Sampling strong tracking nonlinear unscented Kalman filter and its application in eye tracking*

Zhang Zu-Tao(张祖涛)[a)b)†] and Zhang Jia-Shu(张家树)[b)]

[a)] School of Mechanical Engineering, Southwest Jiaotong University, Chengdu 610031, China

[b)] Sichuan Provincial Key Laboratory of Signal & Information Processing, Southwest Jiaotong University, Chengdu 610031, China

(Received 13 January 2010; revised manuscript received 17 May 2010)

The unscented Kalman filter is a developed well-known method for nonlinear motion estimation and tracking. However, the standard unscented Kalman filter has the inherent drawbacks, such as numerical instability and much more time spent on calculation in practical applications. In this paper, we present a novel sampling strong tracking nonlinear unscented Kalman filter, aiming to overcome the difficulty in nonlinear eye tracking. In the above proposed filter, the simplified unscented transform sampling strategy with $n+2$ sigma points leads to the computational efficiency, and suboptimal fading factor of strong tracking filtering is introduced to improve robustness and accuracy of eye tracking. Compared with the related unscented Kalman filter for eye tracking, the proposed filter has potential advantages in robustness, convergence speed, and tracking accuracy. The final experimental results show the validity of our method for eye tracking under realistic conditions.

Keywords: unscented Kalman filter, strong tracking filtering, sampling strong tracking nonlinear unscented Kalman filter, eye tracking

PACC: 4660D

1. Introduction

Recently, the unscented Kalman filter (UKF),[1,2] an efficient derivative-free filter algorithm for obtaining approximate solutions to discrete-time nonlinear optimal filtering problems has been successfully applied to numerous practical nonlinear motion estimation and tracking problems, and outperforms the extended Kalman filter (EKF) in many cases.[3] However, UKF can only achieve good performance under certain assumptions about the system's mathematical model. These assumptions include: 1) sufficient accuracy in modeling the dynamic and observation models; 2) complete information of the noise statistics; 3) proper initial conditions. But in practice this information is usually not totally satisfied. Under these conditions, two factors limit the applications of UKF. One factor is that unscented transform (UT) with $2n+1$ sigma points take so much time on calculation; the other is that the UKF algorithm is not very robust numerically because of assumptions altered. Therefore, simplifying and optimizing UKF algorithms become an important issue in real-time robust nonlinear applications.

With the growing use of HCI, there is a rising concern about the eye tracking. For instance, researchers have utilized eye tracking to study the behaviour in such domains as driver fatigue detection,[4,5] eye typing for helping users with movement disabilities to interact with computers,[6] eye tracking analysis of mental analysis,[7] using eye tracking techniques to study collaboration on physical tasks for medical research, virtual reality (VR) system for measuring inspection methods, and image scanning, etc.[8] Not surprisingly, eye tracking has attracted the interest of many researchers, and eye trackers have been commercially available for many years.[9] Eye tracking has not reached its full potential even though the general-purpose eye tracking technology has been explored for decades. The first obstacle to integrating these techniques into human-computer interfaces is that they have been too expensive for routine use. Currently, a number of eye trackers are available on the market and their prices range from approximately 5000$ to 40000$.[10] The second factor is that it is very difficult to model eye tracking because of the eye motion's

*Project supported by the National Natural Science Foundation of China (Grant No. 60971104), the Fundamental Research Funds for the Cental Universities (Grant No. SWJTU09BR092), and the Young Teacher Scientific Research Foundation of Southwest Jiaotong University (Grant No. 2009Q032).

†Corresponding author. E-mail: zzt@swjtu.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

high nonlinearity. The third factor is that the robustness of eye tracking should be improved because of the variety of head and eyes moving fast, external illumination interference, and realistic lighting conditions. The accuracy of eye tracking cannot satisfy the realistic requirement of HCI.

To tackle some of those problems, we propose a novel sampling strong tracking nonlinear unscented Kalman filter (SSTN-UKF) to eye tracking. In the SSTN-UKF, a set of $n+2$ sigma sampling strategies instead of $2n+1$ sigma points used in the standard UKF, is constructed to approximate the state of eye movement. The reduced sigma points unscented transform (UT) parametrizations can capture the distribution information comparable to that of the $2n+1$ symmetric UT, and reduce computational resources comparable to the $n+2$ UT for eye tracking. At the same time, the suboptimal fading factor of strong tracking filtering (STF) is introduced into time update equations and measurement update equations to improve robustness and accuracy of the algorithm. The theoretical analysis and simulations show that the SSTN-UKF can improve the computational efficiency, robustness and accuracy by STF for real-time eye tracking.

The remainder of this paper is organized as follows. Section 2 is the SSTN-UKF algorithm. Section 3 gives SSTN-UKF based eye tracking performance evaluation and analysis. Some conclusions are drawn in Section 4.

2. Development of the sampling strong tracking nonlinear unscented Kalman filter

2.1. Review of standard UKF

In Refs. [1] and [2] the authors demonstrated the substantial performance gains of the UKF in the context of state estimation for nonlinear control. The UT is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation. The UKF is a straightforward extension of the UT to the recursive estimation,[11] and the UT sigma point selection scheme is applied to this new augmented state to calculate the corresponding sigma matrix.

Consider the general discrete nonlinear system

$$\begin{aligned} x_{k+1} &= F(x_k, u_k) + v_k, \\ y_k &= H(x_k) + n_k, \end{aligned} \tag{1}$$

where x_k represents the unobserved state of the system, u_k is a known exogenous input, and y_k is the observed measurement vector at time k. The process noise v_k drives the dynamic system, and the observation noise is n_k, F and H are state transition matrices.

The UKF equations are given as follows.

(i) Initialization condition is

$$\hat{x} = E[x_0], \quad P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^{\mathrm{T}}], \tag{2}$$

where $\hat{x}$ and P_0 are the initializations of state vector and covariance.

Augmented state vectors are

$$\hat{x}_0^a = E[x^a] = [\hat{x}_0^{\mathrm{T}} \ 0 \ 0]^{\mathrm{T}}, \tag{3}$$

$$P_0^a = E[(x_0^a - \hat{x}_0^a)(x_0^a - \hat{x}_0^a)^{\mathrm{T}}] = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & R^v & \\ 0 & 0 & R^n \end{bmatrix}. \tag{4}$$

(ii) Calculating sigma points are

$$\begin{aligned} \chi_{k-1}^a = &\left[\hat{x}_{k-1}^a \ \hat{x}_{k-1}^a + \gamma\sqrt{P_{k-1}^a} \ \hat{x}_{k-1}^a - \gamma\sqrt{P_{k-1}^a}\right], \\ &k \in \{1, 2, \ldots, 2n+1\}. \end{aligned} \tag{5}$$

(iii) Time updating is represented by

$$\begin{aligned} \chi_{k-1}^x &= F[\chi_{k-1}^x, u_{k-1}, \chi_{k-1}^v], \\ \hat{x}_k^- &= \sum_{i=0}^{2n} W_i^m \chi_{i,k/k-1}^x, \end{aligned} \tag{6}$$

$$P_k^- = \sum_{i=0}^{2n} W_i^{(c)} \left[\chi_{i\,k/k-1}^x - \hat{x}_k\right] \left[\chi_{i\,k/k-1}^x - \hat{x}_k\right]^{\mathrm{T}}, \tag{7}$$

$$\begin{aligned} y_{k|k-1} &= H\left[\chi_{k/k-1}^x, \chi_{k/k-1}^n\right]_{k/k-1}, \\ \hat{y}_k^- &= \sum_{i=0}^{2n} w_i^{(m)} y_{i,k/k-1}^x. \end{aligned} \tag{8}$$

(iv) Measurement update is expressed

$$\begin{aligned} &P_{\tilde{y}_k \tilde{y}_k} \\ &= \sum_{i=0}^{2n} W_i^{(c)} [y_{i,k/k-1} - \hat{y}_k^-][y_{i,k/k-1}^x - \hat{y}_k^-]^{\mathrm{T}}, \end{aligned} \tag{9}$$

$$\begin{aligned} &P_{x_k x_k} \\ &= \sum_{i=0}^{2n} W_i^{(c)} [\chi_{i,k/k-1} - \hat{x}_k^-][y_{i,k/k-1}^x - \hat{y}_k^-]^{\mathrm{T}}. \end{aligned} \tag{10}$$

Given the state model and the measurement model as well as some initial conditions, the state vector $\hat{x}_k$, along with its covariance matrix P_k, can be updated by using the system model (for prediction)

and measurement model (for updating). So the object tracking using UKF algorithm can be concluded as follows:

$$\kappa_k = P_{x_k y_k} P^{-1}_{\hat{x}_k \hat{y}_k}, \tag{11}$$

$$\hat{x}_k = \hat{x}_k^- + \kappa_k (y_k - \hat{y}_k^-), \tag{12}$$

$$P_k = P_k^- - \kappa_k P_{\tilde{y}_k \tilde{y}_k} \kappa_k^{\mathrm{T}}, \tag{13}$$

where $x^a = [x^{\mathrm{T}}\ v^{\mathrm{T}}\ n^{\mathrm{T}}]$, $\chi^a = [(\chi^x)^{\mathrm{T}}\ (\chi^v)^{\mathrm{T}}\ (\chi^n)^{\mathrm{T}}]$, $\gamma = \sqrt{(n+\lambda)}$, λ is a composite scaling parameter, n is dimension of augmented state, R^v = process noise covariance, R^n = measurement noise covariance, w_i = weights calculated as follows:

$$\begin{aligned} w_0^m &= \frac{\lambda}{n+\lambda}; \\ w_0^c &= \frac{\lambda}{n+\lambda} + (n - \alpha^2 + \beta); \\ w_i^m &= w_i^c = \frac{\lambda}{2(n+\lambda)}, \quad i = 1, \ldots, 2n; \\ \lambda &= n(\alpha^2 - 1). \end{aligned} \tag{14}$$

The parameter α determines the spread of the sigma points around $\hat{x}$ and is usually set to $1 \times 10^{-4} \leq \alpha \leq 1$. The constant β is used to incorporate the part of the prior knowledge of distribution of x. For detailed calculation, refer to Ref. [1].

Equation (5) shows that weight selection of sigma points needs massive computation in UT, standard UKF with $2n+1$ sigma points to perform filtering would take much more time on calculation, which could not satisfy the real-time requirement in many practical applications. Therefore, it is necessary to reduce the quantity of sigma points as well as to simplify the complicated process of weight selection.

2.2. The proposed sampling strategy with $n+1$ sigma points

According to the principle of statistical linear regression, to reduce the number of sigma points will improve real-time performance of UKF filtering. The number of sigma point is decreased as considered in Ref. [12], where the minimum number of sigma points required for obtaining the approximate mean and covariance as the n-dimensional random variables is $n+1$. For the $n+1$ sigma points, the sampling strategy UKF can be carried out

$$\begin{aligned} \chi_{k-1}^a = &\left[\hat{x}_{k-1}^a\ \hat{x}_{k-1}^a + \gamma\sqrt{P_{k-1}^a}\ \hat{x}_{k-1}^a - \gamma\sqrt{P_{k-1}^a}\right], \\ &k \in \{1, 2, \ldots, n+1\}. \end{aligned} \tag{15}$$

The $n+1$ sigma points sampling strategy UKF time updating is given by

$$\begin{aligned} \chi_{k-1}^x &= F[\chi_{k-1}^x, u_{k-1}, \chi_{k-1}^v], \\ \hat{x}_k^- &= \sum_{i=0}^{n} W_i^m \chi_{i,k/k-1}^x, \end{aligned} \tag{16}$$

$$P_k^- = \sum_{i=0}^{n} W_i^{(c)} \left[\chi_{i\,k/k-1}^x - \hat{x}_k\right] \left[\chi_{i\,k/k-1}^x - \hat{x}_k\right]^{\mathrm{T}}, \tag{17}$$

$$\begin{aligned} y_{k|k-1} &= H\left[\chi_{k/k-1}^x, \chi_{k/k-1}^n\right]_{k/k-1}, \\ \hat{y}_k^- &= \sum_{i=0}^{n} w_i^{(m)} y_{i,k/k-1}^x. \end{aligned} \tag{18}$$

The $n+1$ sigma points sampling strategy UKF measurement updating is written as

$$\begin{aligned} &P_{\tilde{y}_k \tilde{y}_k} \\ &= \sum_{i=0}^{n} W_i^{(c)} [y_{i,k/k-1} - \hat{y}_k^-][y_{i,k/k-1}^x - \hat{y}_k^-]^{\mathrm{T}}, \end{aligned} \tag{19}$$

$$\begin{aligned} &P_{x_k x_k} \\ &= \sum_{i=0}^{n} W_i^{(c)} [\chi_{i,k/k-1} - \hat{x}_k^-][y_{i,k/k-1}^x - \hat{y}_k^-]^{\mathrm{T}}. \end{aligned} \tag{20}$$

And the rest steps are similar to standard UKF algorithm.

2.3. The proposed sampling strong tracking nonlinear UKF

The STF is proposed by Zhou and Frank[13] to solve the state estimation problem of the conventional nonlinear systems and has been employed in many areas.[13–19] In fact, STF is the extension of the extended Kalman filter with introducing sub-optimal fading factors, and it is especially useful for estimating the states and parameters of nonlinear time-varying random systems. In this paper, therefore, we introduce the sub-optimal fading factor into time update equations and measurement update equations of the proposed simplified UKF with $n+2$ sigma points to improve its robustness. As shown in Fig. 1, firstly, a set of $n+2$ sigma points sampling strategy instead of $2n+1$ sigma points in the standard UKF is constructed to approximate the state. Secondly, sub-optimal fading factors are used as time update equations and measurement update equations to improve robustness of the simplified UKF algorithm. The proposed sampling strong tracking nonlinear UKF (SSTN-UKF) can be concluded as follows:

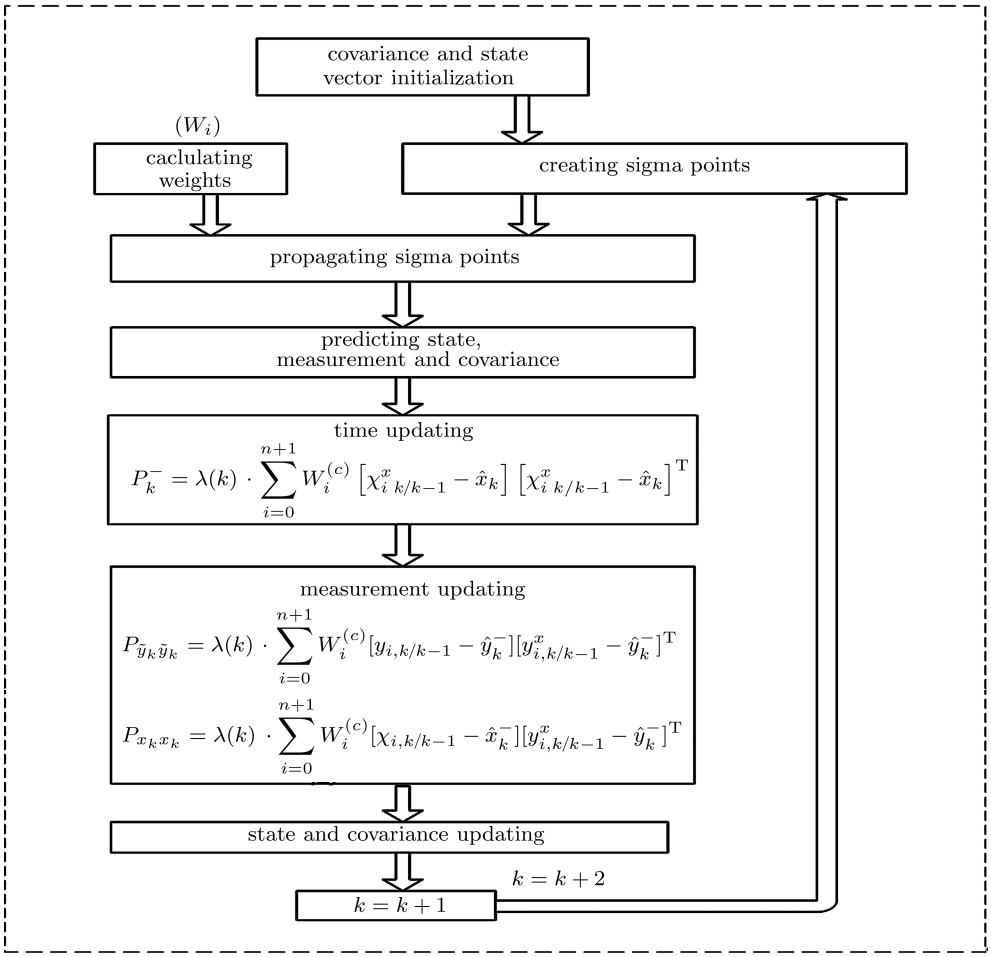


Fig. 1. Structure of the proposed SSTN-UKF algorithm.

Step 1 Covariance and state vector initialization For a nonlinear system in Eq. (1), we obtain

$$\hat{x} = E[x_0], \quad P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^{\mathrm{T}}]. \tag{21}$$

Step 2 Creating sigma points and calculate weights

$$\begin{aligned}
&\chi_0^a = \hat{x};\\
&\chi_i^a = \hat{x} + (\sqrt{(n+\lambda)P_0})_i,\ i = 1, \ldots, n;\\
&\chi_i^a = \hat{x} - (\sqrt{(n+\lambda)P_0})_i,\ i = n, n+1, n+2.
\end{aligned} \tag{22}$$

The matrix χ of $n+2$ sigma vectors χ_i^a is obtained according to the above equation, and $w_i =$ weights as calculated as follows in the SSTN-UKF.

$$\begin{aligned}
&w_0^m = \frac{\lambda}{n+\lambda};\\
&w_0^c = \frac{\lambda}{n+\lambda} + (n - \alpha^2 + \beta);\\
&w_i^m = w_i^c = \frac{\lambda}{2(n+\lambda)}, \quad i = 1, \ldots, n+1;
\end{aligned}$$

$$\lambda = n(\alpha^2 - 1). \tag{23}$$

Step 3 Predicting state, measurement and covariance

$$\begin{aligned}
&\chi_{k-1}^a\\
&= \left[\hat{x}_{k-1}^a \quad \hat{x}_{k-1}^a + \gamma\sqrt{P_{k-1}^a} \quad \hat{x}_{k-1}^a - \gamma\sqrt{P_{k-1}^a}\right].
\end{aligned} \tag{24}$$

For $k \in \{1, 2, \ldots, n+2\}$, a set of $n+2$ sigma points instead of $2n+1$ sigma points are used in the proposed algorithm. From the above equation, we can calculate the state and covariance of sigma points.

Step 4 Sub-optimal fading factors used for time update equations and time updating

$$\begin{aligned}
&\chi_{k-1}^x = F[\chi_{k-1}^x, u_{k-1}, \chi_{k-1}^v],\\
&\hat{x}_k^- = \sum_{i=0}^{n+1} W_i^m \chi_{i,k/k-1}^x,
\end{aligned} \tag{25}$$

$$P_k^- = \lambda(k) \cdot \sum_{i=0}^{n+1} W_i^{(c)} \left[\chi_{i\ k/k-1}^x - \hat{x}_k\right]$$

$$\times \left[\chi_{i\,k/k-1}^{x} - \hat{x}_k\right]^{\mathrm{T}}, \tag{26}$$

where $\lambda(k)$ is the sub-optimal fading factor used to fade the bypast datum and adjust the predictable state estimation covariance matrix. Sub-optimal fading factor $\lambda(k)$ can be directly determined as follows:

$$\lambda(k) = \begin{cases} \lambda_0, & \lambda_0 \geq 1; \\ 1, & \lambda_0 < 1; \end{cases} \tag{27}$$

where

$$\lambda_0 = \mathrm{tr}[N(k+1)]/\mathrm{tr}[M(k+1)], \tag{28}$$

$$\begin{aligned} N(k+1) = {} & V_0(k+1) - G_x(k)F_v(k)\cdot F_v(k)^{\mathrm{T}} \\ & - G_w(k)R_kG_w(k)^{\mathrm{T}}, \end{aligned} \tag{29}$$

$$M(k+1) = G_xF_x(k)\hat{P}_kF_x^{\mathrm{T}}(k)G_x^{\mathrm{T}}(k), \tag{30}$$

$$\begin{aligned} & V_0(k+1) \\ = {} & \frac{1}{k}\sum_{j=1}^{k}\gamma_j\gamma_j^{\mathrm{T}} \\ = {} & \begin{cases} G_x(0)\hat{P}_0G_x(0) + G_w(0)^{\mathrm{T}}, & k=0; \\ \dfrac{\rho V_0(k) + \gamma_j\gamma_j^{\mathrm{T}}}{1+\rho}, & k\geq 1; \end{cases} \end{aligned} \tag{31}$$

where $0 \leq \rho \leq 1$ is the pre-selected forgetting, it may be selected according to the real processes. For fast changing processes, a smaller ρ should be selected, and *vice versa*. As that pointed out in Ref. [13], $\lambda(k)$ is insensitive to the value of ρ

Step 5 Sub-optimal fading factors used to obtain measurement updating equations and measurement updating

$$\begin{aligned} & P_{\tilde{y}_k\tilde{y}_k} \\ = {} & \lambda(k)\sum_{i=0}^{n+1}W_i^{(c)}[y_{i,k/k-1} - \hat{y}_k^-][y_{i,k/k-1}^x - \hat{y}_k^-]^{\mathrm{T}}, \end{aligned} \tag{32}$$

$$\begin{aligned} & P_{x_kx_k} \\ = {} & \lambda(k)\sum_{i=0}^{n+1}W_i^{(c)}[\chi_{i,k/k-1} - \hat{x}_k^-][y_{i,k/k-1}^x - \hat{y}_k^-]^{\mathrm{T}}. \end{aligned} \tag{33}$$

Step 6 State and covariance updating:

$$\kappa_k = P_{x_ky_k}P_{\hat{x}_k\hat{y}_k}^{-1}, \tag{34}$$

$$\hat{x}_k = \hat{x}_k^- + \kappa_k(y_k - \hat{y}_k^-), \tag{35}$$

$$P_k = P_k^- - \kappa_kP_{\tilde{y}_k\tilde{y}_k}\kappa_k^{\mathrm{T}}. \tag{36}$$

The state vector $\hat{x}_k$, along with its covariance matrix P_k, can be updated by propagating $n+2$ sigma points using the system model (for prediction) and measurement model (for updating). So we can use the proposed SSTN-UKF to object track, which can improve the computational efficiency and robustness.

In the proposed SSTN-UKF algorithm, a set of $n+2$ sigma points instead of $2n+1$ ones are used to reduce the matrix operation time and also reduce the algorithm computation, which enhances operating speed and favours the request. At the same time, we introduce sub-optimal fading factor into time update equations and measurement update equations to improve robustness and estimation accuracy of algorithm. The final theoretical analysis and experimental results show the validity of our method for eye tracking.

3. SSTN-UKF based eye tracking performance evaluation and analysis

3.1. Theoretical validation

3.1.1. The computational efficiency of proposed algorithm

According to statistical linear regression, a nonlinear function $y = g(x)$ is evaluated for r sigma points (χ_i, y_i), which is approximated by $\chi_i, i = 1, 2, \ldots r$. The y-regression is the linear regression $y = Ax + b$ that minimizes the sum of the squared errors

$$\min\sum_{i=1}^{r}\left\{e_i^{\mathrm{T}}e_i\right\}, \tag{37}$$

where $e_i = y_i - (Ax_i + b)$.

The solution to expression (37) is $A = P_{xy}^{\mathrm{T}}P_{xx}^{-1}$, $b = \overline{y} - A\overline{x}$. The covariance matrix of deviations e_i is

$$P_{ee} = \frac{1}{r}\sum_{i=1}^{r}e_ie_i^{\mathrm{T}} = P_{yy} - AP_{xx}A^{\mathrm{T}}. \tag{38}$$

Equation (38) illustrates that the computational efficiency of statistical linear regression depends on the value of r. So the SSTN-UKF using $n+2$ sigma points has a better computational efficiency than the standard UKF using $2n+1$ sigma points. The final experimental results in Table 3 show the computational efficiency of SSTN-UKF for eye tracking.

3.1.2. The robustness analysis of SSTN-UKF

Lemma The sufficient condition for filter being robust strong tracking filtering is as follows:[16,20]

$$E[x_k - \hat{x}_k][x_k - \hat{x}_k]^{\mathrm{T}} = \min,$$
$$E[v_k + jv_k^{\mathrm{T}}] = 0, \quad k = 0, 1, 2, \ldots, j = 1, 2, \ldots, \quad (39)$$

where v_k is measurement residual. Zhou *et al.*[16] proved that the sufficient condition for a filter can be called the robust strong tracking only if the time-varying filter gain matrix is selected online such that the state estimation mean-square error is minimized and the innovations remain orthogonal.

For the proposed SSTN-UKF algorithm, we deduce that

$$x_{k-1}^i = \hat{x}_{k-1} + \delta_{k-1}^{(i)}, \quad (40)$$

where δ is the Kronecker δ-function, defined by

$$\delta_{k-1}^{(i)} = \begin{cases} 0, & i = 0; \\ -\delta_{k-1}, & 1 \le i \le n; \\ +\delta_{k-1}, & n+1 \le i \le n+2; \end{cases} \quad (41)$$

$$x_{k/k-1}^i = f(x_{k-1}^i), \quad (42)$$

$$\hat{x}_{k/k-1} = \sum_{i=0}^{n+1} w^i x_{k/k-1}^i. \quad (43)$$

To prove the SSTN-UKF algorithm is robust strong tracking filter, we use the Lipschitz condition defined as

$$\left|F(x_k) - F(x_k^i)\right| \le \gamma \left|x_k - x_k^i\right|. \quad (44)$$

In the above Lipschitz condition, for a nonlinear system in form (1), we have $\gamma > 0$ and γ is independent of y_k.

From Eqs. (26)–(34), we can deduce that

$$y_k = \lambda(k) \cdot \sum_{i=0}^{n+1} w^i H\left[\chi_k\right],$$
$$\hat{y}_{k/k-1} = \lambda(k) \cdot \sum_{i=0}^{n+1} w^i H\left[\chi_{k/k-1}^x, \chi_{k/k-1}^n\right]_{k/k-1}. \quad (45)$$

Using formulae (1) and (40)–(45), we can deduce that

$$\begin{aligned} & |v_k| \\ &= \left|y_k - \hat{y}_{k/k-1}\right| \\ &= \left|\lambda(k) \cdot \left[\lambda(k) \cdot \sum_{i=0}^{n+1} w^i H\left[\chi_k\right] - \sum_{i=0}^{n+1} w^i y_{k/k-1}^i\right]\right| \\ &\le |\lambda(k)| \cdot \sum_{i=0}^{n+1} w^i \cdot \gamma \left|x_k - x_{k/k-1}^i\right| \\ &= |\lambda(k)| \cdot \gamma \cdot \frac{\sum_{i=0}^{n+1} \left|\delta_{k-1}^{(i)}\right|}{n+\xi} \to 0. \end{aligned} \quad (46)$$

Then

$$E[v_k + jv_k^{\mathrm{T}}] = 0. \quad (47)$$

From the above theoretical analysis, we can deduce that the SSTN-UKF is the robust strong tracking filter also. The final experiments show the robustness of the proposed algorithm for the eye tracking.

3.2. Results of experiments

In this subsection, we develop the following eye tracking by using SSTN-UKF. Because the eye motion is of high nonlinearity in the likelihood model, it is very difficult to model human eye movement dynamics. In our tracking system, the following nonlinear equations are used to model the eye movement dynamics:

$$x_{k+1} = Rx_k + Sv_k,$$
$$y_{k+1} = Ty_k + n_k, \quad (48)$$
$$x_k = [x_k, y_k, x_{k-1}, y_{k-1}, \ln \rho_k^2, \ln \lambda_k^2]^{\mathrm{T}}, \quad (49)$$

where ρ and λ are the scaled parameters for noise, R and S are state transition matrices, T is observation matrices.

$$R = \begin{bmatrix} 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (50)$$

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (51)$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (52)$$

The proposed eye tracking experiment is developed in a platform of OPEN CV. Our system uses a ViewQuest VQ680 video camera to capture human images. The experiment is made on a Pentium III 1.7 G CPU with 128 MB RAM. Eye tracking based on the proposed method can reach 10 frames per second. The format of input video is 352×288. Figure 2 represents the eye tracking from test videos by the

proposed SSTN-UKF algorithm under different lighting conditions. The correct rate of eye tracking is shown in Table 1. And with the results of Table 2, we have evaluated the performance of the proposed method and other eye tracking methods in Refs. [21] and [22].

Table 1. Result of eye tracking using SSTN-UKF algorithm.

	video 1	video 2	video3
total frames	7586	12590	9680
tracking failure	13	18	18
correct rate	99.83%	99.94%	99.81%
average correct rate		99.86%	

Note: Correct rate of eye tracking is defined as follows: correct rate $= \frac{\text{total frames} - \text{tracking failure}}{\text{total frames}}$.

Table 3 shows that the complexity of the proposed SSTN-UKF algorithm is reduced, and the expense of CPU time is saved. The SSTN-UKF algorithm simplifies the sigma selection strategy so that it can reduce the matrix operation time and the algorithm computation, and enhance operating speed and favour the request. The related experimental results are listed in Table 3. The complexity of SSTN-UKF algorithm is $O\{(n+2)^3\}$, which is less than $O\{(2n+1)^3\}$ of standard UKF, where n is the dimension of state. The expense of CPU time in Table 3 shows that the proposed SSTN-UKF algorithm has a better computational efficiency than the standard UKF for real-time eye tracking. Especially, as the dimension is higher, the computation reduces more obviously. As shown in Table 4, the root-mean-square error and mean-square error of SSTN-UKF are remarkably lower, and it shows that the filter has a better robustness. So the SSTN-UKF algorithm for nonlinear eye tracking has a better estimation accuracy, real-time performance and robustness.









Fig. 2. Eye tracking by SSTN-UKF algorithm.

Table 2. Comparison of eye tracking algorithms.

algorithm	correct rate	remark
Kalman and mean shift algorithm	99.10%	Ref. [11]
standard UKF	99.50%	Ref. [23]
SSTN-UKF algorithm in this paper	99.86%	Refer to Table 1

Table 3. Comparison of computational complexity.

comparison	standard UKF	SSTN-UKF
complexity	$O\{(2n+1)^3\}$	$O\{(n+2)^3\}$
expense CPU time ($n = 20$)	0.26 s	0.09 s
expense CPU time ($n = 50$)	0.78 s	0.26 s
expense CPU time ($n = 100$)	2.12 s	0.68 s

Table 4. Root-mean-square error and mean-square error of eye tracking filtering algorithms.

algorithm	RMSE (root-mean-square error)	MSE (mean-square error)
Kalman filter algorithm	0.13155	0.164661
UKF tracking algorithm	0.08069	0.06069
SSTN-UKF algorithm in this paper	0.05850	0.03790

4. Conclusion

We have proposed a new eye tracking method which uses the sampling strong tracking nonlinear unscented Kalman filter (SSTN-UKF) algorithm. First, the simplified unscented transform with $n + 2$ sigma points leads to the computational efficiency. Secondly, suboptimal fading factor of strong tracking filtering is introduced into time update equations and measurement update equations to improve the robustness and accuracy of eye tracking. From both theoretical analysis and practical experiments, it is shown that the novel SSTN-UKF algorithm not only has a better computational efficiency, but also is more robustness and accuracy than Kalman filter and standard UKF algorithm for the high stochastic systems such as eye tracking.

References

[1] Julier S J, Uhlmann J K and Durran H F 2000 *IEEE Trans. Automatic Control* **45** 477

[2] Liu F H, Wang W and Zhang Q C 2008 *Chin. Phys.* B **17** 4123

[3] Chen S, Chang S J and Yuan J H 2001 *Acta Phys. Sin.* **50** 674 (in Chinese)

[4] Ji Q, Zhu Z W and Lan P L 2004 *IEEE Trans. Veh. Technol.* **53** 1052

[5] Horng W B, Chen C Y, Chang Y and Fan C H 2004 *Int. Conf. Networking, Sensing and Control* (Taipei: IEEE Press) p. 7

[6] Majaranta P and Raiha K 2002 *Int. Conf. ACM Eye Tracking Research and Applications Symposium* (Louisiana: ACM Press) p. 15

[7] Komogortsev O V and Khan J I 2009 *J. Control Theory Appl.* **7** 14

[8] Noton D and Stark L 1971 *Vision Res.* **11** 929

[9] Takehiko O, Naoki M and Shinjiro K 2003 *Int. Conf. CHI* (Florida: ACM Press) p. 115

[10] Li D H, David W and Derrick J 2005 *Int. Conf. CVPR* (San Diego: ACM Press) p. 79

[11] Wu X D and Song Z H 2008 *Chin. Phys.* B **17** 3241

[12] Julier S J and Uhlmann J K 2002 *Int. Conf. ACC* (Jefferson City: ACM Press) p. 887

[13] Zhou D H and Frank P M 1996 *Int. J. Control* **65** 295

[14] Zhou D H, Sun Y X, Xi Y G and Zhang Z J 1993 *IEEE Trans. Automatic Control* **38** 1270

[15] Zhou D H, Xi Y G and Zhang J Z 1990 *Control and Decision* **5** 1

[16] Zhou D H, Xi Y G and Zhang J Z 1991 *Acta Automatica Sinica* **17** 689 (in Chinese)

[17] Wen C H and Zhou D H 2002 *Multiscale Estimate Theory and Application* (Beijing: Tsinghua University Press) (in Chinese)

[18] Duan Z S and Zhao H C 2004 *J. Sys. Simul.* **16** 1020

[19] Li J W, Lin B L and Huang Y C 2007 *Chin. Phys.* **16** 753

[20] Turk M and Kang S B 2004 *Emerging Topics in Computer Vision* (Boston: Prentice Hall) p. 318

[21] Zhu Z W and Ji Q 2002 *Int. Conf. Pattern Recognition* (Canada: IEEE Press) p. 318

[22] Zhang Z T and Zhang J S 2008 *J. Southwest Jaotong University* **43** 697 (in Chinese)