}}$ with respect to the boundary and D is transparency coefficient of the interface that can be momentum-dependent generally. For a δ-form potential barrier insulating interface with a one-particle potential $U(x) = H\delta(x)$ the transmission amplitude is only a function of the incident angle β between the electron momentum and the normal to the barrier plane and equal to $D(\beta) = \cos^2(\beta)/[Z^2 + \cos^2(\beta)]$. Here, the barrier strength Z is defined as $Z = 2mH$ with m being the electron mass.[30] As transparency coefficient is a smooth function of electron momentum direction, we consider a constant transparency model. Similar relations also hold for $\widehat{s}^{\,l}$ and $\widehat{d}^{\,l}$. In our calculation, the following equations:

$$\Delta_{l,r}(\theta,\varphi) = \frac{1}{2}\Delta_0(T)(1+\sin^4\theta\cos 4(\varphi-\alpha_{l,r}))$$

are used for the left side and the right side of the interface respectively with s+g-wave order parameters being symmetric and $\alpha_{l,r}$ being orientations with respect to the interface.[4,10,,11] It is remarkable that step functions for the spatial dependence for the phase of order parameters are considered as $\Delta_{l,r}(x) = \Delta_{l,r}\exp \pm \mathrm{i}\phi/2$ respectively, where ϕ is phase difference between two bulks of superconductors. For all quasiparticle trajectories the Green functions satisfy the boundary conditions both in the right and the left bulks and at the interface.

3. Analytical and numerical results

We solved this problem and obtained the Green functions of this system analytically. Using our solutions of Eilenberger equations and symmetries of singlet order parameters, we analytically obtained the following expression for the current density (4) at the interface

$$\boldsymbol{j} = \sum_{\omega>0}\left\langle \frac{4\pi e N(0) T \boldsymbol{v}_{\mathrm{F}} \eta D \Delta_l \Delta_r \sin\phi}{2\Omega_l\Omega_r + D(\omega^2 + \Delta_l\Delta_r\cos\phi - \Omega_l\Omega_r)} \right\rangle_{\widehat{\boldsymbol{v}}}, \tag{9}$$

where $\Omega_{l,r} = \sqrt{\omega^2 + \Delta^2_{l,r}}$ with ω being the Matsubara frequency, $\eta = \mathrm{sign}(v_x)$ and ϕ is the phase difference between two bulks. For example, in the case of junction without mis-orientation, $\alpha_l = \alpha_r$, and total transparent interface, $D = 1$, for Josephson current normal to the interface we have

$$j_{\mathrm{J}}(0) = \frac{j_0 T}{T_{\mathrm{c}}} \sum_{\omega>0}\left\langle \frac{\widehat{\boldsymbol{v}}\eta\Delta^2 \sin\dfrac{\phi}{2}\cos\dfrac{\phi}{2}}{\omega^2 + \left(\Delta\cos\dfrac{\phi}{2}\right)^2} \right\rangle_{\widehat{\boldsymbol{v}}}, \tag{10}$$

where $\omega = (2n+1)\pi T$, $\widehat{\boldsymbol{v}} = \boldsymbol{v}_{\mathrm{F}}/v_{\mathrm{F}}$ is the unit vector, and $j_0 = 4\pi e N(0) v_{\mathrm{F}} T_{\mathrm{c}}$. Using

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2 + x^2} \equiv \frac{\pi}{4x}\tanh\frac{\pi x}{2},$$

we obtain

$$j_{\mathrm{J}}(0) = \frac{j_0 \sin\dfrac{\phi}{2}}{4T_{\mathrm{c}}}\left\langle \widehat{\boldsymbol{v}}\eta\Delta \tanh\left(\Delta\cos\frac{\phi}{2}\Big/2T\right)\right\rangle_{\widehat{\boldsymbol{v}}} \tag{11}$$

for finite temperature.

At the limit of zero temperature, $T \to 0$, and $\phi \neq \pi$ we have

$$j_{\mathrm{J}}(0) = \frac{j_0\Delta_0(0)\sin\dfrac{\phi}{2}}{16T_{\mathrm{c}}}\left(1 - \frac{\cos 4\alpha_{l,r}}{24}\right). \tag{12}$$

On the other hand, for the small values of transparency of interface between borocarbides, the current expression is

$$\boldsymbol{j}(0) = j_0 \frac{T}{T_c} \sum_{\omega>0} \left\langle \frac{\widehat{\boldsymbol{v}}\eta\Delta_l\Delta_r D\sin\phi}{2\Omega_l\Omega_r} \right\rangle_{\widehat{\boldsymbol{v}}}, \qquad (13)$$

that is the same as the results of Ref. [31]. As it is clear from Eq. (9), for high values of transparency and low temperatures of junction, current is a nonlinear function of transparency D. But for the low values of transparency and high values of temperature, current is an exactly linear function of transparency (see Eq. (13)). On the other hand, when nonlinearity is removed, the deviation from the sinus disappears. It is well known that the current in this system for the low temperatures does not have the sinusoidal form, while near the critical temperature it tends to the sinusoidal from.[12] Also, this system at the low transparency, has a current–phase dependence similar to the case of high temperature. For the Josephson junction between d-wave superconductors, in addition to the Josephson current, another term of current is observed, called the spontaneous current. The spontaneous current that is parallel to the interface, is plotted as a function of mis-orientation and transparency in Fig. 4. The Josephson current and the spontaneous current are plotted as the functions of ϕ for different mis-orientations at $T/T_c = 0.1$, $D = 1.0$ in Figs. 2–4 and for a low value of transparency $D = 0.5$ in Fig. 5. The effect of mis-orientation on the Josephson current is plotted in Fig. 6. It is seen that by reducing the transparency, currents are decreasing (Fig. 6). For high values of transparency the currents are not sinusoidal function of phase, but by decreasing the transparency they tend to the sinusoidal form as observed in expressions (9) and (13). The period of current–phase plot is interesting. We fixed the left orientation at $\alpha_l = 0$ by increasing the right orientation from 0 to $\pi/4$, the period of the current–phase graphs varies from 2π to π then, if the right orientation changes into $\pi/2$, this period returns to 2π (Figs. 2 and 3). In Fig. 6, the effects of mis-orientation and transparency on the Josephson current at a specified value of phase are plotted for both of d-wave and s+g-wave junctions. It is a periodic function with a period of π. The main difference between high T_c and Borocarbides is sign change of current for high T_c which does not happen to the borocarbide junction in Fig. 6.

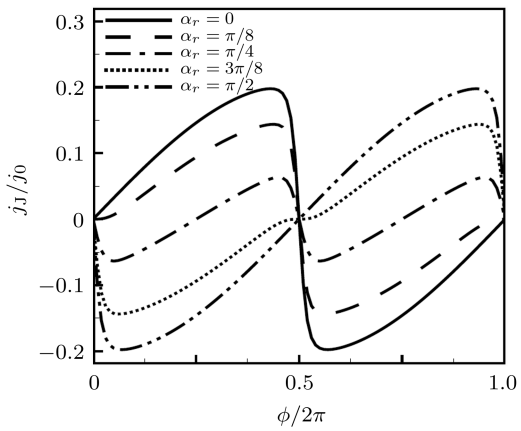


Fig. 2. Curves for Josephson current through the interface versus phase difference between d-wave superconductors for the case of $T/T_c = 0.1$, $D = 1$, $\alpha_l = 0$ and different values of α_r. Currents are plotted in terms of $j_0 = 4\pi eN(0)v_F T_c$. Our results coincide with results of Ref. [29].

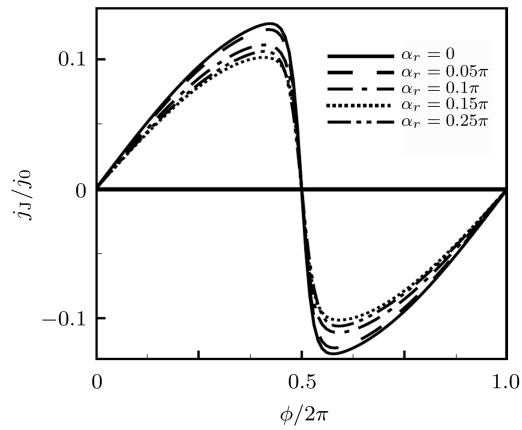


Fig. 3. Curves for Josephson current through the interface versus the phase difference between s+g-wave superconductors for the case of $T/T_c = 0.1$, $D = 1$, $\alpha_l = 0$ and different values of α_r.

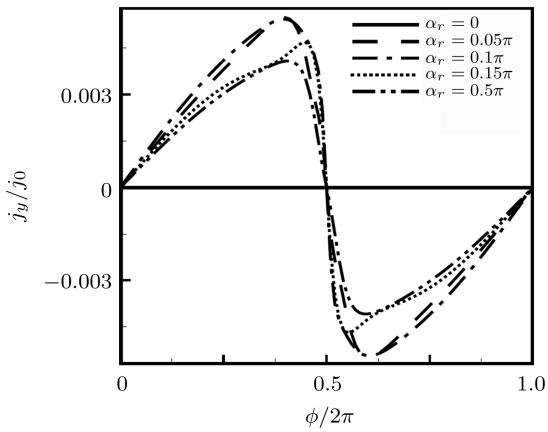


Fig. 4. Curves for spontaneous current j_y versus phase difference between s+g-wave superconductors (ϕ) for $T/T_c = 0.1$, $\alpha_l = 0$, $D = 1.0$ and different values of α_rs. This component of current is flowing parallel to the interface between superconducting bulks.

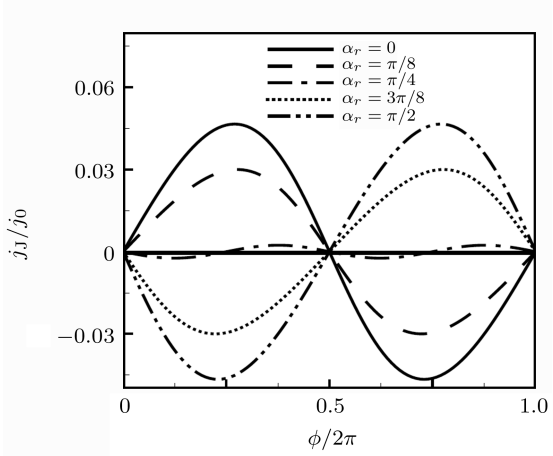


Fig. 5. Curves for Josephson current through the interface versus the Josephson phase difference between d-wave superconductors for $D = 0.5$, $\alpha_l = 0$, $T/T_c = 0.1$ and different values of α_rs.

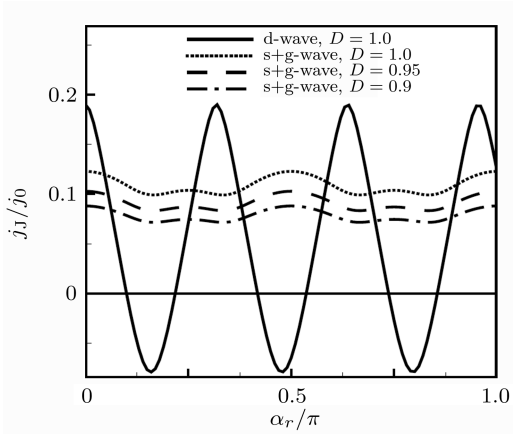


Fig. 6. Curves for Josephson current versus right orientation, α_r, for $\alpha_l = 0$, $T/T_c = 0.1$ and $\phi = 3\pi/4$, different transparency coefficients, with both of d-wave and s+g-wave pair being symmetric.

4. Analytical investigation of system near the critical temperature

For high temperatures, $T_c - T \ll T_c$, the Josephson current has been obtained analytically and exactly as

$$j_J = j_0 D \sin\phi \frac{\Delta_0^2}{{T_c}^2} \frac{102753 + 23368\cos 4\alpha_r}{\pi 1587600} \quad (14)$$

and for the spontaneous current we have

$$j_y = j_0 D \sin\phi \frac{\Delta_0^2}{{T_c}^2} \frac{11552\sin 4\alpha_r}{\pi 1587600} \quad (15)$$

and $j_z = 0$. For temperature dependence of energy gap or order parameter we have to use the gap-self-consistent equation at the bulk of superconductors. After a little simplification we obtain:

$$T_c - T = \sum_{\omega>0}^{\infty} \left(\frac{315\Delta_0 T_c}{379\omega}\right)^2 \frac{\gamma}{\omega}, \quad (16)$$

where

$$\gamma = \frac{1}{2}\oint \mathrm{d}\phi \mathrm{d}\theta (1 + \cos 4\phi \sin^4\theta)^4. \quad (17)$$

Also for the first time we obtain gap dependence on temperature close to the critical temperature for the case of s+g pairing symmetry as follows:

$$\Delta_{0(s+g)}(T) = \frac{1516\pi}{315}\sqrt{\frac{T_c(T_c - T)}{7\zeta(3)}}. \quad (18)$$

From Eqs. (14) and (15) it is clear that for temperatures close to the T_c, the currents are exactly sinusoidal function of phase, quadratic function of order parameter, linear function of $T_c - T$ and the linear function of transparency separately. For high temperatures, the current dependences on the mis-orientation α_r are trigonometric and consequently periodic with a period of $\pi/2$. In addition, we see that for zero mis-orientation, $\alpha_l = \alpha_r = 0$, the Josephson current exists but spontaneous current disappears. This is understandable, because at zero mis-orientation our unconventional junction is like that in the conventional case where only Josephson current exists.

5. Possibility of 0–π transition

The possibilities of 0–π transition in the Josephson junction between borocarbides and the Josephson junction between high T_c superconductors are studied separately. In our results, a spontaneous current parallel to the interface is obtained for both cases of d-wave and s+g-wave Josephson junctions. The 0–π transition happens to the d-wave Josephson junction by mis-orientation as $\alpha_r = \pi/2$ but not to the s+g-wave junction. For the d-wave case the sign reversal of order parameter on the Fermi surface changes from the zero junction to the π junction. But for the s+g-wave, neither mis-orientation nor transparency of interface can change the sign of order parameter, so none of them can produce 0–π transition.

6. Conclusion

The effects of mis-orientation and transparency on the currents through the Josephson junction in the (s+g)–I–(s+g) system as well as in the D–I–D system have been investigated in this paper. It is obtained that for both of d-wave and s+g-wave junctions in addition to standard Josephson current normal to the interface, a spontaneous current parallel to

the interface is flowing.[29,32] For the fixed left orientation, the current is a periodic function of right orientation and *vise versa*. In addition to the mis-orientation between the left and the right order parameters, the angle between order parameter and interface plays a role in the current value because of scattering from the interface. The effect of transparency on current is only a quantitative effect. While for high values of transparency and the low temperatures, current $\boldsymbol{j}(\phi, D)$ is a nonlinear function of transparency and a nonsinus function of phase but for low transparency and high temperature it is a linear function of transparency and sinus of phase. As the most important result for d-wave superconductors, it is obtained that $\boldsymbol{j}(\alpha_l, \alpha_r, \phi + \pi) = \boldsymbol{j}(\alpha_l, \alpha_r + \pi/2, \phi)$. This last sentence means that zero junction can be replaced by the π junction through rotating ab-plane of one of left or right crystals. Also it means that the magnetic Josephson-phase can be cancelled by order parameter mis-orientation. Then we conclude that this is a new reason why we search for a magnetic nature of pairing mechanism in high $T_{\rm c}$ superconductors instead of phonon–electron interaction in conventional superconductors.

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