A numerical analytic method for electromagnetic radiation accompanying with fracture of rocks*

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This paper studies Rabinovitch's compression experiments on granite and chalk and proposes an oscillating dipole model to analyse and simulate the electromagnetic radiation phenomenon caused by fracture of rocks. Our model assumes that the electromagnetic radiation pulses are initiated by vibrations of the charged rock grains on the tips of the crack. The vibrations of the rock grains are stimulated by the pulses of the cracks. Our simulations show comparable results with Rabinovitch's compression experiments. From the simulation results, it verifies an assumption that the crack width is inversely proportional to the circular frequency electromagnetic radiation, which is presented by Rabinovitch *et al*. The simulation results also imply that, by using our oscillating dipole model together with Rabinovitch's two equations about the crack length and crack width, we can quantitatively analyse and simulate the electromagnetic radiation phenomenon, which is induced from the fracture of the rocks.

Keywords: oscillating dipoles, electromagnetic radiation model, fracture of rocks
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1. Introduction

The electromagnetic radiation (EMR) phenomena induced by the fracture of rocks have been investigated for many years by their high potential application in earthquake predictors.[1−3] Some representative measurements can be found, for example, in the papers by Rabinovitch *et al.*,[4,5] who carried out a series of compression experiments on granite and chalk samples. Rabinovitch *et al.* also proposed some numerical fitted formulations[4,5] for their observed EMR pulses induced by rock fractures. However, the origins of the EMR phenomenon are still not well understood. Gershenzon *et al.*[6] proposed a model of moving crack tips, whose negative charges moves with the crack tips while the positive charges accumulate at the surfaces of the cracks, thus dipole radiation occurs from these opposite charges. O'Keefes and Thiel[7,8] gave a capacitors model, whose two sides of the crack constitute parallel plate capacitors and the discharge of capacitors emits the EMR pulses. Rabinovitch *et al.*[5] presented a surface oscillating model, whose EMR is emitted by oscillating dipoles created by ions moving collectively as a surface wave on both sides of the crack and the EMR pulse amplitude decays by interaction with bulk phonons. All of the above-mentioned models are qualitative and they cannot be verified by the simulations. For improving this disadvantage, in this paper, we develop an oscillating dipole model for the EMR induced by rock fractures. In our model, because the rock is a grainy material and lots of positive and negative ions will be created with the cracks[9−11] and the impulses of fracture drive the rock grains on the crack tips to vibrate. The ions associated with the vibrating rock grains move collectively with the grains, thus lots of oscillating dipoles produced.[12] The friction and other resistances in the rocks damp the vibration of the grains, then the dipoles associated with the grains make damped oscillations, this damped oscillation of the dipoles is one of the sources of the EMR pulses. Our model is quantitative and can be verified by simulations. Then according to the Rabinovitch's compression experiments on granite and chalk,[13,14] we programme to verify our model. The model uses a line of oscillating dipoles to simulate the formation of an elliptical crack in the granite and chalk samples, respectively. Our simulations show a good agreement with Rabinovitch's experimental results.[13,14]

2. Model and method

From the Rabinovitch's compression experiments on granite,[13] we assume that a two-dimensional el-

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liptical transverse crack is forming.[15,16] As shown in Fig. 1, the crack extends along the long axis of the ellipse and the vibrating direction of the rock grains parallels to the short axis of the ellipse. From Enomoto's experiments,[10] we have known that the fracture of the rock can create positive and negative ions on the opposite surfaces of the cracks. In our model, the whole crack was divided into many tiny fragments and the vibration of each charged tiny fragment is equivalent to an oscillation of a corresponding dipole. So the whole elliptical crack can be equivalent to a line of oscillating dipoles. Thus we set a line of oscillating electric dipoles with equal spacing along the long axis of the ellipse to simulate the formation of the crack. Each oscillating dipole represents vibrations of charged rock grains in the corresponding tiny crack fragment. The centres of the oscillating dipoles all located at the long axis of the ellipse.

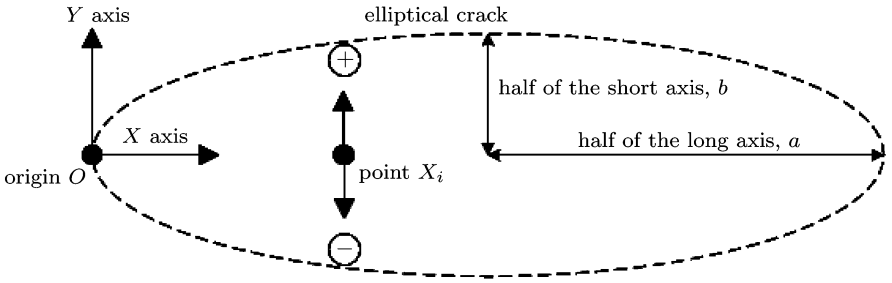


Fig. 1. The elliptical crack and the oscillating directions of the dipoles in the crack.

As shown in Fig. 2, a small loop antenna was located at a distance away from the rock sample to receive the EMR pulses emitted by the crack. The Cartesian coordinates $OXYZ$ on the crack plane was shown in Figs. 1 and 2, the origin O is the starting point of the crack, the positive$X$ axis is along the extending direction of the crack, the positive Y axis is along the initial oscillating direction of the positive charges on the dipole, the positive Z axis is normal to the plane where the small loop antenna situated. We consider one of the dipoles, whose centre at point X_i and its coordinates are $(x_i, 0, 0)$. The dipole's maximum oscillating positions are at $(x_i, \pm y_i, 0)$. The dipole is designated as dipole i. Because the crack is confined by its elliptical boundary, then x_i and y_i must satisfy an ellipse equation:

$$\frac{(x_i - a)^2}{a^2} + \frac{y_i^2}{b^2} = 1, \tag{1}$$

where a and b are the half of the long axis and half of the short axis of the ellipse, respectively. According to the theory of mechanical vibrations,[17] the response of a spring-mass system initially at rest for an excited impulse I is

$$u = \frac{I}{m\omega\sqrt{1-\beta^2}} \mathrm{e}^{-\beta\omega t} \sin(\sqrt{1-\beta^2}\omega t), \tag{2}$$

where u is the displacement of the system, m is the mass of the system, ω is the circular frequency of the vibration, β is the damped factor of the vibration, I is the pulse exerted on the system, t is time. In our model, the vibrations of the tiny crack fragment corresponding to the dipole i is simulated as a spring-mass system, so we have

$$u_i = \frac{I_i}{m_i\omega_n\sqrt{1-\alpha^2}} \mathrm{e}^{-\alpha\omega_n t} \sin(\sqrt{1-\alpha^2}\omega_n t), \tag{3}$$

where u_i is the displacement of the crack fragment, m_i is the mass of the crack fragment, I_i is the pulse exerted on the crack fragment, ω_n is the circular frequency of the vibration, α is the damped factor of the vibration, t is time. Because the ions collectively move with the corresponding rock grains and the oscillating dipole is an abstraction of the oscillating ions, so we can use the maximum oscillating amplitude of the dipole i to substitute the maximum vibrating amplitude of the crack fragment, i.e.,

$$y_i = \frac{I_i}{m_i\omega_n\sqrt{1-\alpha^2}}. \tag{4}$$

Thus the oscillating amplitude of the dipole i is

$$u_i = y_i \mathrm{e}^{-\alpha\omega_n t} \sin(\sqrt{1-\alpha^2}\omega_n t). \tag{5}$$

The electric dipole moment of dipole i is a function of time t[18,19]

$$\boldsymbol{p}_i(t) = 2Qu_i\hat{y} = 2y_iQ\mathrm{e}^{-\alpha\omega_n t} \sin(\sqrt{1-\alpha^2}\omega_n t)\hat{y}, \tag{6}$$

where $\boldsymbol{p}_i$ is the electric dipole moment of the dipole i, $\hat{y}$ is the unit vector in the direction of the positive y axis, $\pm Q$ is the charges on the dipole i. The oscillation of the dipole will generate electromagnetic waves propagated away from the dipole to infinity. We neglect the attenuation of the rock, then the induced magnetic field will be[19]

$$\boldsymbol{B}_i = \frac{\mu_0}{4\pi}\left(\frac{1}{r_i^2}\frac{\mathrm{d}p_i}{\mathrm{d}t} + \frac{1}{cr_i}\frac{\mathrm{d}^2 p_i}{\mathrm{d}t^2}\right)\sin\theta_i\hat{\varphi}, \tag{7}$$

where $\boldsymbol{B}_i$ is the magnetic flux density generated by dipole i, μ_0 is the permeability of the vacuum, $\mathrm{d}p_i/\mathrm{d}t$, $\mathrm{d}^2p_i/\mathrm{d}t^2$ is the first order and second order derivatives of electric dipole moment $p_i(t)$ with respect to time t, respectively, r_i is the distance between point X_i and point A, which is the centre of the loop antenna, c is the speed of light in the vacuum, θ_i is an elevation angle relative to dipole i, $\hat{\varphi}$ is a unit vector in the azimuthal direction of the dipole i (see Fig. 2).

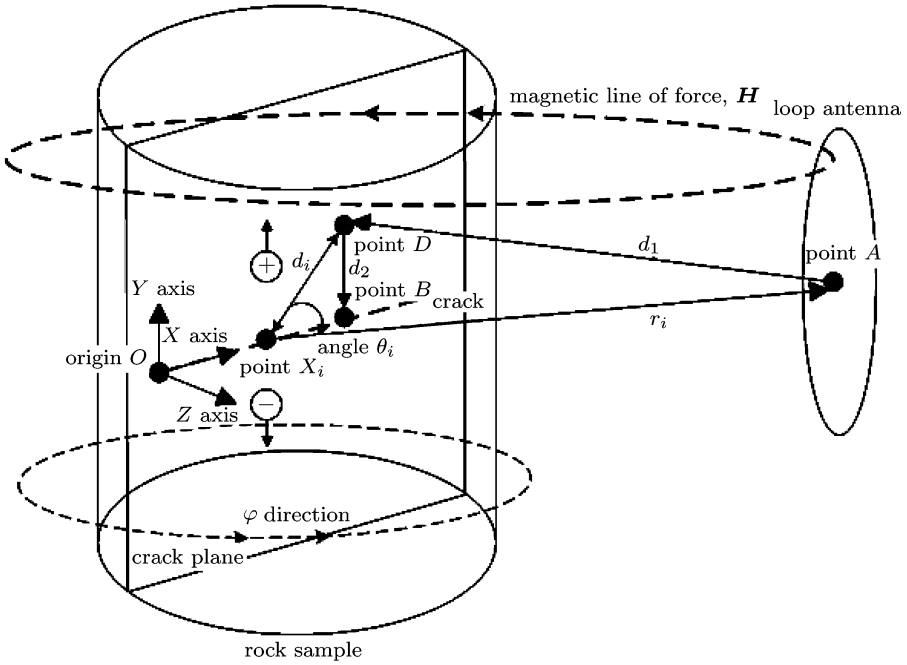


Fig. 2. The EMR model of oscillating dipoles, the point X_i is the centre of the dipole i, the point A is the centre of the loop antenna, the point D is the projective point on the crack plane by the point A, the point B is the projective point on the crack by the point D, d_1 is the distance between the point A and the point D, d_2 is the distance between the point D and point B, d_i is the distance between the point D and point X_i, r_i is the distance between the point X_i and the point A, θ_i is an elevation angle relative to point X_i.

From Figs. 1 and 2, we can obtain the following geometrical relations:

$$r_i = \sqrt{d_1^2 + d_i^2} = \sqrt{d_1^2 + d_2^2 + (x_i - x_b)^2}, \tag{8}$$

$$\sin\theta_i \approx \frac{d_1}{r_i}, \tag{9}$$

where d_1 is the distance between point A and crack plane, d_2 is the distance between point D and point B (see Fig. 2), x_b is the x coordinate of point B, d_i is the distance between point D and point X_i. Because the loop antenna is located in the near fields created by the oscillating dipole i, thus the quantity r_i is far more less than the quantity c, then equation (6) can be simplified as

$$\boldsymbol{B}_i \approx \frac{\mu_0}{4\pi r_i^2}\frac{\mathrm{d}p_i}{\mathrm{d}t}\sin\theta_i\hat{\varphi}. \tag{10}$$

According to Faraday's law of electromagnetic induction,[19] we have

$$\begin{aligned} E_i(t) &= -\frac{\mathrm{d}\Phi_i}{\mathrm{d}t} = -\int_{SA}\frac{\partial \boldsymbol{B}_i}{\partial t}\cdot\mathrm{d}\boldsymbol{A} \\ &= -\left(\frac{2\mu_0 AQy_i\omega_n^2}{4\pi r_i^2}\sin\theta_i\right)\mathrm{e}^{-\alpha\omega_n t} \\ &\quad\times[(2\alpha^2-1)\sin(\sqrt{1-\alpha^2}\omega_n t) \\ &\quad-(2\alpha\sqrt{1-\alpha^2})\cos(\sqrt{1-\alpha^2}\omega_n t)], \end{aligned} \tag{11}$$

where $\Phi_i(t)$ is the magnetic flux through the surface of the loop antenna; $E_i(t)$ is the electromotive force

(EMF) in the loop antenna, SA is the surface of the loop antenna, A is the area of the loop antenna, $E_i(t)$ equals the received EMR pulse voltage $U_i(t)$ in the loop antenna, which is generated by the oscillation of the dipole i. Summing all the EMR pulse voltages generated by the oscillating dipoles associated with the crack, we obtain the total EMR pulse voltage induced by the crack as

$$U(t) = \sum_{i=1}^{N} U_i(t), \tag{12}$$

where $U(t)$ is the total EMR pulse voltage induced by the crack, N is the number of the oscillating dipoles associated with the crack.

3. Results and discussion

According to the researches of the Rabinovitch *et al.*,[5,20] the crack length and crack width relate to the crack velocity and the circular frequency of the EMR pulses as

$$L_{\text{crack}} = T v_{\text{crack}}, \tag{13}$$

$$W_{\text{crack}} = \frac{\pi v_{\text{crack}}}{\omega_n}, \tag{14}$$

where T is the time measured from the beginning of the EMR pulse to the maximum of the EMR pulse envelope, L_{crack} is the crack length, W_{crack} is the crack width, v_{crack} is the crack velocity, ω_n is the circular frequency of the EMR pulse. Rabinovitch *et al.*[4,21] presented that the value of the crack velocity v_{crack} can be substituted by the value of the Rayleigh wave velocity in the same media. In our model, from Fig. 1, we can find that the crack length L_{crack} and the crack width W_{crack} can be expressed by the parameters of the elliptical crack as follows:

$$L_{\text{crack}} = 2a, \quad W_{\text{crack}} = 2b, \tag{15}$$

where a and b are the half lengths of long axis and short axis of the elliptical crack, respectively. Therefore, if we have measured the values of T and ω_n from the EMR pulse received by the loop antenna, and use value of the Rayleigh wave velocity in the rock for v_{crack}, then we can obtain the parameters a and b from Eqs. (13)–(15). Thus we can plot simulation waveforms for the EMR pulses by using Eqs. (11) and (12). Based on the Rabinovitch's compression experiments on the granites and chalks,[4,5] we programme to verify our model. By the papers written by Rabinovitch *et al.*,[13,14] the experimental samples are all standard cylinders of 100 mm in length and 53 mm in diameter. The loop antenna was placed 20 mm away from the centre of the samples with its normal pointing perpendicularly to the axis of cylinder. And in the following simulations, the charges on each of the dipoles were set to equal $Q = \pm 6.2 \times 10^{-11}$ C[10] and the crack velocity v_{crack} was set to equal the Rayleigh wave velocity. Our simulation results are shown in Figs. 3 and 4, respectively. The simulation parameters were listed in Tables 1 and 2, respectively.

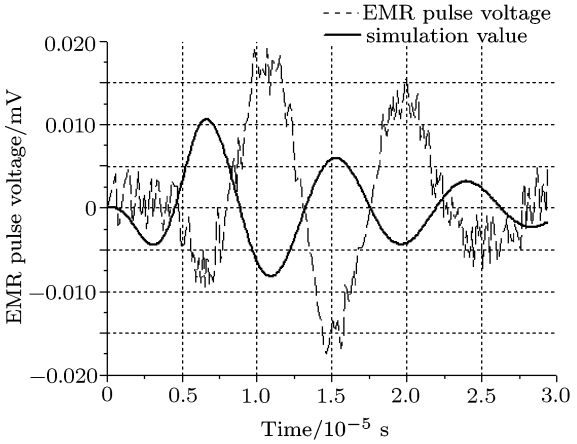


Fig. 3. The simulation result for Rabinovitch's granite compression experiment (Rabinovitch *et al.*, 1998[13]), the dashed line and the solid line are for experimental result and simulated result, respectively.

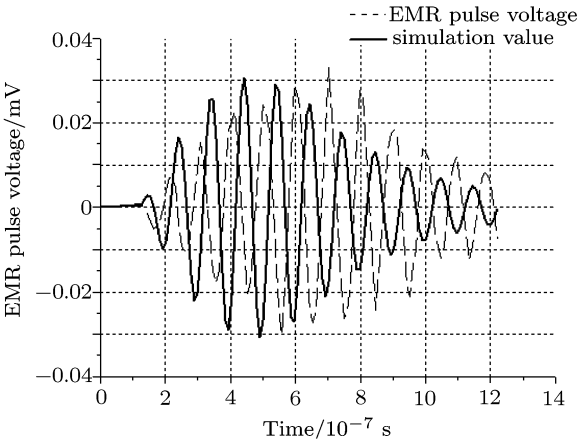


Fig. 4. The simulation result for Rabinovitch's chalk compression experiment (Rabinovitch *et al.*, 2000 [14]), the dashed line and the solid line are for experimental result and simulated result, respectively.

Table 1. Calculated values from Rabinovitch's compression experiments for granite and chalk,[13,14] $\omega_{n,\mathrm{mea}}$ is the measured circular frequency of the EMR pulse, a_{cal}, b_{cal} are the calculated half lengths of long axis and short axis of the elliptical crack, respectively.

type	T/s	$\omega_{n,\mathrm{mea}}$/rad·s^{-1}	L_{crack}/mm	W_{crack}/mm	a_{cal}/mm	b_{cal}/mm
granite	8.0×10^{-6}	6.98×10^{5}	9.42	5.30	4.710	2.650
chalk	4.4×10^{-7}	6.76×10^{7}	1.05	0.11	0.525	0.055

Table 2. Simulation parameters for Rabinovitch's compression experiments of granite and chalk,[13,14] $\omega_{n,\mathrm{cal}}$ is the simulated circular frequency of the EMR pulse, x_b is the x coordinate of point B, S is the spacing between the dipoles, a_{sim}, b_{sim} are the simulated half length of long axis and short axis of the elliptical crack, respectively. Num is the number of cracks, which used in the simulations.

type	Q/C	$\omega_{n,\mathrm{cal}}$/rad·s^{-1}	α	v_{crack}/m·s^{-1}	x_b/mm	d_1/mm	d_2/mm	S/μm	a_{sim}/mm	b_{sim}/mm	Num
granite	6.2×10^{-11}	7.0×10^{5}	0.10	1178	3	20	2	10	4.0	3.00	10
chalk	6.2×10^{-11}	6.0×10^{7}	0.05	2377	2	25	2	22	0.6	0.06	1

From Figs. 3 and 4, we can find a comparable accordance between our simulations and Rabinovitch's experimental results.[13,14] Meanwhile, Tables 1 and 2 also show some differences between the theoretical values by using Rabinovitch's equations (13) and (14) and our simulation parameters. These differences may be caused by the errors of Rabinovitch's equations and our model to the practical crack phenomena. Because the real EMR pulses are induced by lots of different types of cracks and these cracks create at the random times though in Rabinovitch's equations and our model these processes are simplified to the formation of several large elliptical cracks.

However, in our model, from Eq. (1), we can find the maximum oscillating amplitude of the dipole i, namely, y_i, equals half of the crack width of the point X_i. Relating this result with Eq. (4), we can find that this means the crack width of the point X_i is inversely proportional to the EMR circular frequency of the dipole i, namely, ω_n. This relation is accordant with Rabinovitch's equation (14), which means that crack width is inversely proportional to the EMR circular frequency.

From the above discussion, we can find that, by using our model together with Rabinovitch's Eqs. (13) and (14), we can quantitatively analyse and simulate the EMR phenomena associated with the rock fractures.

4. Conclusion

In this paper, we use an assumption that the vibrations of the charged rock grains generate EMR pulses during the fracture of rocks to develop an oscillating dipole model for analysing and simulating the EMR phenomena induced by the rock fracture. Our model is used to simulate the EMR pulses induced in the Rabinovitch's compression experiments on granite and chalk. The simulation results indicated a comparable accordance with Rabinovitch's experimental results. Based on our simulation results, we verified that the crack width is inversely proportional to the circular frequency of the EMR pulses. Meanwhile, the simulations imply, by combining our model with Rabinovitch's equations (13) and (14), we can quantitatively analyse the relationships between the crack parameters and the characteristics of the EMR pulses emitted from the fractured rocks.

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