Topological probability and connection strength induced activity in complex neural networks*

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Recent experimental evidence suggests that some brain activities can be assigned to small-world networks. In this work, we investigate how the topological probability p and connection strength C affect the activities of discrete neural networks with small-world (SW) connections. Network elements are described by two-dimensional map neurons (2DMNs) with the values of parameters at which no activity occurs. It is found that when the value of p is smaller or larger, there are no active neurons in the network, no matter what the value of connection strength is; for a given appropriate connection strength, there is an intermediate range of topological probability where the activity of 2DMN network is induced and enhanced. On the other hand, for a given intermediate topological probability level, there exists an optimal value of connection strength such that the frequency of activity reaches its maximum. The possible mechanism behind the action of topological probability and connection strength is addressed based on the bifurcation method. Furthermore, the effects of noise and transmission delay on the activity of neural network are also studied.

Keywords: topological probability, small world connections, connection strength, neural networks, activity

PACC: 0590, 0520, 0547

1. Introduction

The small-world (SW) effect is generally related to geometrical properties, as size diameter or cliqueness, in network topologies where connectivity among nodes is neither fully local and regular nor totally random but something in between the two extremes. Over the past few years, many large-scale complex network systems have been discovered to have small-world effects, ranging from artificial examples including the power grids, the World Wide Web, and the World Trade Web, etc., to naturally evolved systems such as neural connections, food webs, metabolic networks, etc.[1−3] Nowadays, most studies on SW networks have focused on understanding how the topological probability influences the system's dynamic behaviours. For instance, it is shown that the introduction of topological probability can induce transition from a highly fragmented phase to a uniform phase in social systems,[4] enhance the phase synchronization in complex Chua network,[5] discrete neural networks,[6] etc. On the other hand, the significance of connection strength in complex networks has received considerable attention in recent years. For example, Wei and Luo[7] explored the effect of connection strength on coherence resonance and synchronization of coupled Hindmarsh–Rose (HR) neurons, Kwon *et al.*[8] and Li and Liu[9] exhibited the importance of connection strength to the internal stochastic resonance and the coherence resonance effect in coupled FitzHugh–Nagumo (FN) neurons, respectively.

Neurobiological systems, consisting of a large number of neurons, have always been a fascinating and important subject of research. In a good many of the previous studies, the connectivity between the elements of the neural network was regular (local or global). However, recent experimental evidence shows that the neural network presents the SW property. Hence, introducing the SW topology into neural network investigations is a significant job. On the other hand, the activity of the coupled nonidentical neurons, which is basic for the understanding of dynamical principles and mechanisms behind

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the signal and information processings that occur in neurobiological networks, is one of the most studied phenomena.[10−13] However, to the best of ou knowledge, the studies of the activity of discrete neural networks with SW connections are few in the previous literature. The main aim of this work is to investigate how the activity of the SW neural network depends on the topological probability p and connection strength C. The network element is described by two-dimensional map neuron (2DMN) whose dynamical mechanism and behaviours are different from those of the continuous time neuron.[14−16] Though the single 2DMN model is a simple map, it is capable of describing various types of neural activities observed in real neurons, including the generation of tonic spikes, irregular spiking, both regular and irregular bursts of spikes, etc. The SW neural network is constructed by randomly adding long-range connections to an originally nearest-neighbour coupled network.[17] In order to study the influence of topological probability p and connection strength C on the activity, the parameter regions in the single 2DMN neuron chosen can only sustain a silent state. It is found that with the optimal combination of p and C, the activity is triggered and enhanced in a 2DMN network. To quantitatively study the activity degree of the neuronal network, we introduce an average firing rate to measure the frequency of the active patterns. Furthermore, the dynamics behaviour in the presence of noise and delay of neural networks has received considerable attention in recent years.[18−20] The effects of noise and transmission delay on activity of neural network are also studied in this paper and two main results are obtained: long transmission delays impair the intensity of activity, and p and C induced activity is robust against the changes in noise strength.

2. Network structure

The discrete-time neural network studied in this paper, in which the incoherent noise and the transmission delay are introduced into each original neuron, is described by the following coupled two-dimensional maps:[14−16]

$$\begin{aligned} x_{i,n+1} &= \frac{\alpha_i}{1+x_{i,n}^2} + y_{i,n} \\ &\quad + \frac{C}{M}\sum_{j=1}^{M} a_{ij}x_{j,n-\tau} + \sqrt{D}\xi_{ij,n}, \\ y_{i,n+1} &= y_{i,n} - \sigma_i x_{i,n} - \beta_i, \end{aligned} \tag{1}$$

where indexes $i = 1, \ldots, M$ and $n = 0, \ldots, N$ specify the cell and the discrete-time series, respectively; $x_{i,n}$ and $y_{i,n}$ are the fast and the slow dynamical variables of the i-th neuron, respectively; slow time evolution of $y_{i,n}$ is due to the small value of parameter σ_i; parameter α_i defines the dynamics of the fast variables $x_{i,n}$ of the individual map; τ is the transmission delay; $\xi_{ij,n}$ is Gaussian white noise with the statistical property $\langle \xi_{ij,n} \rangle = 0$ and $\langle \xi_{ij,n}, \xi_{kl,n'} \rangle = \delta_{n-n'}\delta_{ik}\delta_{jl}$; D is the noise intensity; $\frac{C}{M}\sum_{j=1}^{M} a_{ij}x_{j,n}$ is the connection term of the complex neurons, with C being the connection strength; the matrix a_{ij} encodes the connection topology: if a connection exists between neuron i and j, then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$. Also, $a_{ii} = 0$ for all i's. The SW connection patterns are achieved as follows:[17] It starts from a ring lattice with $N = 500$ neurons and $k = 6$ nearest neighbours, and then links are randomly added between non-nearest vertices with probability p. For $p = 0$, it reduces to the originally nearest-neighbour coupled network; for $p = 1$, it becomes a globally coupled network. Note that for a given p there are a lot of network realizations.

The individual dynamics of the 2DMN, which depends on parameter α_i has been investigated fully in Refs. [14]–[16]. The results of those investigations have shown that with parameters α_i changing, the individual 2DMN exhibits silent, periodic and/or chaotic spiking, spiking-bursting behaviours sequencially. In this paper, in order to study the effects of the network topology probability and connection strength on the activity of 2DMNs, we generate $\alpha_i \in [0.8, 1.4]$ at random and set $\beta_i = \sigma_i = 0.002$, such that each neuron in the network is silent and has non-identical properties.

3. Results

The results discussed hereafter are obtained by varying topological probability p and connection strength C. The initial conditions of the system are generated to be $x_{i,0} \in [-3.0, 3.0]$ and $y_{i,0} \in [-4.0, -1.0]$ at random. We obtain the following results.

Figures 1(a)–1(d) show the time series of the fast variable $x_{i,n}$ of any neuron in the 2DMN network for different values of p with fixed $C = 2.5$, $\tau = 0$ and $D = 0$. When $p = 0$, i.e., in the nearest-neighbour network, neuron shows silence without any active pattern, as shown in Fig. 1(a). With the values of p in-

creasing, the firing behaviour can be observed, namely, the active neurons occur in network, which is shown in Fig. 1(b). With the values of p further increasing, the frequency of the firing activity increases rapidly, which can be seen in Fig. 1(c). Whereas, when p is increased to 0.6, i.e. when $p = 0.6$, the firing behaviour disappears in neuron as shown in Fig. 1(d).

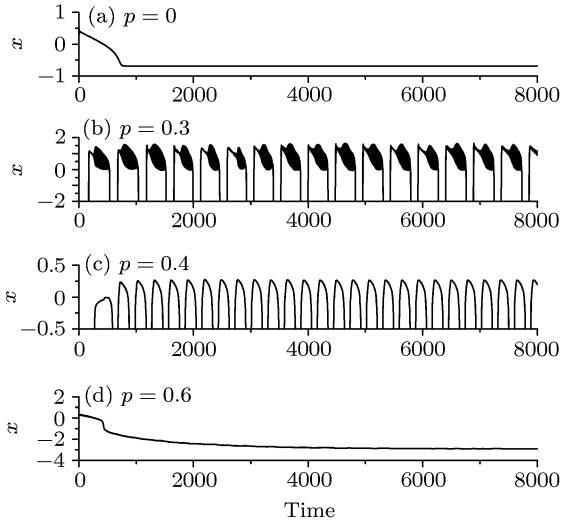


Fig. 1. Time series of the fast variable $x_{i,n}$ of an arbitrary neuron in the 2DMN networks with SW connections for different values of topological probability p with fixed $C = 2.5$, $\tau = 0$ and $D = 0$.

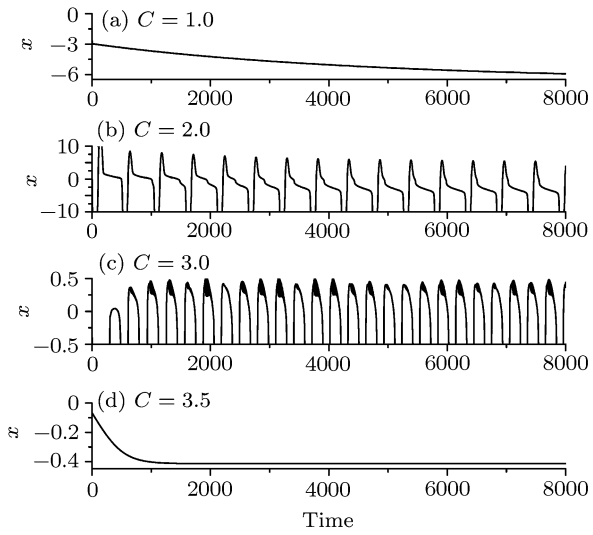


Fig. 2. Time series of the fast variable $x_{i,n}$ of an arbitrary neuron in the 2DMN networks with SW connections for different values of connection strength C with fixed $p = 0.34$, $\tau = 0$ and $D = 0$.

Figures 2(a)–2(d) show the time series of the fast variable $x_{i,n}$ of any neuron in the 2DMN network for different values of C with fixed $p = 0.34$, $\tau = 0$ and $D = 0$. From Figs. 2(a) and 2(d), we can see that when the connection strength level is lower ($C = 1.0$) or higher ($C = 3.5$), there is scarcely any active pattern in neuron. For the intermediate connection strength levels, the firing activity can be induced and enhanced in neuron, more details are shown in Figs. 2(b) and 2(c).

To quantitatively study the activity degree of the neural network, we use the following average firing rate $\overline{\eta}$ as a measure of the frequency of firing behaviour patterns: $\overline{\eta} = \{\frac{1}{M\Delta n}\sum_{t=n_1}^{n_2}\eta_t\}$, where $\{\cdot\}$ denotes the average over 100 different network realizations with the same p; η_t is the number of firing neurons in the whole network at any moment; $\Delta n = n_2 - n_1$ is a time interval; n_1 and n_2 are the initial time and the terminative time of simulation, respectively. In this paper, we set $n_1 = 3000$ and $n_2 = 8000$. It is clear that the higher the firing frequency is, the larger the parameter $\overline{\eta}$ will be.

The dependence of $\overline{\eta}$ on p with fixed $C = 2.5$ is shown in Fig. 3, where one can see that when p is small, the $\overline{\eta}$ approaches zero, namely, there are no active neurons in the neural network. When the p increases to a particular value, i.e., $p = 0.2$, $\overline{\eta}$ is rapidly increased, and it reaches its maximum when p is gradually increased to $p = 0.41$. Whereas, $\overline{\eta}$ is rapidly decreased when p increases further. Figure 4 depicts the dependence of $\overline{\eta}$ on C with fixed $p = 0.34$, from which it is seen that there is an abrupt peak at $C = 3.05$, where the parameter $\overline{\eta}$ is maximal. For a small or large value of C, the average firing rate is drastically decreased. To make an overall inspection, we scan the topological probability p and the connection strength C simultaneously over a relatively wide parameter range. The result is shown in Fig. 5.

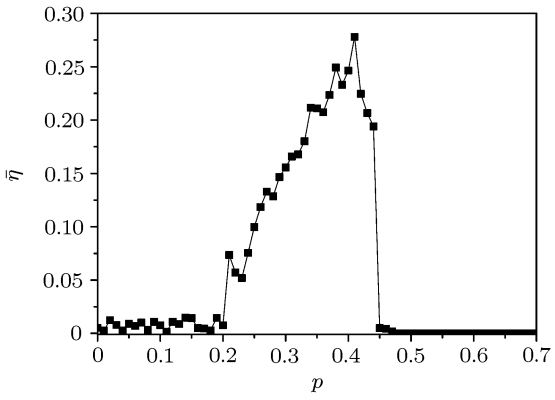


Fig. 3. The measure of frequency of firing activity vs. topological probability p in the case of $C = 2.5$, $\tau = 0$ and $D = 0$.

It is evident that there exist some "optimal islands" in the central area of Fig. 5, where the parameter $\overline{\eta}$ is maximal in the parameter plane. This

reveals the existence of optimal combination of topological probability and connection strength for the occurrence of maximal firing frequency.

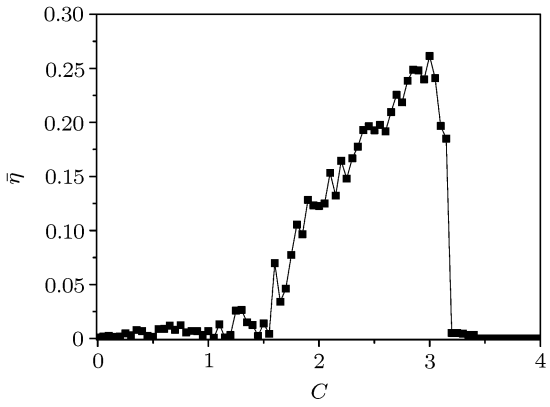


Fig. 4. The measure of frequency of firing activity vs. connection strength C with $p = 0.34$, $\tau = 0$ and $D = 0$.

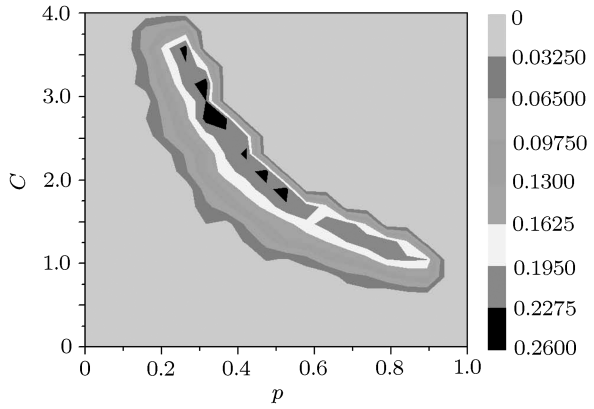


Fig. 5. The contour plot of average firing rate $\overline{\eta}$ as a function of topological probability p and connection strength C.

To study the effects of noise and delay on firing activity of neuronal networks, we have also calculated the average firing rate $\overline{\eta}$ for different noise strengths and delay lengths. We set the parameters $p = 0.38$ and $C = 2.7$ at which the firing activity of neuronal networks is maximal. The dependences of $\overline{\eta}$ on τ and D are shown in Figs. 6 and 7, respectively. From Fig. 6, one can see that the parameter $\overline{\eta}$ decreases with τ increasing, which shows that long transmission delay impairs the intensity of firing activity. From Fig. 7 we find that for different values of D the parameter $\overline{\eta}$ changes slightly, which reveals that the variation of the noise strength exerts a slight influence on firing activity. That is, p and C induced firing activity is robust against the change in noise strength.

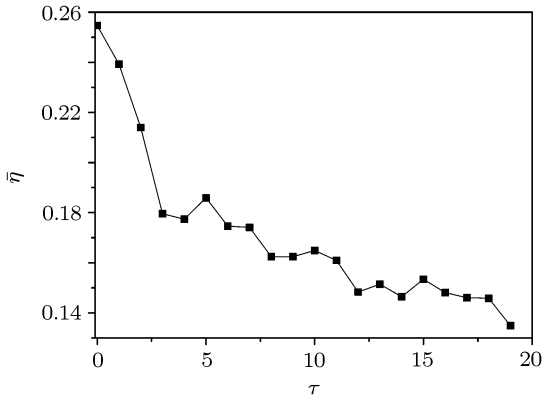


Fig. 6. The measure of frequency of firing activity vs. delay length τ in the case of $p = 0.38$, $C = 2.7$ and $D = 0$.

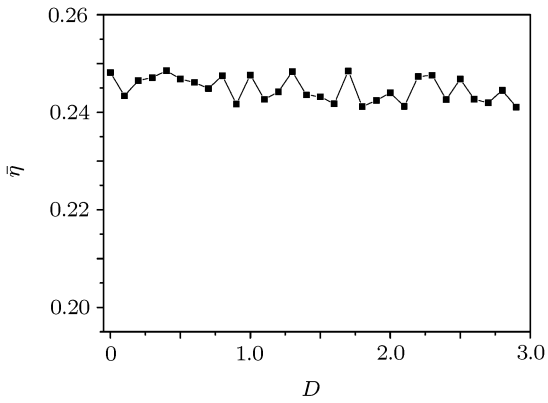


Fig. 7. The measure of frequency of firing activity vs. noise strength D in the case of $p = 0.38$, $C = 2.7$ and $\tau = 0$.

The possible mechanism behind the effects of topological probability and connection strength may be understood as follows. The individual dynamics of the neuron is described by the map of the following form:

$$x_{n+1} = \frac{\alpha}{1 + x_n^2} + y_n, \tag{2}$$

$$y_{n+1} = y_n - \sigma x_n - \beta. \tag{3}$$

As shown in the previous studies,[13–15] y_n changes slowly, the dynamics of x_n can be considered to be independent of map (3) on the assumption that y_n is a parameter $\gamma = y_n$. Therefore, fast dynamics of the neuron can be understood from the analysis of the one-dimensional map

$$x_{n+1} = \frac{\alpha}{1 + x_n^2} + \gamma. \tag{4}$$

The bifurcation diagram of the interspike interval (ISI) versus parameter γ of map (4) is plotted in Fig. 8, from which one can see that if γ is neither too small nor too large the fast map is in a high-frequency repetitive firing state. In this work, for each individual neuron in the complex neural network, the total synaptic input from other neurons is regarded as the parameter γ. With the optimal combination of p and C, the value of synaptic input falls into an intermediate range such that the firing activity of neurons in the neural network is induced and enhanced.

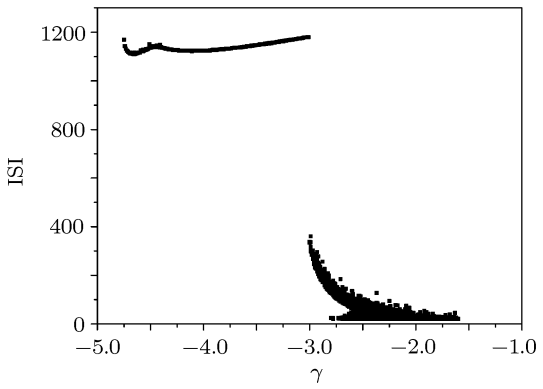


Fig. 8. The bifurcation structure of the interspike interval (ISI) versus parameter γ of the fast maps.

4. Conclusions

The activity of 2DMN networks with SW connections is studied in this paper. In order to assess the effects of topological probability and connection strength on the activity of neural networks, we employ average firing rate $\overline{\eta}$ as a measure of the active frequency. It is found that when the value of connection strength is too small or too large, there are no active neurons in network, no matter what the value of topological probability is. For a given appropriate topological probability, there exists an intermediate range of connection strength such that the firing activity of 2DMN network with SW connections is aroused and enhanced. On the other hand, for a given appropriate connection strength level, there exists an intermediate value of topological probability where the firing activity of coupled neurons reaches its maximum. Furthermore, the effects of noise and transmission delay on firing activity of neural network are also studied. To the best of our knowledge, this is a novel mechanism for the firing activity to emerge in discrete-time neurons. Our results may provide a useful tip on understanding the principles and the mechanisms behind the collective dynamics in coupled real neurobiological systems.

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