Discrete-time movement model of a group of trains on a rail line with stochastic disturbance*

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This paper presents a discrete-time model to describe the movements of a group of trains, in which some operational strategies, including traction operation, braking operation and impact of stochastic disturbance, are defined. To show the dynamic characteristics of train traffic flow with stochastic disturbance, some numerical experiments on a railway line are simulated. The computational results show that the discrete-time movement model can well describe the movements of trains on a rail line with the moving-block signalling system. Comparing with the results of no disturbance, it finds that the traffic capacity of the rail line will decrease with the influence of stochastic disturbance. Additionally, the delays incurred by stochastic disturbance can be propagated to the subsequent trains, and then prolong their traversing time on the rail line. It can provide auxiliary information for rescheduling trains when the stochastic disturbance occurs on the railway.

Keywords: train movement, discrete-time model, simulation, stochastic disturbance

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1. Introduction

The train control problem is a basic problem in the rail traffic, the purpose of which is to determine the velocity and distance profiles when the train traverses on the rail line. Up to now, many researchers[1−7] have studied the optimal control strategy of a train. For instance, Khmelnitsky[1] investigated the operation of a train on a variable grade profile subject to arbitrary speed restrictions. The aim of this problem is to determine a detailed program for traction and brake applications, then minimize the energy consumption on a given route. Liu and Golovitcher[2] presented an analytical process of computing the optimal operation of a rail vehicle to save energy. Effati and Roohparvar[3] presented two methods to solve the energy saving control problem. Howlett[4] investigated how to determine an optimal driving strategy in a train control problem with a generalized equation of motion. Howlett and Pudney[5] investigated train optimal control problem to minimize the energy consumption in different conditions. Howlett and Cheng[6] considered optimal driving strategies for a train on a track with continuously varying gradient. Wang *et al.*[7] designed a novel automatic train operation algorithm based on iterative learning control theory.

It is easy to see that the above researches are mainly focused on the movement of a single train and the interaction with its adjacent trains is not discussed. However, since a group of trains will traverse on the rail line in the real traffic system, it is necessary for us to guarantee the safety of trains' operations. For this purpose, two control systems, i.e., the fixed-block control system and the moving-block control system, are employed on the rail line. In the fixed-block signalling system, the railway will be divided into different physical segments by signal lights. The safe headway can be guaranteed if the following conditions are satisfied: (i) only one train is allowed to traverse on each segment at any time; (ii) there is at least one margin segment between two adjacent trains. The working principle of the moving-block system is much simple, but its realization and reliability are dependent to a deep degree on the advanced communication techniques and equipments. In this control system, the safe headway will be guaranteed by checking the actual headway of adjacent trains in real time and

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then giving corresponding safe operational directions to the following train. When a group of trains traverse on the rail line, the interaction between them needs to be taken into account. Then, it is necessary for us to design the reasonable operational strategies for trains. For this purpose, Li and Gao,[8] Li *et al.*[9] presented cellular automata models to simulate the movements of trains on a rail line. Li *et al.*[10] proposed a new cellular automata model for train movement simulations to reduce energy consumption. Xun *et al.*[11] presented a train-following model for the moving block system with different types of running trains. Li and Guan[12] used a random walk model to simulate the dynamic characteristics of train traffic flow on a railway network. Additionally, for the operational planning of a group of trains, we can refer to Yang *et al.*,[13] Zhou and Zhong,[14] and other references.

We can see from the literature that most researchers consider the train control problem under the certain environment. That is, trains traverse on the rail line without occurrence of disturbance. Nevertheless, there exist a lot of uncertain (or called irregular) events in the real traffic system, such as geological disaster, bad weather, train derailment, equipment failure, etc. These irregular events can disturb, or more seriously, break down the local train traffic flow, and then cause the relevant trains to be late to their destinations. For such a case, it is necessary to reschedule some involved trains in the railway network based on the information of the impacted traffic flow. To investigate the behaviours of traffic flow, Tang *et al.*[15] presented a new car-following model by taking into account the effects of the traffic interruption probability on the car-following behaviour of the following vehicle. However, for the railway traffic, no relevant researches have appeared in literature and the behaviours of trains' movements are unknown when the stochastic disturbance appears. In this paper, we shall explore the dynamic characteristics of train traffic flow when irregular events occur. This is a new idea which has not appeared in literature.

The rest of this paper is organized as follows. In Section 2, two control signalling systems, i.e., the fixed-block signalling system and the moving-block signalling system, are introduced in detail. Also, an improved method of calculating the minimal safe headway is introduced to increase the traffic capacity of the rail line. Section 3 presents a discrete-time dynamic model for the movements of a group of trains, in which the traction operation and the braking operation are performed based on the actual headway of adjacent trains. In Section 4, some numerical simulations are performed to show the dynamic characteristics of train traffic flow when the stochastic disturbance occurs. Finally, the main work of this research is summarized in the conclusion.

2. Train control signalling systems

In the railway traffic system, there are two kinds of signalling systems for the safe operations of trains, that is, the fixed-block signalling system and the moving-block signalling system. As a traditional control method, the fixed-block signalling system, in which the rail line will be divided into different physical segments by signal lights, has been employed more than one hundred years on almost all railways. In this system, only one train is allowed to traverse on each segment at every time, and at least one margin segment should be left between adjacent trains to guarantee their safety. Trains travel on the rail line according to the following operation strategies. That is, if a train arrives at a signal light and its previous train has not left the second previous segment, then this train must stop before the signal light to avoid the possible occurrence of conflicts; otherwise, the train is allowed to enter the next segment. Although the fixed-block signalling system can guarantee the safe operations of trains, the traffic capacity of the rail line cannot be utilized sufficiently. To overcome this drawback, the moving-block signalling system is designed with the development of mobile communication techniques. In this system, the rail line will not be divided into physical segments by signal lights. The safe headway is guaranteed by checking the distance between adjacent trains in real time and then controlling the following train on the basis of its actual headway. The required safe headway between adjacent trains, which generally consists of the braking distance and the safety margin space, is dependent on the current speed of the following train. In this system, if the actual headway between adjacent trains is larger than the minimal safe headway, the following train can perform the traction operation. Otherwise, the following train needs to perform the braking operation. With the development of communication techniques, more and more railways

will be equipped with the moving-blocking systems to utilize the railway traffic sources more efficiently.

In this paper, we shall investigate the characteristics of train traffic flow on the railway with the moving-block signalling system. In literatures, some kinds of moving-block schemes had been presented, including moving space block, moving time block and pure moving block. Among the different schemes, the moving space block is the simplest, in which the minimal safe headway is usually calculated as follows:

$$h_{\min} = \frac{V_{\max}^2}{2\alpha} + SM, \tag{1}$$

where $V_{\max}$ is the maximum speed of the following train, α is the deceleration of the following train, and SM is the safety margin space. The first term on the right-hand side of Eq. (1) is actually the maximum braking distance with the fixed deceleration α (for more details of this kind of headway, we can refer to Li and Gao[8]). However, in the real applications, the deceleration is not a constant indeed, which will vary with respect to the train's current velocity. Furthermore, the braking distances are also different for different velocities. Thus, it is more reasonable that the minimal safe headway should not be treated as a fixed quantity. If the braking distance at speed v is represented by a function $b(v)$, then the real-time minimal safe headway can be reformulated as follows:

$$h_{\min}(v) = b(v) + SM. \tag{2}$$

In applications, this modification can not only guarantee the safety of trains' movements, but also increase the traffic capacity of the railway system. Thus, we shall use this formula to compute the minimal safe headway for two adjacent trains in this research.

3. Discrete time movement model with stochastic disturbance

Suppose that K trains will traverse on a rail line in the same direction. For the convenience of treatment, the trains are denoted by $1, 2, \ldots, K$ in order of departure times. With the moving-block signalling system, we construct the dynamic equation of this group of trains as follows:

$$\begin{aligned}
&s_k(t) - s_{k+1}(t) \geq h_{\min}(v_{k+1}(t)), \quad k = 1, 2, \ldots, K-1,\\
&\mathrm{d}s_k(t)/\mathrm{d}t = v_k(t), \quad k = 1, 2, \ldots, K,\\
&\mathrm{d}v_k(t)/\mathrm{d}t = (F_{km}(v_k(t)) - F_{kb}(v_k(t)) - F_{kr}(v_k(t)))/M_k, \quad k = 1, 2, \ldots, K,\\
&t \in [0, T].
\end{aligned} \tag{3}$$

In this model, notation $s_k(t) \in [0, L]$ denotes the location of train k at time t, which is the distance from the initial station; $t \in [0, T]$ denotes the current time; $v_k(t) \in [0, V_{\max}^k]$ denotes the actual velocity of train k at time t; $F_{km}(v_k(t))$ denotes the traction of train k, which is a function with respect to $v_k(t)$; $F_{kb}(v_k(t))$ denotes the braking strength of train k, which is a function with respect to $v_k(t)$; $F_{kr}(v_k(t))$ denotes the total resistance of train k, which is a function with respect to $v_k(t)$ and consists of wheel resistance, rail resistance, air resistance, etc.; M_k denotes the quality of train k. The first inequality in the model is a headway constraint in the traffic system.

We can discretize the time dimension to simulate the movements of trains, where the increment of time is denoted by Δt. Then the discrete-time form of model (3) can be formulated as follows:

$$\begin{aligned}
&s_k(t + \Delta t) - s_{k+1}(t + \Delta t) \geq h_{\min}(v_{k+1}(t + \Delta t)), \quad k = 1, 2, \ldots, K-1,\\
&s_k(t + \Delta t) = s_k(t) + v_k(t)\Delta t, \quad k = 1, 2, \ldots, K,\\
&v_k(t + \Delta t) = v_k(t) + \Delta t(F_{km}(v_k(t)) - F_{kb}(v_k(t)) - F_{kr}(v_k(t)))/M_k, \quad k = 1, 2, \ldots, K,\\
&t, t + \Delta t \in [0, T].
\end{aligned} \tag{4}$$

After considering the average velocity in time increment Δt, we can rewrite the above model as follows for obtaining more accurate traveling distance:

$$s_k(t + \Delta t) - s_{k+1}(t + \Delta t) \geq h_{\min}(v_{k+1}(t + \Delta t)), \quad k = 1, 2, \ldots, K-1,$$

$$
\begin{aligned}
&s_k(t+\Delta t) = s_k(t) + \frac{1}{2}(v_k(t) + v_k(t+\Delta t))\Delta t, \quad k = 1, 2, \ldots, K, \\
&v_k(t+\Delta t) = v_k(t) + \Delta t(F_{km}(v_k(t)) - F_{kb}(v_k(t)) - F_{kr}(v_k(t)))/M_k, \quad k = 1, 2, \ldots, K, \\
&t, t+\Delta t \in [0, T]. \qquad (5)
\end{aligned}
$$

In this paper, we shall use model (5) to determine the physical characteristics of trains in different time, including acceleration, velocity, location, etc. To simulate the movements of trains, it is necessary for us to design the detailed operational strategies at different simulation times. Note that a train can be influenced directly by its previous adjacent train. We only introduce the traversing strategies for two adjacent trains in the following.

Suppose that train $k-1$ is the previous train and train k is its following train. Then, the operation of train k completely depends on the movement of train $k-1$. On the journey of train k, the relevant operations include traction operation, braking operation, entering and dwelling at a station. On the journey of a train, we need to compute the braking distance to obtain the minimal safe headway at each simulation time. Then, we can determine what kind of operation should be employed at this simulation time. The movement of train k is according to the following strategies at simulation time t.

Step 1 If it is the time for train k to brake for entering the station, then perform the braking operation in the following simulation times until it arrives at the station;

Step 2 If train k will be influenced by stochastic disturbance, then perform the braking operation for a random generated T simulation times;

Step 3 If the actual headway is larger than the minimal safe headway, then perform the traction operation at simulation time t;

Step 4 If the actual headway is less than or equal to the minimal safe headway, then perform the braking operation at simulation time t.

In the following, we give some assumptions in the process of simulation.

(I) In the distance interval $[0, SM]$, stochastic disturbance will not appear to influence the movement of the corresponding train.

(II) When a train detects the site where it should brake for entering the station, stochastic disturbance will not occur to influence its movement.

(III) To investigate the train traffic flow more easily, we suppose that there is only one platform at each station for loading and unloading passengers.

(IV) When a train arrives at its destination, it will not be considered any more and its following train will become the head train in the train traffic flow.

In the following, the detailed operational strategies will be designed for the movements of trains in the discrete-time movement model.

Strategy 1 Traction operation

At time t, if the actual headway between train $k-1$ and train k is larger than the minimal safe headway, then the following train k will perform the traction operation. In the simulation process, the time will be discretized and the time increment is denoted by Δt. In time interval $[t, t+\Delta t]$, we need to compute the traveling distance of train k and determine the location of this train on the rail line. When the traction operation is employed, the train will accelerate if the traction strength is larger than the resistance. Generally, there is a speed limit on each railway section for each train. We use the following method to control train's speed within the range of allowed velocity. Suppose that the velocity of train k at time t is $v_k(t)$ and the acceleration is $a_k(t)$, then the velocity at time $t+\Delta t$ is

$$
v_k(t+\Delta t) = \begin{cases} v_k(t) + a_k(t)\Delta t, & \text{if } v_k(t) + a_k(t)\Delta t \le V_{\max}^k, \\ v_k(t), & \text{otherwise.} \end{cases} \qquad (6)
$$

In this formula, if the velocity at time $t+\Delta t$ obtained by acceleration is larger than the maximal velocity, the present velocity of train k is supposed to be equal to the velocity at time t. That is, the train will traverse with a uniform speed in this time interval and the acceleration is zero. We use the following process to compute the traveling distance in time interval $[t, t+\Delta t]$ and determine its location on the rail line.

Step 1 Compute the traction strength $F_{km}(v_k(t))$ of train k;

Step 2 Compute the resistance $F_{kr}(v_k(t))$ of train k;

Step 3 Compute the acceleration $a_k(t)$ of train k by $(F_{km}(v_k(t)) - F_{kr}(v_k(t)))/M_k$;

Step 4 Compute the velocity of train k at time $t+\Delta t$ by Eq. (6);

Step 5 Compute traveling distance in this time interval by $\Delta S_k(t) = v_k(t)\Delta t + a_k(t)\Delta t^2/2$;

Step 6 Let $S_k(t+\Delta t) = S_k(t) + \Delta S_k(t)$.

Strategy 2 Braking operation

Generally, if the actual headway at time t is less than or equal to the minimal safe headway, the following train k needs to perform the braking operation in time interval $[t, t+\Delta t]$. To simulate the braking operation, we also discretize the time dimension by a time increment Δt. In this operation, the combination of strength acting on the train is the sum of the braking strength and the total resistance. When the braking operation is employed, the train will decelerate. The following method will be used to control train's speed. Suppose that the speed at time t is $v_k(t)$, then the speed at time $t+\Delta t$ should be

$$v_k(t+\Delta t) = \max\{v_k(t) + a_k(t)\Delta t, 0\}. \tag{7}$$

If $v_k(t+\Delta t) = 0$, then $a_k(t)$ will be regarded as zero in the simulation time t approximately. The traversing distance in time interval $[t, t+\Delta t]$ and the location of train k on the rail line can be simulated as follows:

Step 1 Compute the braking strength $F_{kb}(v_k(t))$ of train k;

Step 2 Compute the resistance $F_{kr}(v_k(t))$ of train k;

Step 3 Compute the acceleration $a_k(t)$ of train k by $-(F_{kb}(v_k(t)) + F_{kr}(v_k(t)))/M_k$;

Step 4 Compute the velocity of train k at time $t+\Delta t$ by Eq. (7);

Step 5 Compute traveling distance in this time interval by $\Delta S_k(t) = v_k(t)\Delta t + a_k(t)\Delta t^2/2$ if $v_k(t+\Delta t) > 0$ or $\Delta S_k(t) = 0$ if $v_k(t+\Delta t) = 0$;

Step 6 Let $S_k(t+\Delta t) = S_k(t) + \Delta S_k(t)$.

Strategy 3 Braking distance

In the process of simulating a train's movement, braking distance should be frequently calculated to check the headway constraint or determine whether it is time to brake for entering the station. Suppose that the speed of train k is V at time t, then its braking distance refers to the distance that train k traverses from speed V to zero by the braking operation. Note that the combination of strength in the braking operation is the sum of braking strength and resistance. Thus, according to the physical characteristics, the braking distance at velocity V is the same as the distance traveled by the traction operation from speed zero to V with the same combination of strength. We use the following procedure to simulate the braking distance:

Step 1 Let $S = 0$, $t = 0$, $v_k(0) = 0$;

Step 2 Compute the braking strength $F_{kb}(v_k(t))$ and the resistance $F_{kr}(v_k(t))$ of train k;

Step 3 Compute the acceleration $a_k(t)$ of train k by $(F_{kb}(v_k(t)) + F_{kr}(v_k(t)))/M_k$;

Step 4 Compute the velocity of train k after time Δt by $v_k(t+\Delta t) = v_k(t) + a_k(t)\Delta t$;

Step 5 Compute the traveled distance by $S = S + v_k(t)\Delta t + a_k(t)\Delta t^2/2$, let $t \leftarrow t+\Delta t$. If $v_k(t) < V$, go to Step 2; otherwise, output S.

Strategy 4 Stochastic disturbance

Due to the uncertainty of the real traffic system, the operation of each train can be disturbed when some uncertain (or called irregular) events occur. In the model, we suppose that there is a disturbance probability $P_{\rm d}$ at each simulation time for each train. To investigate the characteristics of train traffic flow, we suppose that once a train is influenced by an irregular event, the relevant train should perform braking operation in the following T simulation times, where T is a randomly generated integer in a given range $[0, U]$. The procedure of determining an irregular event is listed as follows:

Step 1 At time t, randomly generate a number s in $[0, 1]$;

Step 2 If $s \le P_{\rm d}$, randomly generate an integer T in $[0, U]$;

Step 3 Perform the braking operation in the next T simulation times.

Strategy 5 The operation of entering the station

Note that there are some stations on the rail line. We suppose that all trains need to stop at each station to unload and load passengers. Since there exists only one platform at each station for trains' operations, only one train is allowed to enter the station at each simulation time. That is, when the station is occupied by a train, its following train needs to stop at least a safe headway distance away from this train. In the simulation, it is necessary for us to determine whether it is the suitable time to brake for entering the station. At simulation time t, if the current braking distance is equal to or approximately equal to (because of simulation error) the distance from the current location to the next station, then time t is regarded as the suitable time and the braking operation will be performed continuously until the train arrives at the

station. Additionally, in the process of braking for entering the station, we assume that irregular events will not occur to influence the movements of trains.

4. Numerical simulation

In this section, some numerical simulations will be performed to investigate the characteristics of train traffic flow under the stochastic disturbance. The tested rail line consists of four stations, denoted by 1, 2, 3, 4, respectively, and three sections, denoted by $(1,2),(2,3),(3,4)$, respectively. The length of each section is listed as follows: $l_{12} = 20$ km, $l_{23} = 20$ km and $l_{34} = 20$ km. All trains will traverse from station 1 to station 4. At each station, the dwelling time for unloading and loading passengers is assumed to be 100 s. We suppose that the quality of each train is 665 t although it can vary slightly after unloading or loading passengers. The maximum speed on each railway section is $V_{\max}^k = 200$ km/h, $k = 1, 2, \ldots, K$.

Without loss of generality, we suppose that the locomotives of the involved trains are of the same types. The traction (kN) varies with respect to velocity according to the following equation:

$$F_{\rm km}(v) = \begin{cases} 360, & \text{if } 0 \le v \le 180 \text{ km/h}, \\ -\dfrac{6}{7}(v-180)+360, & \text{if } 180 \text{ km/h} \le v \le 200 \text{ km/h}. \end{cases}$$

The resistance (kN) with respect to velocity varies according to the following function

$$F_{\rm kr}(v) = 11.4 + 0.101v + 0.001269v^2.$$

The braking force (kN) varies according to the following formula:

$$F_{\rm kb}(v) = \begin{cases} 300 - 0.2v, & \text{if } 0 \le v \le 100 \text{ km/h}, \\ 280 - 1.2(v-100), & \text{if } 100 \text{ km/h} < v \le 200 \text{ km/h}. \end{cases}$$

On the rail line, the moving-block signalling system will be employed. The numerical simulation is performed in a computer by Visual C++ software. In the simulation, the time dimension is discretized and the time increment is supposed to be $\Delta t = 1$ s. For the convenience of analysing the characteristic behaviours of train traffic flow, only head trains will be considered to be influenced by the stochastic disturbance. If a head train arrives at station 4, then this train runs out of the rail traffic system and will not be considered any more. For this case, its following train will be treated as a new head train in the simulation system. In the model, we suppose that the probability of occurrence of stochastic disturbance is 1.0% at each simulation time. When an irregular event occurs, the disturbance time (unit: second) is randomly generated in interval $[0, 300]$.

We test the simulation algorithm under two traveling modes to show the influence of stochastic disturbance. That is,

Traveling Mode 1 (TM1): Trains traverse on the rail line without stochastic disturbance;

Traveling Mode 2 (TM2): Trains traverse on the rail line with stochastic disturbance.

We use two indexes to show the traffic capacity of the rail line in the given simulation time, that is, the number of trains beginning their journeys and the number of trains finishing their journeys. The corresponding results are listed in Table 1, where the meanings of relevant notations are given at the end of the table.

Table 1. The simulation results.

ST/s	SM/m	TBJ1	TFJ1	TBJ2	TFJ2
10000	1000	67	42	65	33
10000	2000	49	34	45	25
10000	3000	41	30	37	22
10000	4000	36	27	32	20
10000	5000	33	25	28	17
10000	6000	30	24	24	14

ST: The total simulation time; SM: The predetermined safety margin space in the minimal safe headway; TBJ1: The number of trains that begin their journeys in TM1; TFJ1: The number of trains that finish their journeys in TM1; TBJ2: The number of trains that begin their journeys in TM2; TFJ2: The number of trains that finish their journeys in TM2.

In the simulation, we consider the traffic capacity of the rail line within 10000 simulation times with the different safety margin spaces. To show the results conveniently, we list the relevant traffic capacities in Fig. 1. It is easy to see from this figure that the smaller the safety margin space is, the larger the traffic capacity will be. This result just coincides with the real conditions of the rail traffic system. Due to the influence of stochastic disturbance, the traffic capacities of two traveling modes are different. Generally speaking, more trains can begin or finish their journeys in the given simulation time if no stochastic disturbance occurs. However, when stochastic disturbance appears, the head trains have to perform the braking operation, which causes the increase of the total traveling time on the rail line. According to the operational strategy, the corresponding delay will be propagated to the following trains. This fact makes the traffic capacity decrease. For instance, when the safety margin space is set to be 3000 m, 41 trains can begin their journeys in the given simulation time when no disturbance is considered. Meanwhile, a total of 30 trains arrive at their destinations. However, if the stochastic disturbance occurs, only 37 trains can begin their journeys, in which only 22 trains reach their destinations.

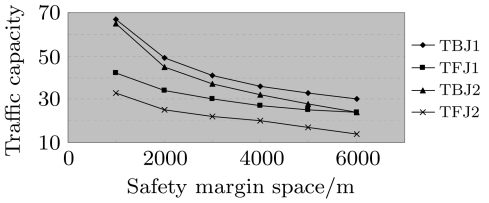


Fig. 1. The traffic capacity of the rail line.

When there exists stochastic disturbance, it is sufficient to decrease the safety margin space of Eq. (2) if we want to improve the traffic capacity. For instance, if no stochastic disturbance occurs, when the safety margin space is randomly generated in interval $[2180, 2404]$, 33 trains can finish their journeys in 10000 simulation times. However, if the stochastic disturbance appears, it is necessary for us to generate the safety margin space in interval $[860, 1034]$ in order to guarantee the same traffic capacity for the traveling mode 2 in 10000 simulation times. This fact also implies that we can compensate for the loss of the traffic capacity by increasing the density of trains on the rail line. The corresponding results can serve as auxiliary information for rescheduling trains when the stochastic disturbance occurs.

To show the dynamic characteristics of the movements of trains, we list two kinds of traffic flow under different traveling modes in Fig. 2, where the safety margin space is supposed to be 3000 m.

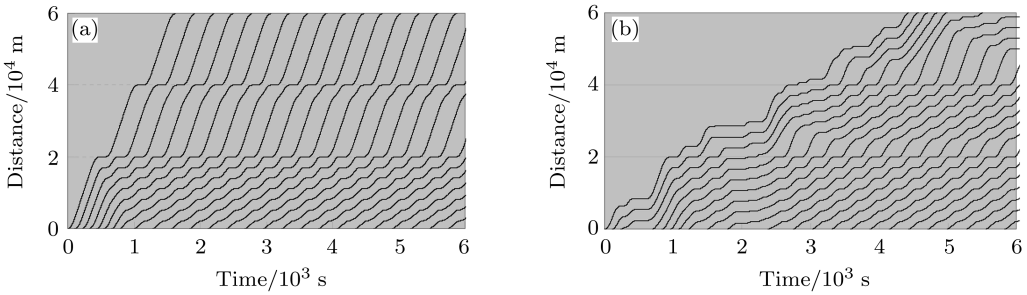


Fig. 2. The train traffic flow.

Figures 2(a) and 2(b) represent the train traffic flow with respect to the traveling modes 1 and 2, respectively. In Fig. 2(a), the x-axis denotes the simulation time and the y-axis denotes the corresponding location on the rail line. On the y-axis, the locations 0 m, 2×10^4 m, 4×10^4 m and 6×10^4 m denote the predetermined stations 1, 2, 3 and 4, respectively. It is easy to see from this figure that all trains can move almost freely on the journey from station 2 to station 4. This fact shows that without the stochastic disturbance, the minimal safe headway between adjacent trains can generally be guaranteed since the dwelling time at stations 2 and 3 is large enough. However, on the journey from station 1 to station 2, the traf-

fic flow is different completely. Note that the safety margin space is 3000 m. A following train will begin its journey when its previous adjacent train departs the initial station 3000 m away. We can see from the figure that when train 1 arrives at station 1, it will dwell at this station for 100 s. Before its departure, the second train has to be delayed before this station according to the moving strategy. For train 3, it will be delayed two times on this section due to the delay of train 2. This tendency will also be propagated to the following trains, which is shown as the wave shapes of traffic flows in the figure.

In the real traffic conditions, some irregular events always occur and thus impact the movements of the subsequent trains. In Fig. 2(b), for analysing the dynamic characteristics of train traffic flow conveniently, we only consider the stochastic disturbance acting on the head trains. In this figure, it is easy to see that all trains will dwell at stations 2 and 3 for 100 seconds for unloading and loading passengers. Before each station, each train will brake for entering the station and then dwell at it for 100 seconds. To keep a safe distance from the previous train, the following train has to brake and stop before the station until the previous train leaves the station. After the departure of the previous train, the following train will accelerate and then brake for entering the station. This law holds for all trains according to the operational strategy when they enter stations 2 and 3. As for the stochastic disturbance, for instance, at simulation time 344 s, the head train is impacted by stochastic disturbance and then perform braking operation for 169 seconds. At simulation time 409 s, the train stops at a site with 8353.76 m away from the initial station. Thus, to keep a safe distance with the head train, the second train has to brake and then stop at the distance 5364.21 m. To avoid the conflict with train 2, train 3 also performs the braking operation and then stops at the distance 2382.96 m. This delay law will be propagated and all the subsequent trains will be influenced. For the other stochastic disturbance, the similar propagation process of delays can also be seen. In the real applications, if the delay time of a train on the rail line is too large, it can be considered to be rescheduled.

Comparing with Fig. 2(a), we find that the characteristic behaviours of Fig. 2(b) change drastically in the simulation. In the first 6000 s of simulation process, 26 trains begin their journeys if no disturbance appears. However, if the disturbance is considered, only 23 trains can begin their journeys. This fact implies that the stochastic disturbance can impact the characteristic of the rail system in a direct way. On the journey of train 1, the irregular events occur 17 times in time window $[0, 6000]$, where the corresponding disturbance time is shown in Fig. 3.

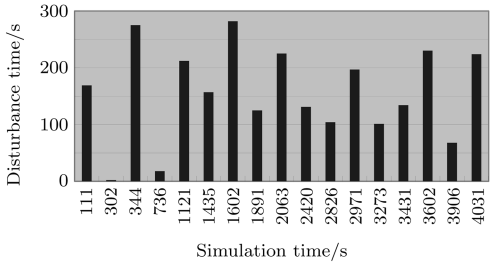


Fig. 3. The disturbance time at each simulation time.

In the stochastic disturbance, the braking time of six irregular events exceeds 200 s. Additionally, the braking time of eight irregular events is in the interval $[100, 200]$ and the braking time of the rest irregular events is not larger than 100 s.

In addition, we give Fig. 4 to show the variation of accelerations for train 1. In Fig. 4(a), the train traverses on the rail line with the traveling mode 1 and the variation of accelerations is regular. At the initial stage of the traction operation, the acceleration is the largest due to the largest combination of strength. With the increase of the velocity, the acceleration will decrease gradually. When the train reaches its maximal speed, it will traverse with a unform speed and the acceleration equals zero. When an appropriate time for braking to enter the station is found, the train will perform the braking operation until it reaches the station. In this process, the acceleration turns negative and the speed will decrease gradually. When the speed turns zero, the train will dwell at the station 100 s and the acceleration in this time interval turns zero. On the next two railway sections, the above process will be repeated and the acceleration varies in a similar way. However, when the stochastic disturbance occurs, the variation of accelerations turn much more complex. Although the accelerations can also be divided into three classes, that is, positive accelerations, negative accelerations and zero accelerations, the general regularity of variation of accelerations has disappeared. Actually, this fact coincides with the real conditions of the rail traffic system. When the acceleration is positive, the train is performing the traction operation. If the acceleration is negative, the train is performing the braking operation. If the acceleration equals

zero, then the train is stopping at some site of rail line or moving with a uniform speed. For instance, in simulation time interval $[1, 110]$, the train moves by the traction operation and the acceleration is positive. At simulation time 111, the train is disturbed by stochastic disturbance and begins to perform the braking operation. Meanwhile, the acceleration turns negative. At simulation time 231 s, the train is stopping at the site where the distance from the initial station is 6378.34 m. For this case, the acceleration equals zero.

In the simulation process, we need to guarantee the minimal safe headway between the adjacent trains. In Fig. 5, we give different headways of trains at simulation time 4433 s. At this time, a total of 18 trains enter the railway line for their journeys. In the figure, the x-axis denotes the indexes of trains and the y-axis denotes the simulated headway between a train and its following train. For instance, the headway of train 1 is 3285.58 m at the time 4433 s, which denotes the simulated headway between train 1 and train 2. In the simulation, it is reasonable that some headway errors appear between different trains due to the existence of the simulation error. For instance, the simulated headway of train 6 in the figure is 2982.6 m at time 4433 s. Since train 6 and its following train are stopping on the railway at this time, the minimal safe headway of train 6 is 3000 m indeed. If we use the following formula to compute the error of the simulated actual headway:

$$\text{error} = \frac{|\text{minimal_headway} - \text{actual_headway}|}{\text{minimal_headway}},$$

the headway error of train 6 is only 0.58%, which is also acceptable for the real applications.

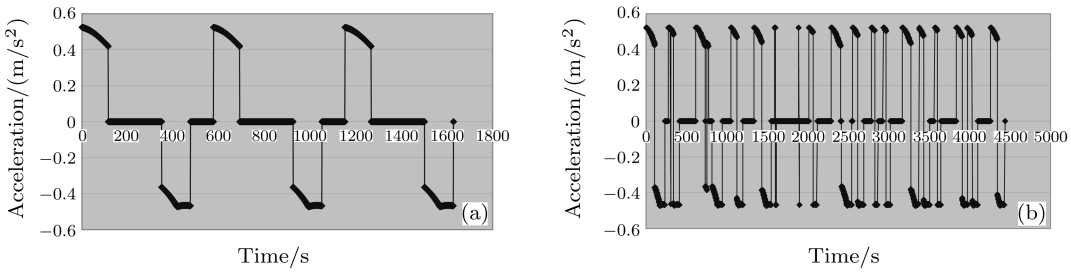


Fig. 4. Time-acceleration curves of train 1.

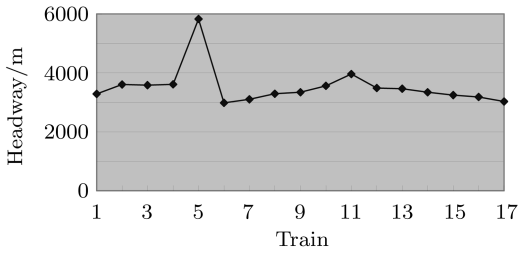


Fig. 5. Headway for different trains.

5. Conclusions

In this paper, a discrete-time dynamic model was constructed to describe the movements of a group of trains on a rail line. In order to simulate trains' movements in a computer, the detailed operational strategies were introduced in the model, including traction operation, braking operation, headway checking, etc. To provide auxiliary information for rescheduling trains when irregular events occur, the model was considered under the stochastic disturbance, in which the train will perform the braking operation when stochastic disturbance occurs. In the operational strategy, the traversing mode of a train is completely dependent on its previous train. That is, if the actual headway is larger than the minimal safe headway, the following train will perform the traction operation; otherwise, the following train needs to perform the braking operation.

To test the effectiveness of the discrete-time dynamic model and analyse the characteristics of the train traffic flow with the stochastic disturbance, a numerical example was simulated in Section 4. The computational results indicate that the discrete-time dynamic model can well describe the movements of a group of trains. The results also show that the stochastic disturbance can not only decrease the traffic capacity of the rail line but also make more delays in the train traffic flow. The law of delay propagation was also analysed in this part.

It is worth while pointing out that the actual traffic environment is always in the railway network. With the stochastic disturbance, the dynamic characteristics of train traffic flow in the railway network can be investigated in the future researches.

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