Effects of memory on scaling behaviour of Kardar–Parisi–Zhang equation*

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In order to describe the time delay in the surface roughing process the Kardar–Parisis–Zhang (KPZ) equation with memory effects is constructed and analysed using the dynamic renormalization group and the power counting mode coupling approach by Chattopadhyay [2009 *Phys. Rev.* E **80** 011144]. In this paper, the scaling analysis and the classical self-consistent mode-coupling approximation are utilized to investigate the dynamic scaling behaviour of the KPZ equation with memory effects. The values of the scaling exponents depending on the memory parameter are calculated for the substrate dimensions being 1 and 2, respectively. The more detailed relationship between the scaling exponent and memory parameter reveals the significant influence of memory effects on the scaling properties of the KPZ equation.

Keywords: memory effect, scaling analysis, self-consistent mode coupling theory

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1. Introduction

The physical significance of the Kardar–Parisi–Zhang (KPZ) equation has become widely recognized in nonequilibrium statistical mechanics, and reads[1]

$$\frac{\partial h(\boldsymbol{r},t)}{\partial t} = \nu\nabla^2 h(\boldsymbol{r},t) + \frac{\lambda}{2}[\nabla h(\boldsymbol{r},t)]^2 + \eta(\boldsymbol{r},t). \quad (1)$$

This equation reflects the competition between the surface tension smoothing force $\nu\nabla^2 h$ and the noise $\eta(x,t)$, which is the Gaussian white noise with zero mean and the short-range correlation described by

$$\langle\eta(\boldsymbol{r},t)\eta(\boldsymbol{r}',t')\rangle = 2D\delta^d(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')\,, \quad (2)$$

where d is the substrate dimension and D specifies the noise amplitude. As a nonlinear generalization of the diffusion equation, it describes many diverse processes, of which the study of stochastically growing surfaces has attracted extensive attention in recent years. The KPZ equation can be applied to the growth of many rough surfaces such as crystal growth, vapour deposition, fluid flow in porous media, and biological growth.[1−3]

The KPZ equation is generally related only to short-range (or local) nature of interaction in the nonlinear process that describes the lateral growth. However, in many actual growth processes the long-range interactions play a nontrivial role in scaling properties. In order to incorporate these long-range interactions into the kinetic roughening of surface, Mukherji and Bhattacharjee[4] proposed the nonlocal KPZ equation with a nonlinear term representing the long-range interactions correlating each site with all other sites. The equation can be described as

$$\frac{\partial h(\boldsymbol{r},\,t)}{\partial t} = \nu\nabla^2 h(\boldsymbol{r},\,t) + \frac{1}{2}\int \mathrm{d}\boldsymbol{r}'\,\vartheta(\boldsymbol{r}')\nabla h(\boldsymbol{r}+\boldsymbol{r}',\,t) \cdot \nabla h(\boldsymbol{r}-\boldsymbol{r}',\,t) + \eta(\boldsymbol{r},t), \quad (3)$$

where the kernel function $\vartheta(\boldsymbol{r}')$ has a short-range part $\sim\lambda_0\delta(\boldsymbol{r})$ and a long-range part $\sim\lambda_\rho r^{\rho-d}$. Several analytic approximations such as scaling analysis (SA),[5] dynamic renormalization group (DRG),[4] and classical self-consistent mode coupling approach (CSCMC)[6,7] were employed to investigate the scaling properties of the nonlocal KPZ equation.

Memory effects in intrinsically nonlinear diffusive dynamics have been a hot subject in recent years. This

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is partly due to the fact that along with dimensionality and associated symmetries, memory, or equivalently time delay, has now been shown to have a highly nontrivial effect both experimentally and theoretically. Recently, Chattopadhyay[8] proposed the continuum model to study a memory-dependent dynamical equation of motion. Instead of a site: site interaction at the same time instant t in nonlocal KPZ equation, in the KPZ equation with memory effects, the fluctuation at a site $\boldsymbol{r}$ at time t interacting with the fluctuation of the same site at some other time $t+t'$ was considered. The consequent equation of motion reads

$$\frac{\partial h(\boldsymbol{r},t)}{\partial t} = \nu\nabla^2 h(\boldsymbol{r},t) + \frac{1}{2}\int \mathrm{d}t'\vartheta(t')\nabla h(\boldsymbol{r},t+t') \cdot \nabla h(\boldsymbol{r},t-t') + \eta(\boldsymbol{r},t), \quad (4)$$

where the first term on the right-hand side is the diffusion term, and the memory kernel $\vartheta(t)$ is power-law correlated and is defined through the relation $\vartheta(t) = \beta_0\delta(t) + \beta_\theta t^{\theta-1}$. In Fourier space, $\vartheta(\omega) = \beta_0 + \beta_\theta\omega^{-\theta}$. Here, θ is just the exponent that describes time decay, and its specific value depends on the physical system at hand. For $\beta_\theta = 0$, conventional KPZ dynamics follows, while for all other values of $\beta_\theta \neq 0$ the system has a non-negligible memory. $\eta(\boldsymbol{r},t)$ is the Gaussian white noise with zero mean and the short-range correlation described by Eq. (2).

Chattopadhyay[8] investigated analytically the scaling properties of the equation by DRG and mode coupling theory in which the mode coupling equations were analysed using the self-consistent power counting scheme (PCS). From these two methods, he found the relationship between scaling exponents and the memory parameter, which showed that the memory effects can impose significant influence on the scaling properties of KPZ equation and the nature of the perturbation generated by the presence of memory has a marked quantitative difference from that generated by other external perturbations. However, the results, in particular the impact of memory effects on roughness exponent, obtained from the two analytical methods, cannot perfectly match each other. In our opinion, the PCS method is not a complete self-consistent mode-coupling approach, and further theoretical analysis is required to clarify the confusion. SA and CSCMC are two reliable methods which have achieved satisfactory results in analysis of Langevin type equations such as KPZ equation and the deformations of the KPZ equation. In this paper, we employ SA and CSCMC to analyse the scaling properties of the KPZ equation with memory effects. We obtain the consistent relationship between the scaling exponents and the memory parameter θ, and the results indicate that the existence of memory effects introduces a new phase into the KPZ equation which reveals more detailed impact of the memory effects on KPZ equation.

2. Scaling analysis of KPZ equation with memory effects

The scaling approach as a Flory-type approach introduced by Hentschel and Family[9] is very similar to the scaling arguments used by Kolmogorov in the analysis of fully developed turbulence, and could be readily applied to Langevin-type growth equations to determine their scaling exponents in any dimension for different scaling regimes. The validity of a scaling regime can be found in a self-consistent manner from the region of length scales, to which the intrinsic assumptions are applied. This approach, in fact, may be considered as a nonequilibrium equivalence of the Flory theory for equilibrium scaling. Some of us promotionally applied the SA method to some typical growth equations such as nonlocal KPZ equation, Lai-Das Sarma-Villain equation, and so on.[5,10–13]

In the spirit of scaling approach, we assume that at long time $t \gg t_l$ (characteristic time $t_l \sim l^z$), and averaged over length scale l, the typical magnitude of the fluctuations in $h(\boldsymbol{r},t)$ scales as h_l, and that these fluctuations last for times of the order t_l. The idea now is to estimate the magnitudes of the individual terms. Basically for any equation such as Eq. (4) to show time scaling, each separate term, when coarse grains disperse over length scale l, must be of the same order of magnitude or negligible. The scaling behaviour can arise only under these circumstances. The various terms in Eq. (4) may be estimated as $\langle|\partial h/\partial t|\rangle_l \sim h_l/t_l$, $\nu\left\langle\left|\nabla^2 h\right|\right\rangle_l \sim \nu h_l/l^2$, and the SR part of the nonlinear term $(\beta_0/2)\left\langle\left|\nabla h^2\right|\right\rangle_l \sim \beta_0 h_l^2/l$, while the LR part may be estimated as $(\beta_\theta/2)\left\langle\left|\int \mathrm{d}t'\vartheta(t')\nabla h(\boldsymbol{r},t+t')\cdot\nabla h(\boldsymbol{r},t-t')\right|\right\rangle_l \sim \beta_\theta t^\theta h^2/l^2$. As for the noise term expressed in Eq. (4),

we can estimate its fluctuations on length scale l and time scale t_l as $\langle|\eta|\rangle_{l,\mathrm{wc}} \sim (D/l^d t_l)^{1/2}$ for smooth surfaces, and $\langle|\eta|\rangle_{l,\mathrm{sc}} \sim (D/h_l^d t_l)^{1/2}$ for rough surfaces, respectively. On the sufficiently large length scale the nonlinear term in Eq. (4) will dominate over the diffusion term, and so the diffusion term is negligible.

In the SR limit, equating the inertial term $\langle|\partial h/\partial t|\rangle_l$ with the nonlinear term, i.e. $h_l/t_l \sim \beta_0 h_l^2/l$, we have $h_l \sim (1/\beta_0)l/t_l$.

In the weak-coupling region where the dimensionless parameter $g_0 \equiv \beta_0^d D/\nu^{d+1} \ll 1$, the inertial term equals the noise fluctuation estimated for smooth surfaces, $\langle|\partial h/\partial t|\rangle_l \sim \langle|\eta|\rangle_{l,\mathrm{wc}}$, which yields $h_l \sim (Dt_l/l^d)^{1/2}$.

Thus we can obtain a fluctuation of h_l scale in time as

$$h_l \sim (D/\beta_0^{d+2})^{1/(d+4)} t_l^{(2-d)/(d+4)} \sim l^{(2-d)/3}, \quad (5)$$

and the scaling exponents in this region can be obtained to be

$$\alpha = \frac{2-d}{3}, \quad \beta = \frac{2-d}{d+4}, \quad z = \frac{d+4}{3}. \quad (6)$$

In the strong-coupling region $g_0 \gg 1$, equating the inertial term to the noise fluctuation estimated for rough surfaces $\langle|\partial h/\partial t|\rangle_l \sim \langle|\eta|\rangle_{l,\mathrm{sc}}$ gives $h_l \sim (Dt_l/l^d)^{1/3}$. Then we can estimate the local derivative h_l and scaling exponents respectively

$$h_l \sim (D/\beta_0^{d+1})^{-1/(d+4)} t_l^{1/(d+2)} \sim l^{2/(d+3)},$$
$$\alpha = \frac{2}{d+3}, \quad \beta = \frac{1}{d+2}, \quad z = \frac{d+2}{d+3}. \quad (7)$$

From Eqs. (6) and (7), we find that normal KPZ equation is recovered.

In the LR limit, equating the inertial term $\langle|\partial h/\partial t|\rangle_l$ with the long-range part of the nonlinear term $(\beta_\theta/2)\left\langle\left|\int \mathrm{d}t' \vartheta(t')\nabla h(\boldsymbol{r}, t+t')\cdot\nabla h(\boldsymbol{r}, t-t')\right|\right\rangle_l$, i.e., $h_l/t_l \sim \beta_\theta h^2/l^{-\theta+1}$, we obtain $\Upsilon_l \sim (1/\lambda_\rho)l^{1-\rho}/t_l$. Similarly, in the weak-coupling region ($g_\rho \equiv \beta_\theta^d D/\nu^{d+1} \ll 1$), equating the noise fluctuation estimated for smooth surfaces to the inertial term, we can easily obtain

$$h_l \sim (D/\beta_\theta^{d+1})^{-1/(d+4)} t_l^{(2-d-d\theta)/(d+4)} \sim l^{(2-d-d\theta)/(2\theta+3)},$$
$$\alpha = (2-d-d\theta)/(2\theta+3),$$
$$\beta = (2-d-d\theta)/(d+4),$$
$$z = (d+4)/(2\theta+3). \quad (8)$$

While in the strong-coupling region ($g_\rho \gg 1$), equating the noise fluctuation estimated for rough surfaces to the inertial term yields

$$h_l \sim (D/\beta_\theta^{d+1})^{1/(d+4)} t_l^{1/(d+2)} \sim l^{2/(d+d\theta+2\theta+3)},$$
$$\alpha = 2/(d+d\theta+2\theta+3), \quad \beta = 1/(d+2),$$
$$z = 2(d+2)/(d+d\theta+2\theta+3). \quad (9)$$

It can be seen that, in the case of time decay exponent $\theta = 0$, equations (8) and (9) in the LR limit, can be reduced to Eqs. (6) and (7) in the SR part, respectively.

3. CSCMC theory of KPZ equation with memory effects

The CSCMC, as a non-perturbative theory, essentially consists in a resummation of the perturbation theory, such that all propagator renormalizations are properly taken into account, while the vertex corrections are neglected entirely. This is seemingly an arbitrary and uncontrolled procedure, however, CSCMS have had success in applications to many areas of condensed matter theory.[14−16] In all those fields, it has been found that CSCMC is capable of describing experiments in a quantitative manner. In the same way, when applied to the dynamic growing functions, CSCMC also has achieved satisfactory results. Frey *et al.*[17] gave systematic analysis of CSCMC by using the field theoretical formulation of Langevin dynamics. Tu[18] solved numerically the mode coupling equations by direct integration to determine the upper critical dimension of the KPZ equation. Colaiori and Moore[19,20] analysed CSCMC for the KPZ equation in the strong coupling regime. By constructing an appropriate exponential ansatz consistent with the asymptotic forms of the correlation and response functions, they discussed the upper critical dimension, dynamic exponents, and scaling functions. From the analytic approximation process itself and the satisfactory results, it can be said that CSCMC is more reasonable and reliable than other numerical methods applied to the mode-coupling equations before. Hu and Tang[6] generalized the application of CSCMC to the nonlocal KPZ equation and determined the scaling exponents as a function of nonlocal parameter. Owing to

the great success in the Langevin equations, we think, the CSCMC approach can be reasonably employed to analyse the scaling properties of KPZ equation with memory effects.

In Fourier space, equations (4) and (2) can be expressed, respectively, as

$$h(\boldsymbol{k},\omega) = h_0(\boldsymbol{k},\omega) + \frac{1}{2}G_0(\boldsymbol{k},\omega)\int_{q,\Omega}\left[-\vartheta(2\omega)\left(\frac{1}{2}\boldsymbol{k}+\boldsymbol{q}\right)\left(\frac{1}{2}\boldsymbol{k}-\boldsymbol{q}\right) \times h\left(\frac{1}{2}\boldsymbol{k}+\boldsymbol{q},\frac{1}{2}\omega+\Omega\right)h\left(\frac{1}{2}\boldsymbol{k}-\boldsymbol{q},\frac{1}{2}\omega-\Omega\right)\right], \tag{10}$$

$$\langle\eta(\boldsymbol{k},\omega)\eta(\boldsymbol{k}',\omega')\rangle = 2D(2\pi)^{d+1}\delta^d(\boldsymbol{k}+\boldsymbol{k}')\delta(\omega+\omega'), \tag{11}$$

where $G_0(\boldsymbol{k},\omega) = 1/(\nu k^2 - \mathrm{i}\omega)$ is the bare response function, the correlation function and the response function are defined in Fourier space as

$$C(\boldsymbol{k},\omega) = \langle h(\boldsymbol{k},\omega)h^*(\boldsymbol{k},\omega)\rangle\,, \tag{12}$$

$$G(\boldsymbol{k},\omega) = \delta^{-d}(\boldsymbol{k}+\boldsymbol{k}')\delta^{-1}(\omega+\omega')\left\langle\frac{\partial h(\boldsymbol{k},\omega)}{\partial\eta(\boldsymbol{k}',\omega')}\right\rangle, \tag{13}$$

where $\langle * \rangle$ indicates an average over η. In the CSCMC approximation,[17,21,22] the correlation function and the response function are the solutions of two coupled equations respectively

$$G^{-1}(\boldsymbol{k},\omega) = G_0^{-1}(\boldsymbol{k},\omega) - \int_{q,\Omega}\vartheta^2(\omega)(\boldsymbol{q}\cdot\boldsymbol{k})[\boldsymbol{q}(\boldsymbol{k}-\boldsymbol{q})]C(\boldsymbol{q},\Omega)G(\boldsymbol{k}-\boldsymbol{q},\omega-\Omega), \tag{14}$$

$$C(\boldsymbol{k},\omega) = C_0(\boldsymbol{k},\omega) + \frac{1}{2}\left|G(\boldsymbol{k},\omega)\right|^2\int_{q,\Omega}\vartheta^2(\omega)[\boldsymbol{q}(\boldsymbol{k}-\boldsymbol{q})]^2C(\boldsymbol{q},\Omega)C(\boldsymbol{k}-\boldsymbol{q},\omega-\Omega), \tag{15}$$

where $C_0(\boldsymbol{k},\omega) = 2D_0\left|G(\boldsymbol{k},\omega)\right|^2$ is the bare correlation function, in the strong-coupled regime, the response function and the correlation function have the following scaling forms respectively:

$$G(\boldsymbol{k},\omega) = k^{-z}g(\omega/k^z), \quad C(\boldsymbol{k},\omega) = k^{-\Delta}n(\omega/k^z), \tag{16}$$

where $g(x)$ is a complex function and $n(x)$ is a real function. Substituting the above scaling forms into Eqs. (14) and (15), and keeping only the leading terms in the limit $\omega,\ k\to 0$ while keeping ω/k^z finite, we can find that the exponents Δ and z have to satisfy the following relation:

$$\Delta + z(1+2\theta) = d+4, \tag{17}$$

the Galilean invariance of Eq. (4) makes the scaling exponents α and z satisfy the following identical equation[23]

$$\alpha + z(1+\theta) = 2. \tag{18}$$

From Eqs. (17) and (18), we obtain

$$\Delta = 2\alpha + d + z. \tag{19}$$

This exponent relation indicates that the correlation function obtained here has the same scaling form as the one in the KPZ equation. Substituting the above scaling forms into the mode coupling equations (14) and (15), we can find that in the strong coupling region, the scaling functions $g(x)$ and $n(x)$ ought to satisfy the following equations respectively:

$$g^{-1}(x) = -\mathrm{i}x + I_1(x), \tag{20}$$

$$n(x) = \left|g(x)\right|^2 I_2(x), \tag{21}$$

where $x = \omega/k^z$, and $I_1(x)$ and $I_2(x)$ are given respectively by

$$I_1(x) = P\int_0^\pi \mathrm{d}\theta \sin^{d-2}\theta \int_0^\infty \mathrm{d}q\, q^{(1-2\alpha)}(\cos^2\theta - q\cos\theta) r^{-z} x^{-2\theta} \int \mathrm{d}y n(y) g\left[\frac{x-yq^z}{r^z}\right], \tag{22}$$

$$I_2(x) = \frac{P}{2}\int_0^\pi \mathrm{d}\theta \sin^{d-2}\theta \int_0^\infty \mathrm{d}q\, q^{(1-2\alpha)} r^{-(2\alpha+d+z)} x^{-2\theta}(\cos\theta - q)^2 \int \mathrm{d}y\, n(y) n\left[\frac{x-yq^z}{r^z}\right] \tag{23}$$

with $P = \beta_\theta^2 S_{d-1}/(2\pi)^{d+1}$, $r^2 = q^2 - 2q\cos\theta + 1$, S_d being the surface area of the d-dimensional sphere.

The solution $z = z(d)$ requires the consistency of Eqs. (20) and (21) on matching both sides at an arbitrarily chosen value of x. Because of the nonlocalities of Eqs. (20) and (21), the matching condition depends on the forms of the functions $g(x)$ and $n(x)$ for all x's. The key approach to obtaining the dynamic exponent z is to assume that the scaling functions $g(x)$ and $n(x)$ have simple power forms which exhibit the same asymptotic behaviours as the ones in Eqs. (20) and (21) in the scaling limit $x \to \infty$[20] respectively. As $x \to \infty$, the integrals $I_1(x)$ and $I_2(x)$ in Eqs. (22) and (23) are controlled by regions where $q \sim x^{1/z}$, so by simple power counting, the following large x asymptotic behaviours for $g(x)$ and $n(x)$ can be obtained respectively:

$$n(x) \sim x^{-1-\beta/z}, \tag{24}$$

$$g_R(x) = \mathrm{Re}[\,g(x)] \sim x^{-1-2/z}, \quad g_I(x) = \mathrm{Im}[\,g(x)] \sim x^{-1}, \tag{25}$$

where $\beta = 4 + d - 2\theta z - 2z$.

In Fourier space, equations (20) and (21) can be written as

$$\frac{\hat{g}_R}{|g|^2}(p) = \hat{I}_1(p), \tag{26}$$

$$\frac{\hat{n}}{|g|^2}(p) = \hat{I}_2(p), \tag{27}$$

where $\hat{I}_1(p)$ and $\hat{I}_2(p)$ are the Fourier transforms of $I_1(x)$ and $I_2(x)$ respectively, and expressed as

$$\hat{I}_1(p) = 2\pi P\int_0^\infty \mathrm{d}\theta \sin^{d-2}\theta \int_0^\infty \mathrm{d}q\, q^{1-2\alpha-2\theta z}(\cos^2\theta - q\cos\theta)\hat{n}(pq^z)\hat{g}_R(pr^z), \tag{28}$$

$$\hat{I}_2(p) = \pi P\int_0^\infty \mathrm{d}\theta \sin^{d-2}\theta \int_0^\infty \mathrm{d}q\, q^{1-2\alpha-2\theta z} r^{-(2\alpha+d)}(\cos\theta - q)^2 \hat{n}(pq^z)\hat{n}(pr^z). \tag{29}$$

According to Ref. [19], the scaling functions $\hat{g}(p)$ and $\hat{n}(p)$ are assumed respectively to have the following power forms:

$$\hat{g}(p) = 2C\theta(p)\exp(-|p|^{2/z}), \tag{30}$$

$$\hat{n}(p) = A\exp(-|Bp|^{\beta/z}), \tag{31}$$

where $C = 1/2$, A and B are parameters depending on d, z, and θ. In x space, for large x limit, these hypothetic forms can be easily translated into the following forms:

$$g(x) \simeq D x^{-1-2/z} + \mathrm{i}x^{-1}, \tag{32}$$

$$n(x) \simeq D x^{-1-\beta/z}, \tag{33}$$

where D is a constant. Obviously the ansatzes of the scaling functions have the same asymptotic behaviours as those of Eqs. (20) and (21) in large x limit as expected. At the same time, in large x limit, equivalently $p \to 0$ in Fourier space, $|g(x)|^{-2} \approx x^2$, so the terms on the left-hand sides in both Eqs. (26) and (27) are dominated by the terms $\mathrm{d}^2\hat{g}_R/\mathrm{d}p^2$ and $\mathrm{d}^2\hat{n}/\mathrm{d}p^2$ respectively. Accordingly, one may have

$$\frac{z-2}{z^2} = \lim_{p\to 0}|p|^{2-2/z}\,\hat{I}_1(p), \tag{34}$$

$$-\frac{AB^{\beta/z}\beta(\beta-z)}{z^2} = \lim_{p\to 0}|p|^{2-\beta/z}\,\hat{I}_2(p). \tag{35}$$

By performing the ansatzes, Eqs. (30) and (31) of the scaling functions in Eqs. (28) and (29) and taking the limit $p \to 0$, the following coupled equations are given:

$$PAS_d' = \frac{d(z-2)}{\pi z^2}\frac{1}{I(B,z)}, \tag{36}$$

$$PAS'_d = B^{\beta/z}\frac{\beta(z-\beta)}{\pi z^2 G(B,z,\theta)}, \tag{37}$$

where

$$S'_d = \int_0^\pi \mathrm{d}\theta \sin^{d-2}\theta, \quad \beta = 4 + d - 2\theta z - 2z,$$
$$I(B,z) = \int_0^\infty \mathrm{d}s s^{2z-3}(1-2s^2)\exp(-B^{\beta/z}s^\beta - s^2),$$
$$G(B,z,\theta) = \int_0^\infty \mathrm{d}s s^{-5+4z-d+2\theta z}\exp^2(-B^{\beta/z}s^\beta).$$

In Eqs. (36) and (37), A is a constant, making discussion similar to that in Refs. [6] and [19], in the simplest scenario, the relation $B = B(z,d)$ can be taken to be $B = [2(2-z)]^{-1}$. So the dynamic exponent z depending on dimension d and memory parameter θ can be figured out by solving Eqs. (36) and (37). At first, the values of dynamic exponent z depending on memory parameter θ in the cases of 1- and 2-dimensions, i.e., $d = 1$, 2, are obtained by calculating Eqs. (36) and (37) numerically. And then utilizing the equation $\alpha + z(1+\theta) = 2$ which originates from the Galilean invariance of Eq. (4), the values of roughness exponent α depending on memory parameter θ in the cases of 1- and 2-dimensions, i.e., $d = 1$, 2, are obtained.

4. Discussion and conclusion

Figure 1 shows the plot of dynamic exponent z versus memory parameter θ obtained from CSCMC and SA compared with PCS results[8] for $d = 1$. As can be seen in Fig. 1, the critical exponent z in the CSCMC case increases slowly when θ changing form 0 to 0.1, and then declines when $\theta > 0.1$. However, the values are decreasing monotonically in corresponding SA and PCS results.

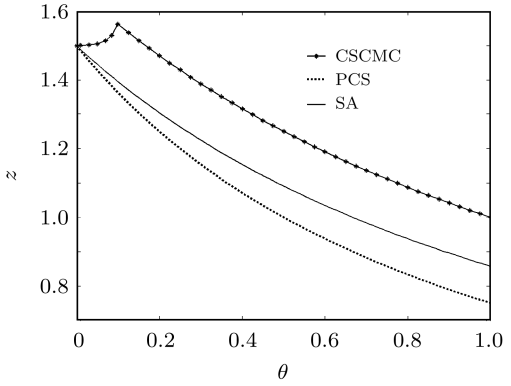


Fig. 1. Dynamic exponents each as a function of memory parameter obtained from CSCMC and SA for $d = 1$, compared with PCS results.

In Fig. 2, we show the relations between z and θ in the case of $d = 2$ based on CSCMC, SA and PCS. Similar to the case of $d = 1$, z decreases monotonically with θ in PCS and SA, and there is an inflection point in CSCMS, differ from the case of $d = 1$, the inflexion appears at $\theta = 0.2$.

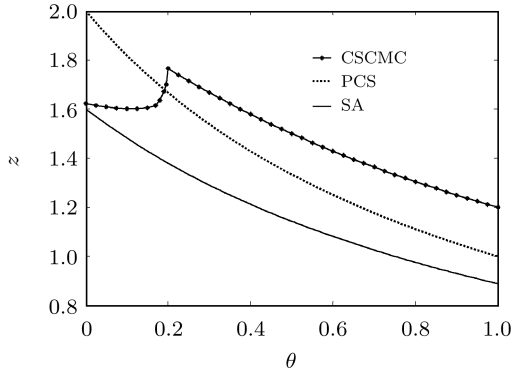


Fig. 2. Dynamic exponents each as a function of memory parameter obtained from CSCMC and SA for $d = 2$, compared with PCS results.

Figures 3 and 4 show the curves for roughness exponent α against memory parameter θ obtained from SA, CSCMS and PCS in the cases where $d = 1$ and 2 respectively. As can be seen easily, the values of α obtained here decrease with θ in CSCMC and SA. However, the values of α remain independent of θ from PCS in the cases where $d = 1$ and $d = 2$. The reason for the difference between PCS and CSCMC results is that the former method is not a complete self-consistent mode coupling approach but only of analysing the coupling equations by power counting scheme. Using CSCMC, our results show the inflexions ($\theta = 0.1$ for $d = 1$ and $\theta = 0.2$ for $d = 2$) signify that the KPZ equation with memory effects can obtain a new phase which is similar to the long range correlation in the nonlocal KPZ equation.[6]

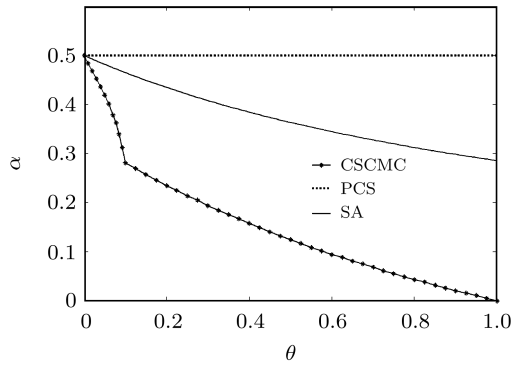


Fig. 3. Roughness exponents each as a function of memory parameter obtained from CSCMC and SA for $d = 1$, compared with PCS results.

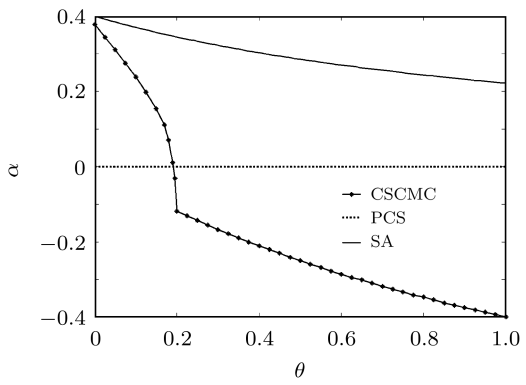


Fig. 4. Roughness exponents each as a function of memory parameter obtained from CSCMC and SA for $d = 2$, compared with PCS results.

In summary, we investigated the scaling properties of the KPZ equation with memory effects to determine dynamic scaling based on SA approach and CSCMC theory. The variations of the scaling exponents with the memory parameter θ from the two methods match each other at a semiquantitative level. The results reveal that memory effects can lead to nontrivial scaling properties of the surface fluctuations in the KPZ equation. Our analysis also shows that SA and CSCMC theories employed here are applicable and effective approaches to determining the dynamic scaling behaviour. As a matter of fact, the necessary omissions in the CSCMC theory and the relatively rough approximation in SA approach cause the difficulty in providing a physical insight into the scaling behaviour of the KPZ equation including memory effects, which needs more research in this direction, by analytical study, numerical study, and introducing new discrete growth models.

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