Generalized photon-added coherent state and its quantum statistical properties*

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In this paper, we propose a class of the generalized photon-added coherent states (GPACSs) obtained by repeatedly operating the combination of Bosonic creation and annihilation operatoes on the coherent state. The normalization factor of GPACS is related to Hermite polynomial. We also derive the explicit expressions of its statistical properties such as photocount distribution, Wigner function and tomogram and investigate their behaviour as the photon-added number varies graphically. It is found that GPACS is a kind of nonclassical state since Wigner function exhibits the negativity by increasing the photon-added number.

Keywords: generalized photon-added coherent state, photocount distribution, Wigner function, tomogram

PACC: 4250, 0365

1. Introduction

Nonclassical states play a crucial role in understanding the fundamentals of quantum physics and have many applications in quantum information processing.[1] The nonclassical behaviour of quantum states can be well described in terms of sub-Poissonian statistics of photon numbers.[2] Another manifestation of nonclassicality occurs when the noise of one quadrature component of a radiation field is below that of the vacuum.[3] Besides, the negativity of Wigner function is indeed a good indicator of the nonclassical character of the state.[4] A simple analysis of the Wigner distribution of the quantum state can provide the information about the nonclassicality itself.

On the other hand, there has been increasing interest in the study of various nonclassical states because many schemes have been proposed to generate some nonclassical states. For example, the photon-added coherent state (PACS), introduced first by Agarwal and Tara,[5] is an interesting intermediate state between the Fock state and the coherent state, which is obtained by repeatedly operating the photon creation operator on a coherent state. Such a state was considered as a nonlinear coherent state[6] and some extensions were also shown.[7−11] PACS exhibits some nonclassical effects like amplitude squeezing, sub-Poissonian character, nonclassical quasi-probability distribution and higher-order properties.[5,6,12] Its statistical properties were discussed in detail in dissipative channel[13] and thermal environment[14] separately. Recently, by using parametric down-conversion, the Zavatta's group[15] prepared experimentally the single PACS, and its nonclassical behaviour was detected through homodyne tomography technology.[16] Another scheme was proposed by Kalamidas *et al.*[17] to generate the two PACSs. In addition, a simple scheme was proposed to generate an arbitrary photon-added coherent state of a travelling optical field by combining an array of degenerate parametric amplifiers and corresponding single-photon detectors.[18]

In the present paper, enlightened by the above studies, we will propose another photon-added coherent state, i.e.,

$$|\alpha, m\rangle = N_{m,\alpha}\Omega^m |\alpha\rangle , \tag{1}$$

where $\Omega = a^\dagger + a$ with $\left[a, a^\dagger\right] = 1$, m is an integer, $|\alpha\rangle$ is the coherent state, and $N_{m,\alpha}$ is the normalization factor. Note that the operator Ω is Hermitian $\Omega = \Omega^\dagger$. For convenience, $|\alpha, m\rangle$ is called the generalized photon-added coherent state (GPACS). We shall derive the normalization factor $N_{m,\alpha}$ of GPACS

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and study the behaviour of quantum statistical properties of GPACSs as the value of m varies graphically. Conclusions and discussions are involved in the last section.

2. Normalization of GPACS

Based on the definition of GPACS (see Eq. (1)), we now calculate the normalization constant $N_{m,\alpha}$, which is the key to the analysis of the quantum statistical properties of GPACS. For this purpose, we first derive the normal ordering form of Ω^m. Recall the generating function of Hermite polynomial $\mathrm{H}_m(x)$[19]

$$\sum_{m=0}^{\infty}\frac{t^m}{m!}\mathrm{H}_m(x)=\mathrm{e}^{2tx-t^2},$$
$$\mathrm{H}_m(x)=\frac{\partial^m}{\partial t^m}\ \mathrm{e}^{2tx-t^2}\Big|_{t=0}, \tag{2}$$

where

$$\mathrm{H}_m(x)=\sum_{l=0}^{[m/2]}\frac{(-1)^l\,m!}{l!(m-2l)!}(2x)^{m-2l}, \tag{3}$$

and the Baker–Hausdorff formula[20]

$$\mathrm{e}^{A+B}=\mathrm{e}^{A}\,\mathrm{e}^{B}\,\mathrm{e}^{[A,B]/2}=\mathrm{e}^{B}\,\mathrm{e}^{A}\,\mathrm{e}^{-[B,A]/2}. \tag{4}$$

For the operator $\Omega=a^\dagger+a$ with $\left[a,a^\dagger\right]=1$, using Eqs. (2) and (4) we easily obtain

$$\begin{aligned}\mathrm{e}^{\lambda\Omega}&=\mathrm{e}^{\lambda\left(a+a^\dagger\right)}=:\mathrm{e}^{\lambda\Omega+\lambda^2/2}:\\&=\sum_{m=0}^{\infty}\frac{\left(\frac{-\mathrm{i}\lambda}{\sqrt{2}}\right)^m}{m!}:\mathrm{H}_m\left(\frac{\mathrm{i}\Omega}{\sqrt{2}}\right):,\end{aligned} \tag{5}$$

where : : denotes the normally ordered product of Bose operators. Comparing the terms with the same powers of λ in the expansion of $\mathrm{e}^{\lambda\Omega}=\sum_{m=0}^{\infty}(\lambda^m/m!)\Omega^m$, on both sides of Eq. (5), we have the neat formula

$$\Omega^m=\left(\frac{-\mathrm{i}}{\sqrt{2}}\right)^m:\mathrm{H}_m\left(\frac{\mathrm{i}\Omega}{\sqrt{2}}\right):, \tag{6}$$

which is just the normal ordering expansion of Ω^m. Thus, the GPACS is expressed as

$$|\alpha,m\rangle=N_{m,\alpha}\left(\frac{-\mathrm{i}}{\sqrt{2}}\right)^m\mathrm{H}_m\left[\frac{\mathrm{i}\left(a^\dagger+\alpha\right)}{\sqrt{2}}\right]|\alpha\rangle, \tag{7}$$

which indicates that the GPACS is equivalent to a Hermite-excited coherent state, where in the last step we have used the eigenequation $a\,|\alpha\rangle=\alpha\,|\alpha\rangle$.

Using Eq. (6), we see the normalization condition

$$\begin{aligned}1&=\left(N_{m,\alpha}\right)^2\langle\alpha|\,\Omega^{2m}\,|\alpha\rangle\\&=\left(N_{m,\alpha}\right)^2\left(\frac{-\mathrm{i}}{\sqrt{2}}\right)^{2m}\langle\alpha|:\mathrm{H}_{2m}\left(\frac{\mathrm{i}\Omega}{\sqrt{2}}\right):|\alpha\rangle\\&=\left(N_{m,\alpha}\right)^2\left(-\frac{1}{2}\right)^m\mathrm{H}_{2m}\left[\frac{\mathrm{i}\left(\alpha^*+\alpha\right)}{\sqrt{2}}\right].\end{aligned} \tag{8}$$

So the compact form of $N_{m,\alpha}$ is found to be

$$\left(N_{m,\alpha}\right)^{-2}=\left(-\frac{1}{2}\right)^m\mathrm{H}_{2m}\left[\frac{\mathrm{i}\left(\alpha^*+\alpha\right)}{\sqrt{2}}\right], \tag{9}$$

which is related to a Hermite polynomial.

From Eq. (6), we also give the quantity

$$\langle\beta|\,\Omega^m\,|\alpha\rangle=\left(\frac{-\mathrm{i}}{\sqrt{2}}\right)^m\mathrm{H}_m\left[\frac{\mathrm{i}\left(\beta^*+\alpha\right)}{\sqrt{2}}\right]\langle\beta|\,\alpha\rangle, \tag{10}$$

which is very useful for the following calculations, where $|\beta\rangle$ is a coherent state and $\langle\beta|\,\alpha\rangle=\exp\left(-\left|\beta\right|^2/2-\left|\alpha\right|^2/2+\beta^*\alpha\right)$.

3. Photocount distribution of GPACS

The quantum mechanical photon counting formula was derived first by Kelley and Kleiner.[21] As shown in Ref. [22], for the case of a single radiation mode the probability distribution $P(n)$ of registering n photoelectrons in the time interval T is given by

$$P(n)=\mathrm{Tr}\left[\rho:\frac{\left(\xi a^\dagger a\right)^n}{n!}\,\mathrm{e}^{-\xi a^\dagger a}:\right], \tag{11}$$

where $\xi\propto T$ is called the quantum efficiency (a measure) of the detector, ρ is a single-mode density operator of the light field concerned. When $\xi=1$, $P(n)$ becomes the photon number distribution for a given state. In Ref. [23], by virtue of the technique of integration within an ordered product (IWOP) of operators, equation (11) is reformed as

$$\begin{aligned}&P(n)\\&=\frac{\xi^n}{(\xi-1)^n}\int\frac{\mathrm{d}^2z}{\pi}\,\mathrm{e}^{-\xi|z|^2}\mathrm{L}_n\left(|z|^2\right)Q\left(\sqrt{1-\xi}z\right),\end{aligned} \tag{12}$$

where $Q(\beta)=\langle\beta|\,\rho\,|\beta\rangle$ is the Q-function, and $\mathrm{L}_n(x)$ is the Laguerre polynomials. Once the Q-function of ρ is known, it is easy to calculate photocount distribution from Eq. (12).

Next, we study the photocount distribution of GPACS optical field. From Eqs. (1) and (10), we can obtain directly the Q-function of GPACS,

$$\begin{aligned} Q(\beta) &= \langle\beta|\,\rho\,|\beta\rangle \\ &= (N_{m,\alpha})^2\,\langle\beta|\,\Omega^m\,|\alpha\rangle\,\langle\alpha|\,\Omega^m\,|\beta\rangle \\ &= (N_{m,\alpha})^2\left(-\frac{1}{2}\right)^m \mathrm{e}^{-|\beta-\alpha|^2}\mathrm{H}_m\!\left(\frac{\mathrm{i}\,(\beta^*+\alpha)}{\sqrt{2}}\right)\mathrm{H}_m\!\left(\frac{\mathrm{i}\,(\alpha^*+\beta)}{\sqrt{2}}\right) \\ &= (N_{m,\alpha})^2\left(\frac{1}{2}\right)^m \mathrm{e}^{-|\beta-\alpha|^2}\left|\mathrm{H}_m\!\left(\frac{\mathrm{i}\,(\beta^*+\alpha)}{\sqrt{2}}\right)\right|^2, \end{aligned} \tag{13}$$

where, in the last step, we have used

$$\mathrm{H}_m(-x) = (-1)^m\,\mathrm{H}_m(x), \tag{14}$$

which is directly derived from Eq. (3). Then substituting Eq. (13) into Eq. (12) and using Eq. (2) and the two-variable Hermite polynomials expression of Laguerre polynomials[19]

$$\mathrm{L}_n(zz^*) = \frac{(-1)^n}{n!}\mathrm{H}_{n,n}(z,z^*) = \frac{(-1)^n}{n!}\frac{\partial^{2n}}{\partial t^n\partial t'^n}\exp\left(-tt'+tz+t'z^*\right)|_{t=t'=0}, \tag{15}$$

we obtain the final result (see Appendix)

$$P(n) = D_{mn}\sum_{l=0}^{m}\sum_{j,k=0}^{n}\frac{B^{2l+j+k}A^{*n-j}A^{n-k}\mathrm{H}_{m-l-j}\left(\dfrac{-C^*}{2}\right)\mathrm{H}_{m-l-k}\left(\dfrac{C}{2}\right)}{l!j!k!\,(n-j)!\,(m-l-j)!\,(n-k)!\,(m-l-k)!}, \tag{16}$$

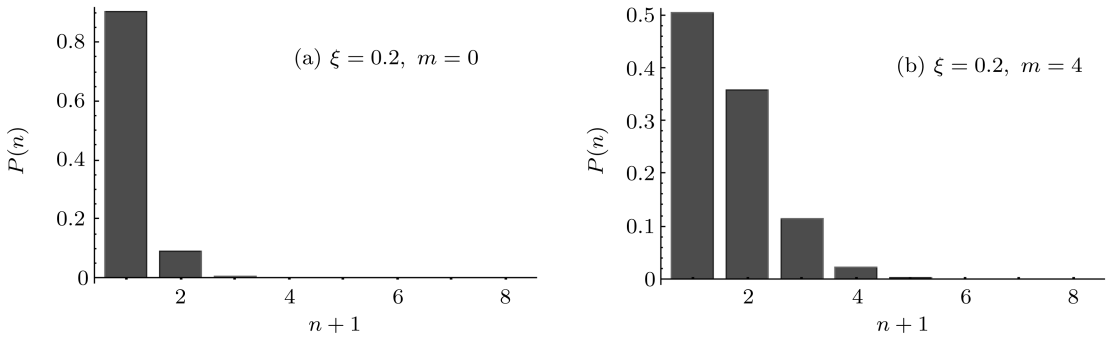
where

$$\begin{aligned} &A \equiv \sqrt{1-\xi}\alpha, \\ &B \equiv \mathrm{i}\sqrt{2-2\xi}, \\ &C \equiv \mathrm{i}\sqrt{2}\left[\alpha+(1-\xi)\,\alpha^*\right], \\ &D_{mn} \equiv \left(-\frac{1}{2}\right)^m\frac{n!\,(m!)^2\,\xi^n\,(N_{m,\alpha})^2\,\mathrm{e}^{-\xi|\alpha|^2}}{(1-\xi)^n}. \end{aligned} \tag{17}$$

Especially, when $m=0$, equation (17) reduces to

$$P(n) = \frac{\mathrm{e}^{-\xi|\alpha|^2}}{n!}\left(\xi\,|\alpha|^2\right)^n,$$

which is just a Poissonion distribution of the coherent state in agreement with the results of Ref. [22]. As a result of Eq. (16), photocount distributions of GPACS are plotted for $\alpha = 0.5 + \mathrm{i}0.5$ and several values of parameters ξ and m in Fig. 1. By increasing parameter m, we are able to move the peak from zero photons to nonzero photons (see Figs. 1(a)–1(c)). It is seen from Figs. 1(b) and 1(d) that for a given m the larger the parameter ξ is, the wider the tail of photon number distribution becomes.



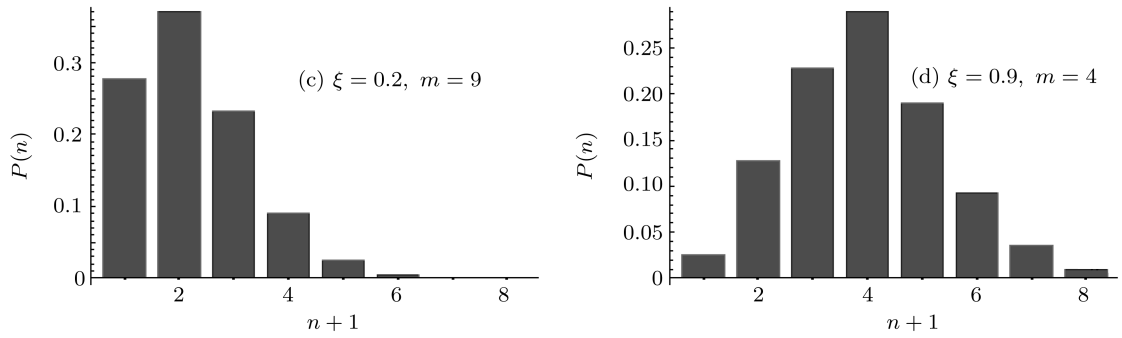


Fig. 1. Photocount distributions of GPACS as a function of $n+1$ for $\alpha = 0.5+\mathrm{i}0.5$ and several different values of parameters ξ and m.

4. Wigner function of GPACS

The Wigner function is a powerful tool to investigate the nonclassicality of optical fields. Its partial negativity implies the highly nonclassical properties of quantum states and is often used to describe the decoherence of quantum states.[13,14,24] In this section, we derive the analytical expression of Wigner function for GPACS. For a single-mode system, Wigner function in the coherent state $|\beta\rangle$ representation is given by

$$\mathrm{W}(z) = \frac{1}{\pi^2}\,\mathrm{e}^{2|z|^2}\int \langle -\beta|\,\rho\,|\beta\rangle\ \mathrm{e}^{-2(\beta z^* - \beta^* z)}\,\mathrm{d}^2\beta, \tag{18}$$

where $z = (q+\mathrm{i}p)/\sqrt{2}$. When $\rho = (N_{m,\alpha})^2\,\Omega^m\,|\alpha\rangle\,\langle\alpha|\,\Omega^m$, according to the integral identity

$$\int \frac{\mathrm{d}^2 z}{\pi}\exp\left(\zeta\,|z|^2 + \xi z + \eta z^*\right) = -\frac{1}{\zeta}\exp\left[-\frac{\xi\eta}{\zeta}\right],\quad \mathrm{Re}\,(\zeta) < 0, \tag{19}$$

by using Eqs. (2) and (10), the Wigner function of GPACS is given by

$$\begin{aligned}
\mathrm{W}(q,p) &= \mathrm{W}(z)\\
&= \frac{(N_{m,\alpha})^2\ \mathrm{e}^{2|z|^2}}{\pi}\int\frac{\mathrm{d}^2\beta}{\pi}\,\langle-\beta|\,\Omega^m\,|\alpha\rangle\,\langle\alpha|\,\Omega^m\,|\beta\rangle\ \mathrm{e}^{-2(\beta z^* - \beta^* z)}\\
&= \frac{(N_{m,\alpha})^2\ \mathrm{e}^{2|z|^2-|\alpha|^2}}{\pi}\left(-\frac{1}{2}\right)^m\frac{\partial^{2m}}{\partial t^m\partial\tau^m}\exp\left(\mathrm{i}\sqrt{2}t\alpha - t^2 + \mathrm{i}\sqrt{2}\tau\alpha^* - \tau^2\right)\\
&\quad\times\int\frac{\mathrm{d}^2\beta}{\pi}\exp\left[-|\beta|^2 + \left(\alpha^* - 2z^* + \mathrm{i}\sqrt{2}\tau\right)\beta + \left(2z - \alpha - \mathrm{i}\sqrt{2}t\right)\beta^*\right]\Big|_{t=\tau=0}\\
&= \frac{(N_{m,\alpha})^2\ \mathrm{e}^{-2|z-\alpha|^2}}{\pi}\left(-\frac{1}{2}\right)^m\frac{\partial^{2m}}{\partial t^m\partial\tau^m}\exp\left[2t\tau + 2Et - 2E^*\tau - t^2 - \tau^2\right]\Big|_{t=\tau=0}\\
&= \frac{(N_{m,\alpha})^2\ \mathrm{e}^{-2|z-\alpha|^2}}{\pi}\left(-\frac{1}{2}\right)^m\frac{\partial^{2m}}{\partial t^m\partial\tau^m}\sum_{l=0}^{\infty}\frac{2^l t^l\tau^l}{l!}\exp\left[2Et - 2E^*\tau - t^2 - \tau^2\right]\Big|_{t=\tau=0}\\
&= \frac{(N_{m,\alpha})^2\ \mathrm{e}^{-2|z-\alpha|^2}}{\pi}\left(-\frac{1}{2}\right)^m\sum_{l=0}^{\infty}\frac{(-1)^{l+m}}{2^l l!}\frac{\partial^{2l}}{\partial E^l\partial E^{*l}}\mathrm{H}_m(E)\mathrm{H}_m(E^*)\\
&= \frac{(N_{m,\alpha})^2\ \mathrm{e}^{-2|z-\alpha|^2}}{\pi}\left(-\frac{1}{2}\right)^m\sum_{l=0}^{m}\frac{(-1)^{l+m}\,2^l\,(m!)^2}{l!\,[(m-l)!]^2}\,|\mathrm{H}_{m-l}(E)|^2\,,
\end{aligned} \tag{20}$$

where

$$E \equiv \frac{\mathrm{i}\,(2z^* + \alpha - \alpha^*)}{\sqrt{2}}, \tag{21}$$

and in the last step we have used the well-known differential relations of $\mathrm{H}_m(x)$

$$\frac{\partial}{\partial x^l}\mathrm{H}_m(x) = \frac{2^l m!}{(m-l)!}\mathrm{H}_{m-l}(x). \tag{22}$$

Equation (20) seems to be a new result, related to

single-variable Hermite polynomials. In particular, when $m = 0$, equation (20) reduces to

$$\mathrm{W}_{m=0}(q,p) = \frac{1}{\pi^2}\,\mathrm{e}^{-2|z-\alpha|^2}, \tag{23}$$

which is just the Wigner function of coherent state, a Gaussian in phase space.

Next we shall see how the variation of the parameter m affects the Wigner function $\mathrm{W}(q,p)$. Using Eq. (20) we plot in Fig. 2 $\mathrm{W}(q,p)$ of GPACS in phase space as a function of q and p with $\alpha = 0.5 + \mathrm{i}0.5$ for several different values of m. It is interesting to note that Wigner function distribution of GPACS is sensitive to parameter m. There exists an upward peak, and the number of downward minor peaks is equal to the value of m. From Fig. 2, as $m \geq 1$, Wigner function presents a partial negative region. In other words, the GPACS may become more nonclassical by increasing m. This is a useful way to conditionally manipulate a nonclassical state.

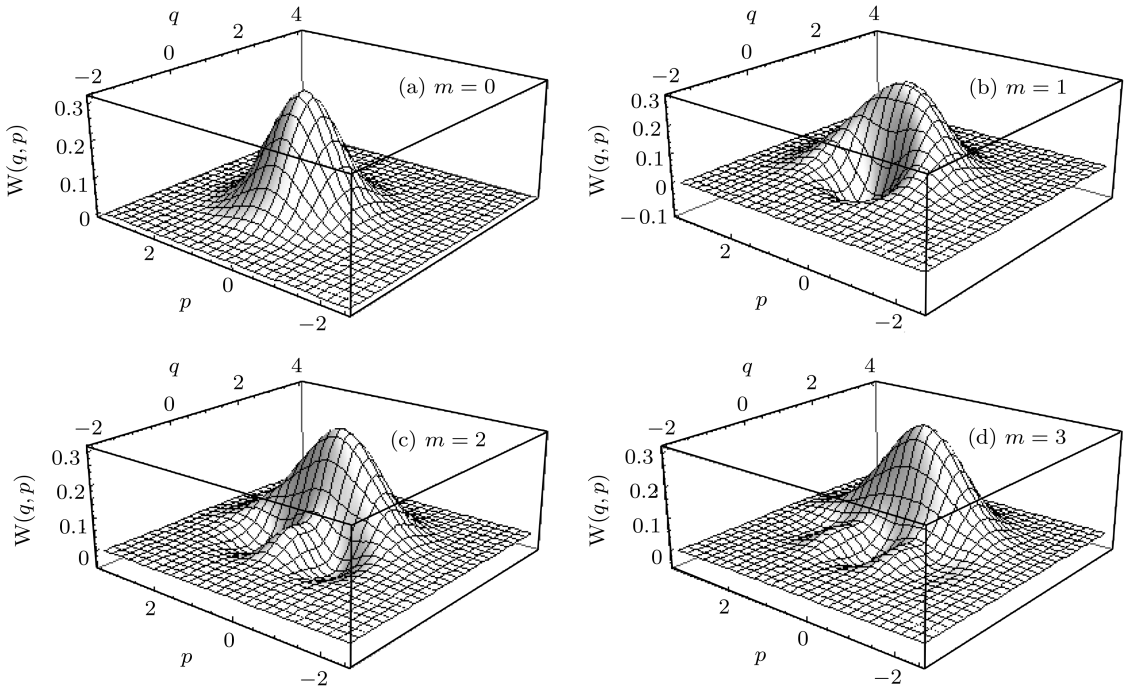


Fig. 2. Wigner functions of GPACS in phase space as a function of q and p with $\alpha = 0.5 + \mathrm{i}0.5$ and several different values of m.

5. Tomogram of GPACS

In recent years, the use of tomograms in quantum mechanics and quantum optics makes it possible to describe a quantum state with a positive probability distribution.[25] A direct description of quantum states by means of quantum tomogram is interesting from both the theoretical and the experimental points of view. In this section, we derive the tomogram of GPACS, which is defined as

$$T(x,f,g) = \int \mathrm{d}q'\,\mathrm{d}p'\,\delta\left(x - fq' - gp'\right)\mathrm{W}(q',p'). \tag{24}$$

However, it will be very difficult to evaluate the integration if we directly substitute Eq. (20) into Eq. (24). Fortunately, in Refs. [26]–[28], one can prove that the Radon transform of Wigner operator $\Delta\left(q',p'\right)$ is just a pure density operator

$$\begin{aligned} &|x\rangle_{f,gf,g}\,\langle x| \\ &= \int \mathrm{d}q'\,\mathrm{d}p'\,\delta\left(x - fq' - gp'\right)\Delta\left(q',p'\right), \end{aligned} \tag{25}$$

where

$$|x\rangle_{f,g} = M\exp\left(\frac{\sqrt{2}\,xa^{\dagger}}{G} - \frac{G^{*}a^{\dagger 2}}{2G}\right)|0\rangle \tag{26}$$

with

$$\begin{aligned} &G = f - \mathrm{i}g, \\ &M = \left[\pi\left(f^2+g^2\right)\right]^{-1/4}\exp\left[-\frac{x^2}{2\left(f^2+g^2\right)}\right] \end{aligned} \tag{27}$$

is named the intermediate coordinate–momentum representation, which obeys the eigen-equation

$$(fQ+gP)\left|x\right\rangle_{f,g}=x\left|x\right\rangle_{f,g} \tag{28}$$

with $Q=\left(a+a^{\dagger}\right)/\sqrt{2}$, $P=\left(a-a^{\dagger}\right)/\left(\mathrm{i}\sqrt{2}\right)$, and f and g are real. Thus, the tomogram of GPACS is

$$\begin{aligned}
&T(x,f,g)\\
&={}_{f,g}\left\langle x\right|\left.\alpha,m\right\rangle\\
&=M\left\langle 0\right|\exp\left(\frac{\sqrt{2}qa}{G^{*}}-\frac{Ga^{2}}{2G^{*}}\right)\left|\alpha,m\right\rangle\\
&=N_{m,\alpha}M\left(\frac{-\mathrm{i}}{\sqrt{2}}\right)^{m}\mathrm{e}^{-|\alpha|^{2}/2}\frac{\partial^{m}}{\partial t^{m}}\exp\left(t\mathrm{i}\sqrt{2}\left(z^{*}+\alpha\right)-t^{2}\right)\int\frac{\mathrm{d}^{2}z}{\pi}\exp\left(-|z|^{2}+\frac{\sqrt{2}\,xz}{G^{*}}-\frac{Gz^{2}}{2G^{*}}+z^{*}\alpha\right)\Bigg|_{t=0}\\
&=N_{m,\alpha}M\left(\frac{-\mathrm{i}}{\sqrt{2}}\right)^{m}\exp\left(-\frac{|\alpha|^{2}}{2}\right)\frac{\partial^{m}}{\partial t^{m}}\exp\left\{\frac{\sqrt{2}\,x}{G^{*}}\left(\alpha+\mathrm{i}t\sqrt{2}\right)-\frac{G}{2G^{*}}\left(\alpha+\mathrm{i}t\sqrt{2}\right)^{2}+\mathrm{i}\sqrt{2}\alpha t-t^{2}\right\}\Bigg|_{t=0}\\
&=N_{m,\alpha}M\left(\frac{-\mathrm{i}}{\sqrt{2}}\right)^{m}\exp\left(-\frac{|\alpha|^{2}}{2}-\frac{G}{2G^{*}}\alpha^{2}+\frac{\sqrt{2}\,x\alpha}{G^{*}}\right)\\
&\quad\times\frac{\partial^{m}}{\partial t^{m}}\exp\left\{\left(\frac{2\mathrm{i}x}{G^{*}}-\frac{\mathrm{i}\sqrt{2}\alpha G}{G^{*}}+\mathrm{i}\sqrt{2}\alpha\right)t-\left(1-\frac{G}{G^{*}}\right)t^{2}\right\}\Bigg|_{t=0},
\end{aligned} \tag{30}$$

which is an alternative approach to the derivation of the tomogram of quantum states. Using Eqs. (2) and (7) and the completeness relation of the coherent state, we have the tomogram amplitude of $|\alpha,m\rangle$, i.e.

$$\begin{aligned}
T(x,f,g)&=\left\langle\alpha,m\right|\Delta\left(x,f,g\right)\left|\alpha,m\right\rangle\\
&=\int\mathrm{d}q'\,\mathrm{d}p'\delta\left(x-fq'-gp'\right)\left\langle\alpha,m\right|\Delta\left(q',p'\right)\left|\alpha,m\right\rangle\\
&=\left|{}_{f,g}\left\langle x\right|\left.\alpha,m\right\rangle\right|^{2},
\end{aligned} \tag{29}$$

where we have used the following integral formula:

$$\int\frac{\mathrm{d}^{2}z}{\pi}\exp\left\{\zeta\left|z\right|^{2}+\xi z+\eta z^{*}+fz^{2}+gz^{*2}\right\}=\frac{1}{\sqrt{\zeta^{2}-4fg}}\exp\left\{\frac{-\zeta\xi\eta+\xi^{2}g+\eta^{2}f}{\zeta^{2}-4fg}\right\} \tag{31}$$

with $\mathrm{Re}(\zeta\pm f\pm g)<0$ and $\mathrm{Re}\big((\zeta^{2}-4fg)/(\zeta\pm f\pm g)\big)<0$. Further, using Eq. (2) again leads to

$${}_{f,g}\left\langle x\right|\left.\alpha,m\right\rangle=N_{m,\alpha}M\left(-\mathrm{i}\sqrt{\frac{1}{2}-\frac{G}{2G^{*}}}\right)^{m}\exp\left(-\frac{|\alpha|^{2}}{2}-\frac{G}{2G^{*}}\alpha^{2}+\frac{\sqrt{2}\,x\alpha}{G^{*}}\right)\mathrm{H}_{m}(K), \tag{32}$$

where

$$K\equiv\frac{\mathrm{i}\left(\sqrt{2}\,x-\alpha G+\alpha G^{*}\right)}{G^{*}\sqrt{\left(2-\dfrac{2G}{G^{*}}\right)}}. \tag{33}$$

Finally, inserting Eq. (32) into Eq. (29), we derive the analytical expression of the tomogram of GPACS as

$$T(x,f,g)=\left|N_{m,\alpha}M\left(-\mathrm{i}\sqrt{\frac{1}{2}-\frac{G}{2G^{*}}}\right)^{m}\exp\left(-\frac{|\alpha|^{2}}{2}-\frac{G}{2G^{*}}\alpha^{2}+\frac{\sqrt{2}\,x\alpha}{G^{*}}\right)\mathrm{H}_{m}(K)\right|^{2}. \tag{34}$$

The influence of m on tomograms $T(x,\theta)$ of GPACS is shown in Fig. 3 with $\alpha=0.5+\mathrm{i}0.5$, $f=\cos\theta$ and $g=\sin\theta$. There are four plots for $T(x,\theta)$ versus x and θ for $m=0,1,4,7$, corresponding to Figs. 3(a)–3(d) respectively. Tomograms are real and positive distribution functions. When $m=0$ (Fig. 3(a)), it is just the tomogram of well-known coherent state. One can see from Fig. 1 that the tomogram $T(x,\theta)$ of GPACS presents π-periodic variations with coordinate and depends on parameter m.

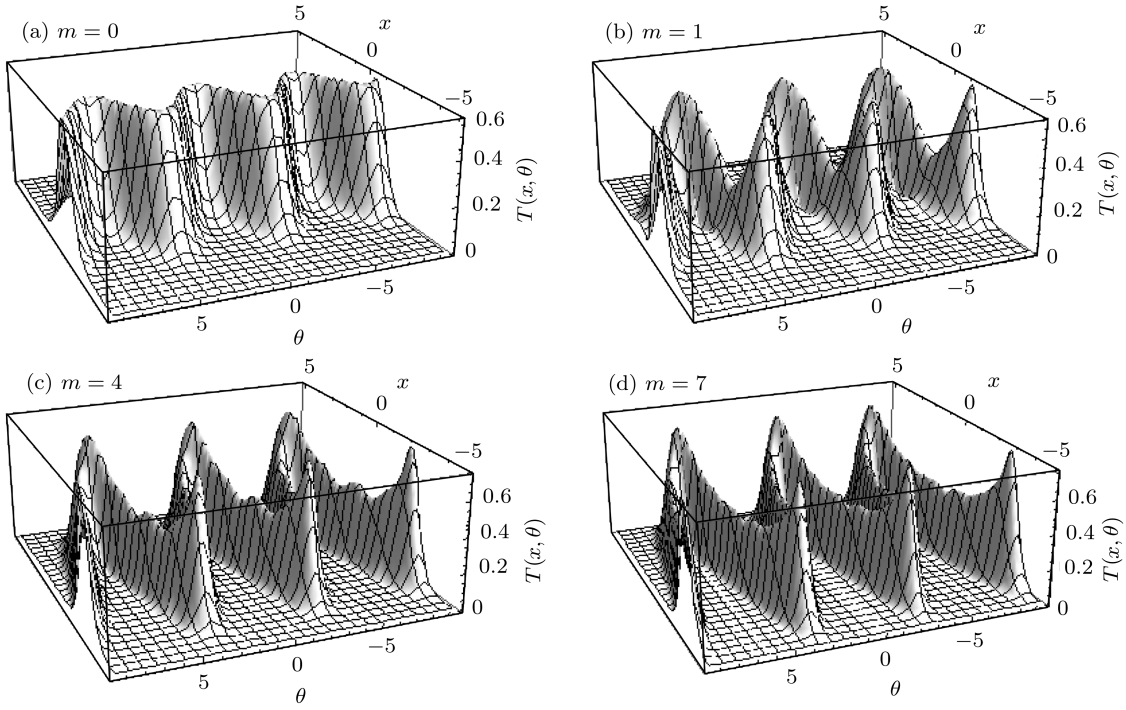


Fig. 3. Tomograms of GPACS each as a function of x and θ with $\alpha = 0.5 + \mathrm{i}0.5$ and several different values of m.

6. Conclusions and discussion

We presented a class of the generalized photon-added coherent states (GPACSs), which are obtained by repeatedly operating the combination of $a^\dagger + a$ on the coherent state $|\alpha\rangle$. First, we derived the normalization factor of GPACS, which is related to a Hermite polynomial, and found that the GPACS is actually regarded as a Hermite-excited coherent state. Then, we deduced the explicit expressions of its statistical properties such as photocount distribution, Wigner function and tomogram, and we also investigated their behaviours as photon-added number m varies graphically. It is found that these properties are affected by the value of m more significatly and Wigner function presents a partial negative region by increasing m, which implies that the GPACS is a kind of nonclassical state.

In addition, it is worth while mentioning that the GPACS may be generated via using cavity quantum electrodynamics (QED) and quantum measurement. The idea of the method is to prepare the electromagnetic field in coherent state $|\alpha\rangle$ and then to subject this state to the interaction with a two-level atom in a resonator. Suppose that the two-level excited atom passes through the cavity, analogous to that proposed in Ref. [5]. The atom makes a transition from the excited state $|e\rangle$ to the ground state $|g\rangle$ by emitting a photon. Under the nonrotating wave approximation, the interaction Hamiltonian has the following form ($\hbar = 1$):

$$H = g\left(a + a^\dagger\right)\left(\sigma_- + \sigma_+\right), \tag{35}$$

where g is the coupling constant, σ_+ and σ_- are the Pauli operators corresponding to the two-level atom, a and $a^\dagger$ are the Bosonic creation and annihilation operators of the field mode. Then the initial state of the atom-field system is $|\alpha\rangle|e\rangle$. If the interaction time is sufficiently small, such that $gt \ll 1$, then we expand the exponential to the first order and we can obtain the system state vector with the Hamiltonian in Eq. (35)

$$\begin{aligned}|\psi(t)\rangle \;&=\; \exp\left(-\mathrm{i}Ht\right)|\alpha\rangle|e\rangle \simeq |\alpha\rangle|e\rangle \\ &\quad -\mathrm{i}gt\left(a + a^\dagger\right)|\alpha\rangle|g\rangle\,.\end{aligned} \tag{36}$$

Thus if the atom is detected in the ground state $|g\rangle$ after it has passed through the cavity, then the state of the field is reduced to the $\left(a + a^\dagger\right)|\alpha\rangle$ with $m = 1$. If the cavity state is initially at $\left(a + a^\dagger\right)|\alpha\rangle$, by repeating the above process, we obtain in principle the unnormalized state, $\left(a + a^\dagger\right)^m|\alpha\rangle$.

Finally, we point out the main differences between GPACS and PACS as follows: i) the normalization factor of GPACS is Hermite polynomial, while PACS's is Laguerre polynomial; ii) from their explicit expressions of Wigner function, the former is related to a single-variable Hermite polynomial, the latter corresponds to a Laguerre polynomial; iii) from the point of view of generating via using cavity QED, the former does not need the rotating wave approximation, the latter is under rotating wave approximation.

Appendix: Derivation of Eq. (16)

Substituting Eq. (13) into Eq. (12) and using Eqs. (2) and (15), we have

$$\begin{aligned} P(n) = & \frac{\xi^n}{(\xi-1)^n}\int\frac{\mathrm{d}^2z}{\pi}\,\mathrm{e}^{-\xi|z|^2}\mathrm{L}_n\left(|z|^2\right)Q\left(\sqrt{1-\xi}z\right) \\ = & \left(-\frac{1}{2}\right)^m\frac{\xi^n\left(N_{m,\alpha}\right)^2\mathrm{e}^{-|\alpha|^2}}{n!\,(1-\xi)^n}\frac{\partial^{2n}}{\partial t^n\partial t'^n}\frac{\partial^{2m}}{\partial\tau^m\partial\tau'^m}\,\mathrm{e}^{\sqrt{2}\mathrm{i}\alpha\tau+\alpha^*\sqrt{2}\mathrm{i}\tau'-tt'-\tau^2-\tau'^2} \\ & \times\int\frac{\mathrm{d}^2z}{\pi}\exp\left[-|z|^2+\left(t+B\tau'+A^*\right)z+\left(t'+B\tau+A\right)z^*\right]\Big|_{t=t'=\tau=\tau'=0}, \end{aligned} \tag{A1}$$

where

$$A\equiv\sqrt{1-\xi}\alpha\ \text{ and }\ B\equiv\mathrm{i}\sqrt{2-2\xi}. \tag{A2}$$

Using Eq. (19) yields

$$\begin{aligned} P(n) = & \left(-\frac{1}{2}\right)^m\frac{\xi^n\left(N_{m,\alpha}\right)^2\mathrm{e}^{-|\alpha|^2}}{n!\,(1-\xi)^n}\frac{\partial^{2n}}{\partial t^n\partial t'^n}\frac{\partial^{2m}}{\partial\tau^m\partial\tau'^m} \\ & \times\exp\left[\sqrt{2}\mathrm{i}\alpha\tau+\alpha^*\sqrt{2}\mathrm{i}\tau'-tt'-\tau^2-\tau'^2+\left(t+B\tau'+A^*\right)\left(t'+B\tau+A\right)\right]\Big|_{t=t'=\tau=\tau'=0} \\ = & \left(-\frac{1}{2}\right)^m\frac{\left(N_{m,\alpha}\right)^2\xi^n\,\mathrm{e}^{-\xi|\alpha|^2}}{n!\,(1-\xi)^n}\frac{\partial^{2n}}{\partial t^n\partial t'^n}\frac{\partial^{2m}}{\partial\tau^m\partial\tau'^m}\,\mathrm{e}^{B^2\tau'\tau+Bt'\tau'+Bt\tau} \\ & \times\exp\left[At+A^*t'-\tau^2+C\tau-\tau'^2-C^*\tau'\right]|_{t=t'=\tau=\tau'=0}, \end{aligned} \tag{A3}$$

where we have set

$$C=\mathrm{i}\sqrt{2}\left[\alpha+(1-\xi)\,\alpha^*\right]. \tag{A4}$$

Then expanding the term $\mathrm{e}^{B^2\tau'\tau+Bt'\tau'+Bt\tau}$ into the form of the series and using Eq. (2) again, we finally obtain

$$\begin{aligned} P(n) = & \left(-\frac{1}{2}\right)^m\frac{\left(N_{m,\alpha}\right)^2\xi^n\,\mathrm{e}^{-\xi|\alpha|^2}}{n!\,(1-\xi)^n}\frac{\partial^{2n}}{\partial t^n\partial t'^n}\frac{\partial^{2m}}{\partial\tau^m\partial\tau'^m}\sum_{l,j,k}^{\infty}\frac{B^{2l+j+k}}{l!j!k!}\tau^{l+k}\tau'^{l+j}t'^jt^k \\ & \times\exp\left[At+A^*t'-\tau^2+C\tau-\tau'^2-C^*\tau'\right]|_{t=t'=\tau=\tau'=0} \\ = & \left(-\frac{1}{2}\right)^m\frac{\left(N_{m,\alpha}\right)^2\xi^n\,\mathrm{e}^{-\xi|\alpha|^2}}{n!\,(1-\xi)^n}\frac{\partial^{2n}}{\partial t^n\partial t'^n}\frac{\partial^{2m}}{\partial\tau^m\partial\tau'^m}\sum_{l,j,k}^{\infty}\frac{B^{2l+j+k}}{l!j!k!} \\ & \times\frac{\partial^{j+k}}{\partial A^{*j}\partial A^k}\frac{\partial^{2l+j+k}}{\partial\left(-C^*\right)^{l+j}\partial C^{l+k}}\exp\left[At+A^*t'-\tau^2+C\tau-\tau'^2-C^*\tau'\right]|_{t=t'=\tau=\tau'=0} \\ = & \left(-\frac{1}{2}\right)^m\frac{\left(N_{m,\alpha}\right)^2\xi^n\,\mathrm{e}^{-\xi|\alpha|^2}}{n!\,(1-\xi)^n}\sum_{l,j,k}^{\infty}\frac{B^{2l+j+k}}{l!j!k!2^{l+k}2^{l+j}}\frac{\partial^jA^{*n}}{\partial A^{*j}}\frac{\partial^kA^n}{\partial A^k}\frac{\partial^{l+k}\mathrm{H}_m\left(\dfrac{C}{2}\right)}{\partial\left(\dfrac{C}{2}\right)^{l+k}}\frac{\partial^{l+j}\mathrm{H}_m\left(\dfrac{-C^*}{2}\right)}{\partial\left(\dfrac{-C^*}{2}\right)^{l+j}} \\ = & D_{mn}\sum_{l=0}^{m}\sum_{j,k=0}^{n}\frac{B^{2l+j+k}A^{*n-j}A^{n-k}\mathrm{H}_{m-l-j}\left(\dfrac{-C^*}{2}\right)\mathrm{H}_{m-l-k}\left(\dfrac{C}{2}\right)}{l!j!k!\,(n-j)!\,(m-l-j)!\,(n-k)!\,(m-l-k)!}, \end{aligned} \tag{A5}$$

where D_{mn} is just given in Eq. (17) and we have used the differential relations of $\mathrm{H}_m(x)$ of Eq. (22). Thus we complete the derivation of Eq. (16) in detail.

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