Anti-synchronization of a new hyperchaotic system via small-gain theorem*

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(Received 8 February 2010; revised manuscript received 25 April 2010)

Based on the small-gain theorem, the anti-synchronization between two identical new hyperchaotic systems is investigated, moreover, the general sufficient conditions to achieve anti-synchronization between the new hyperchaotic system and the new hyperchaotic Lorenz system are obtained via small-gain theorem. Numerical simulations are performed to verify and illustrate the analytical results.

Keywords: hyperchaotic system, anti-synchronization, small-gain theorem

PACC: 0545

1. Introduction

Since the first hyperchaotic system (Rössler system)[1] was presented for a model of particular chemical reaction, many hyperchaotic systems have been developed such as the hyperchaotic Lorenz system[2,3] and the hyperchaotic Chen system.[2,3] Recently, there has been increasing interest in chaos synchronization of the hyperchaotic systems.[4−6] So far, different types of chaos synchronization of hyperchaotic systems have been studied, such as generalized synchronization,[7] lag synchronization,[8] anti-synchronization.[9−11] Among cycling chaotic systems, anti-synchronization is one of the most interesting problems.[9−11]

The aim of this study is to investigate the chaos anti-synchronization of a new hyperchaotic system which was introduced by Chen *et al.*[12] This new hyperchaotic system is a four-dimensional autonomous system, which is derived from the Chen–Lee system.[13] Chen *et al.*[12] studied the chaos hybrid projective synchronization problems of the new hyperchaotic system via Lyapunov's direct method. In this study, a class of nonlinear controllers is designed to achieve anti-synchronization between the new hyperchaotic system and the new hyperchaotic Lorenz system based on small-gain theorem.

The rest of this paper is organized as follows: problem formulation of small-gain theorem and the new hyperchaotic system are introduced in Sections 2 and 3, respectively. In Sections 4 and 5, the anti-synchronization between two identical new hyperchaotic systems and the two different hyperchaotic systems are achieved by using the small-gain theorem method. Finally conclusions are drawn in Section 6.

2. Problem formulation

Consider a nonlinear dynamic system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \tag{1}$$

with state $\boldsymbol{x} \in \boldsymbol{R^n}$, input $\boldsymbol{u} \in \boldsymbol{R^m}$, in which $\boldsymbol{f}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0}$ and $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$ is locally Lipschitz on $\boldsymbol{R^n} \times \boldsymbol{R^m}$. The input function $\boldsymbol{u} : [0, \infty) \to \boldsymbol{R^m}$ of system (1) can be any piecewise continuous bounded function.

Recall that a function $\gamma : \boldsymbol{R^+} \to \boldsymbol{R^+}$ is of class K, if it is continuous, positive-definite, vanishing at zero and strictly increasing. By K_∞ we denote the subclass of $\boldsymbol{K}$ consisting of all functions $\gamma \in \boldsymbol{K}$ with $\gamma(s) \to +\infty$ as $s \to +\infty$. A function $\beta : \boldsymbol{R^+} \times \boldsymbol{R^+} \to \boldsymbol{R^+}$ is of class KL, if for each fixed $t \in \boldsymbol{R^+}$ the function $\beta(\cdot, t)$ is of class K and for each fixed $s \in \boldsymbol{R^+}$, the function $\beta(s, \cdot)$ is decreasing and $\lim_{t\to\infty} \beta(s, t) = 0$.

System (1) is input-to-state stable (ISS)[14,15] if there exists a class KL function $\beta(\cdot, \cdot)$ and a class K function $\gamma(\cdot)$, called a gain function, such that, for any input $\boldsymbol{u}(\cdot) \in \boldsymbol{L_\infty^m}$ and any initial state $\boldsymbol{x}(0) \in \boldsymbol{R^n}$, the response $\boldsymbol{x}(t)$ of system (1) exists on $[0, \infty)$ and satisfies

$$\|\boldsymbol{x}(\boldsymbol{t})\| \le \beta(\|\boldsymbol{x}(0)\|, t) + \gamma(\|\boldsymbol{u}(\cdot)\|_\infty) \text{ for all } t \ge 0. \tag{2}$$

*Project supported by the Fundamental Research Funds for the Central Universities of China (Grant No. CDJRC 10100001).

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Input-to-state stable (ISS) provides a way of expressing the dependence of the state of nonlinear systems on the magnitude of the input, and it has been employed in studies concerning the stabilization, the robust control of nonlinear systems.[16,17]

A smooth function $V : \boldsymbol{R^n} \to \boldsymbol{R}$ is an ISS-Lyapunov function[14,15] for system (1) if V is proper, positive-definite, that is, there exist class $\boldsymbol{K}_\infty$ functions $\underline{\alpha}(\cdot)$, $\bar{\alpha}(\cdot)$, such that

$$\underline{\alpha}(\|\boldsymbol{x}\|) \leq V(\boldsymbol{x}) \leq \bar{\alpha}(\|\boldsymbol{x}\|) \text{ for all } \boldsymbol{x} \in \boldsymbol{R^n}, \quad (3)$$

and there exists a positive-definite function $\alpha(\cdot)$, a class K function $\chi(\cdot)$ such that the following implication holds:

$$\|\boldsymbol{x}\| \geq \chi(\|\boldsymbol{u}\|) \Rightarrow \nabla V(\boldsymbol{x}) f(\boldsymbol{x}, \boldsymbol{u}) \leq -\alpha(\|\boldsymbol{x}\|) \text{ for all } \boldsymbol{x} \in \boldsymbol{R^n}. \quad (4)$$

According to Refs. [14] and [15], system (1) is ISS if and only if there exists an ISS-Lyapunov function and the following equation holds

$$\gamma(r) = \alpha^{-1} \circ \bar{\alpha} \circ \chi(r). \quad (5)$$

For interconnected systems:

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}_1), \\ \dot{\boldsymbol{x}}_2 &= \boldsymbol{f}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}_2), \end{aligned} \quad (6)$$

where, for $i = 1, 2$, $\boldsymbol{x}_i \in \boldsymbol{R}^{n_i}$, $\boldsymbol{u}_i \in \boldsymbol{R}^{m_i}$, and $\boldsymbol{f}_i : \boldsymbol{R}^{n_1} \times \boldsymbol{R}^{n_2} \times \boldsymbol{R}^{m_i} \to \boldsymbol{R}^{n_i}$ is locally Lipschitz.

Assume that, for $i = 1, 2$, there exists an ISS-Lyapunov function V_i for the $\boldsymbol{x}_i$ subsystem such that the following assumptions hold:

(i) there exist class $\boldsymbol{K}_\infty$ functions $\underline{\alpha}_{i1}(\cdot)$, $\bar{\alpha}_{i2}(\cdot)$ such that

$$\underline{\alpha}_{i1}(\|\boldsymbol{x}_i\|) \leq V_i(\boldsymbol{x}_i) \leq \bar{\alpha}_{i2}(\|\boldsymbol{x}_i\|) \text{ for all } \boldsymbol{x}_i \in \boldsymbol{R}^n; \quad (7)$$

(ii) there exist functions $\alpha_i(\cdot) \in K_\infty$, $\chi_i \in K$ such that

$$\|\boldsymbol{x}_1\| \geq \chi_1(\|\boldsymbol{u}_1\|)$$

implies

$$\nabla V_1(\boldsymbol{x}_1) f_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}_1) \leq -\alpha_1(\|\boldsymbol{x}_1\|), \quad (8)$$

and $\|\boldsymbol{x}_2\| \geq \chi_2(\|\boldsymbol{u}_2\|)$ implies

$$\nabla V_2(\boldsymbol{x}_2) f_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}_2) \leq -\alpha_2(\|\boldsymbol{x}_2\|). \quad (9)$$

Based on the above assumptions, Jiang *et al.*[15] established a Lyapunov-type nonlinear small-gain theorem for the interconnected system (6) as follows.

Theorem 1 (Small-gain theorem) Assume that, for $i = 1, 2$, the $\boldsymbol{x}_i$ subsystem has an ISS-Lyapunov function V_i satisfying inequalities (7)–(9). If $\gamma_1(\gamma_2(r)) < r$ for all $r > 0$, system (6) is ISS. In particular, the zero solution of system (6) with no input (i.e. $u = 0$) is globally asymptotically stable.

By small-gain theorem, we can not only test the global asymptotic stability of nonlinear systems, but also achieve anti-synchronization between the hyperchaotic systems in the following sections.

3. System description

Very recently, Chen *et al.* presented a new hyperchaotic dynamical system[12] based on Chen–Lee system[13] as follows:

$$\begin{aligned} \dot{x} &= -yz + a_1 x, \\ \dot{y} &= xz + b_1 y, \\ \dot{z} &= (1/3)xy + c_1 z + 0.2w, \\ \dot{w} &= d_1 x + 0.5yz + 0.05w, \end{aligned} \quad (10)$$

where x, y, z and w are state variables, a_1, b_1, c_1 and d_1 are system parameters. System (10) is chaotic as shown in Fig. 1 when $a_1 = 5.0$, $b_1 = -10$, $c_1 = -3.8$, $d_1 = 1.2$.

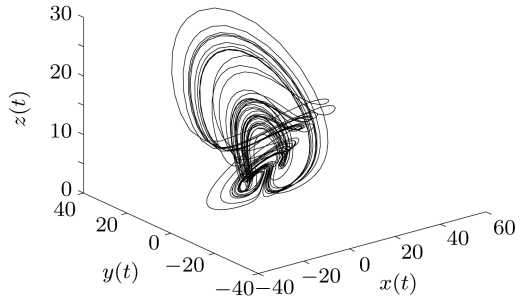


Fig. 1. Chaotic attractor ($a_1 = 5.0$, $b_1 = -10$, $c_1 = -3.8$, $d_1 = 1.2$).

In 2007, Jia constructed a new hyperchaotic Lorenz system[6] by introducing state feedback control to the familiar Lorenz system. The new hyperchaotic Lorenz system is described by the following differential equations:

$$\begin{aligned} \dot{x} &= -a_2(y - x) + w, \\ \dot{y} &= -xz + r_2 x - y, \\ \dot{z} &= xy - b_2 z, \\ \dot{w} &= -xz + d_2 w, \end{aligned} \quad (11)$$

where a_2, r_2, b_2 and d_2 are system parameters. This system shows hyperchaotic behaviour when $a_2 = 10$, $r_2 = 28$, $b_2 = 8/3$ and $d_2 = 1.3$.[6]

4. Anti-synchronization of two identical new hyperchaotic systems

Now, we assume that we have two identical new hyperchaotic systems[12] where the master (drive) system with the subscript 1 and the slave (response) system having identical equations denoted by the subscript 2. The master and slave systems are described by the following equations:

$$\begin{aligned}\dot{x}_1 &= -y_1z_1 + a_1x_1,\\ \dot{y}_1 &= x_1z_1 + b_1y_1,\\ \dot{z}_1 &= (1/3)x_1y_1 + c_1z_1 + 0.2w_1,\\ \dot{w}_1 &= d_1x_1 + 0.5y_1z_1 + 0.05w_1,\end{aligned} \tag{12}$$

and

$$\begin{aligned}\dot{x}_2 &= -y_2z_2 + a_1x_2 + u_1,\\ \dot{y}_2 &= x_2z_2 + b_1y_2 + u_2,\\ \dot{z}_2 &= (1/3)x_2y_2 + c_1z_2 + 0.2w_2 + u_3,\\ \dot{w}_2 &= d_1x_2 + 0.5y_2z_2 + 0.05w_2 + u_4.\end{aligned} \tag{13}$$

Our objective is to find suitable controllers u_i $(i = 1, 2, 3, 4)$ to ensure the drive system (12) and the response system (13) approach anti-synchronization.

Define the error vector as

$$\begin{aligned}e_1 &= x_1 + x_2, \quad e_2 = y_1 + y_2,\\ e_3 &= z_1 + z_2, \quad e_4 = w_1 + w_2.\end{aligned} \tag{14}$$

The error dynamical system can be written as fellows:

$$\begin{aligned}\dot{e}_1 &= a_1e_1 - y_1z_1 - y_2z_2 + u_1,\\ \dot{e}_2 &= b_1e_2 + x_1z_1 + x_2z_2 + u_2,\\ \dot{e}_3 &= c_1e_3 + 0.2e_4 + (1/3)x_1y_1 + (1/3)x_2y_2 + u_3,\\ \dot{e}_4 &= 0.05e_4 + d_1e_1 + 0.5y_1z_1 + 0.5y_2z_2 + u_4.\end{aligned} \tag{15}$$

When suitable controllers are designed, the two hyperchaotic systems will approach anti-synchronization for any initial conditions. In order to achieve this objective, we choose the following controllers:

$$\begin{aligned}u_1 &= y_1z_1 + y_2z_2 - P_1e_1,\\ u_2 &= -x_1z_1 - x_2z_2 + e_3 - P_2e_2,\\ u_3 &= -(1/3)x_1y_1 - (1/3)x_2y_2 - P_3e_3,\\ u_4 &= -0.5y_1z_1 - 0.5y_2z_2 - P_4e_4,\end{aligned} \tag{16}$$

where P_i $(i = 1, 2, 3, 4)$ are control inputs.

Combining systems (15) and (16) yields

$$\begin{aligned}\dot{e}_1 &= (a_1 - P_1)e_1,\\ \dot{e}_2 &= (b_1 - P_2)e_2 + e_3,\\ \dot{e}_3 &= (c_1 - P_3)e_3 + 0.2e_4,\\ \dot{e}_4 &= d_1e_1 + (0.05 - P_4)e_4.\end{aligned} \tag{17}$$

We regard system (17) as the interconnection of two systems described by the upper two equations and the lower two equations, respectively. The upper subsystem

$$\begin{aligned}\dot{e}_1 &= (a_1 - P_1)e_1,\\ \dot{e}_2 &= (b_1 - P_2)e_2 + e_3\end{aligned} \tag{18}$$

can be viewed as a system with states e_1, e_2 and input e_3. We choose $V(e_1, e_2) = (e_1^2 + e_2^2)/2$ and it can be obtained that

$$\begin{aligned}&\dot{V}(e_1, e_2)\\ &= e_1 \cdot \dot{e}_1 + e_2 \cdot \dot{e}_2\\ &= (a_1 - P_1)e_1^2 + (b_1 - P_2)e_2^2 + e_2e_3\\ &\le \max\{a_1 - P_1, b_1 - P_2\}(e_1^2 + e_2^2) + |e_2| \cdot |e_3|\\ &\le \max\{a_1 - P_1, b_1 - P_2\}(e_1^2 + e_2^2)\\ &\quad + (|e_1| + |e_2|) \cdot |e_3|,\end{aligned}$$

since

$$(|e_2| + |e_1|)^2 \le 2(e_1^2 + e_2^2),$$

we have

$$\begin{aligned}&\dot{V}(e_1, e_2)\\ &\le \max\{a_1 - P_1, b_1 - P_2\}(e_1^2 + e_2^2)\\ &\quad + 2(e_1^2 + e_2^2)^{1/2} \cdot |e_3|.\end{aligned}$$

If the following inequality holds

$$2 + \max\{a_1 - P_1, b_1 - P_2\} < 0, \tag{19}$$

we can select a sufficiently small positive number ε_1, such that

$$2 + \max\{a_1 - P_1, b_1 - P_2\} + \varepsilon_1 < 0.$$

Let

$$\chi_1(r) = \frac{2}{-\max\{a_1 - P_1, b_1 - P_2\} - \varepsilon_1}r,$$

and assume

$$\left\|(e_1, e_2)^T\right\| \geq \chi_1(\|e_3\|),$$

which yields

$$\dot{V}(e_1, e_2) \leq -\varepsilon_1(e_1^2 + e_2^2).$$

If we take class K_∞ functions

$$\underline{\alpha}(r) = \bar{\alpha}(r) = \frac{1}{2}r^2, \quad \alpha(r) = \varepsilon_1 r^2,$$

it is deduced that $V(e_1, e_2)$ satisfies conditions (3) and (4), thus it is an ISS-Lyapunov function for system (18). Hence system (18) is input-to-state stable, and in view of formula (5), we have

$$\begin{aligned}\gamma_1(r) &= \underline{\alpha}^{-1} \circ \bar{\alpha} \circ \chi_1(r) \\ &= \frac{2}{-\max\{a_1 - P_1, b_1 - P_2\} - \varepsilon_1} r.\end{aligned}$$

For the lower subsystem

$$\begin{aligned}\dot{e}_3 &= (c_1 - P_3)e_3 + 0.2e_4, \\ \dot{e}_4 &= d_1 e_1 + (0.05 - P_4)e_4\end{aligned} \tag{20}$$

can be viewed as a system with states $e_3,\ e_4$, and input e_1. Consider the positive-definite function $V(e_3, e_4) = (e_3^2 + e_4^2)/2$ and it can be obtained that

$$\begin{aligned}&\dot{V}(e_3, e_4) \\ &= e_3 \cdot \dot{e}_3 + e_4 \cdot \dot{e}_4 \\ &= (c_1 - P_3)e_3^2 + 0.2e_3e_4 + d_1e_1e_4 + (0.05 - P_4)e_4^2 \\ &\leq (c_1 - P_3)e_3^2 + 0.1(e_3^2 + e_4^2) + d_1 |e_1| \cdot |e_4| \\ &\quad + (0.05 - P_4)e_4^2 \\ &\leq [\max\{c_1 - P_3, 0.05 - P_4\} + 0.1](e_3^2 + e_4^2) \\ &\quad + d_1 |e_1| \cdot |e_4|,\end{aligned}$$

since

$$(|e_3| + |e_4|)^2 \leq 2(e_3^2 + e_4^2),$$

we have

$$\begin{aligned}\dot{V}(e_3, e_4) \leq\ & [\max\{c_1 - P_3, 0.05 - P_4\} + 0.1](e_3^2 + e_4^2) \\ & + 2d_1(e_3^2 + e_4^2)^{1/2} |e_1|.\end{aligned}$$

If the following inequality holds

$$2d_1 + \max\{c_1 - P_3, 0.05 - P_4\} + 0.1 < 0, \tag{21}$$

we can select a sufficiently small positive number ε_2, such that

$$2d_1 + \max\{c_1 - P_3, 0.05 - P_4\} + 0.1 + \varepsilon_2 < 0.$$

Let

$$\chi_2(r) = \frac{2d_1}{-\max\{c_1 - P_3, 0.05 - P_4\} - 0.1 - \varepsilon_2} r,$$

and assume

$$\left\|(e_3, e_4)^T\right\| \geq \chi_2(\|e_1\|),$$

which yields

$$\dot{V}(e_3, e_4) \leq -\varepsilon_2(e_3^2 + e_4^2).$$

If we take class K_∞ functions

$$\underline{\alpha}(r) = \bar{\alpha}(r) = \frac{1}{2}r^2, \quad \alpha(r) = \varepsilon_2 r^2,$$

it is deduced that $V(e_2, e_3)$ satisfies conditions (3) and (4), thus it is an ISS-Lyapunov function for system (20). Hence system (20) is input-to-state stable, and in view of formula (5), we have

$$\begin{aligned}\gamma_2(r) &= \underline{\alpha}^{-1} \circ \bar{\alpha} \circ \chi_2(r) \\ &= \frac{2d_1}{-\max\{c_1 - P_3, 0.05 - P_4\} - 0.1 - \varepsilon_2} r,\end{aligned}$$

therefore,

$$\begin{aligned}&\gamma_2(\gamma_1(r)) \\ &= \frac{2}{-\max\{a_1 - P_1, b_1 - P_2\} - \varepsilon_1} \\ &\quad \cdot \frac{2d_1}{-\max\{c_1 - P_3, 0.05 - P_4\} - 0.1 - \varepsilon_2} r \leq r \\ &\quad \text{for all} \quad r > 0.\end{aligned}$$

In view of Theorem 1, the equilibrium point (0,0,0,0) of error system (15) is globally asymptotically stable if conditions (19) and (21) hold. Therefore, anti-synchronization is achieved between the two identical new hyperchaotic systems via small-gain theorem.

To verify and demonstrate the effectiveness of the proposed method, we will display the numerical results for anti-synchronizing new hyperchaotic systems. The fourth order Runge–Kutta method is used to solve the systems (12) and (13) with time step size 0.001. We select the parameters of new hyperchaotic system as $a_1 = 5.0$, $b_1 = -10$, $c_1 = -3.8$, $d_1 = 1.2$, so these systems exhibit chaotic behaviours. We set $P_1 = P_2 = P_3 = P_4 = 50$, which satisfy the conditions (19) and (21). The initial conditions of the drive system and the response system are $(10, 10, 10, 10)$ and $(10, 20, 30, 5)$, respectively. Figure 2 displays the anti-synchronization errors of systems (12) and (13). It can be seen from the figure that the anti-synchronization errors converge to zero rapidly.

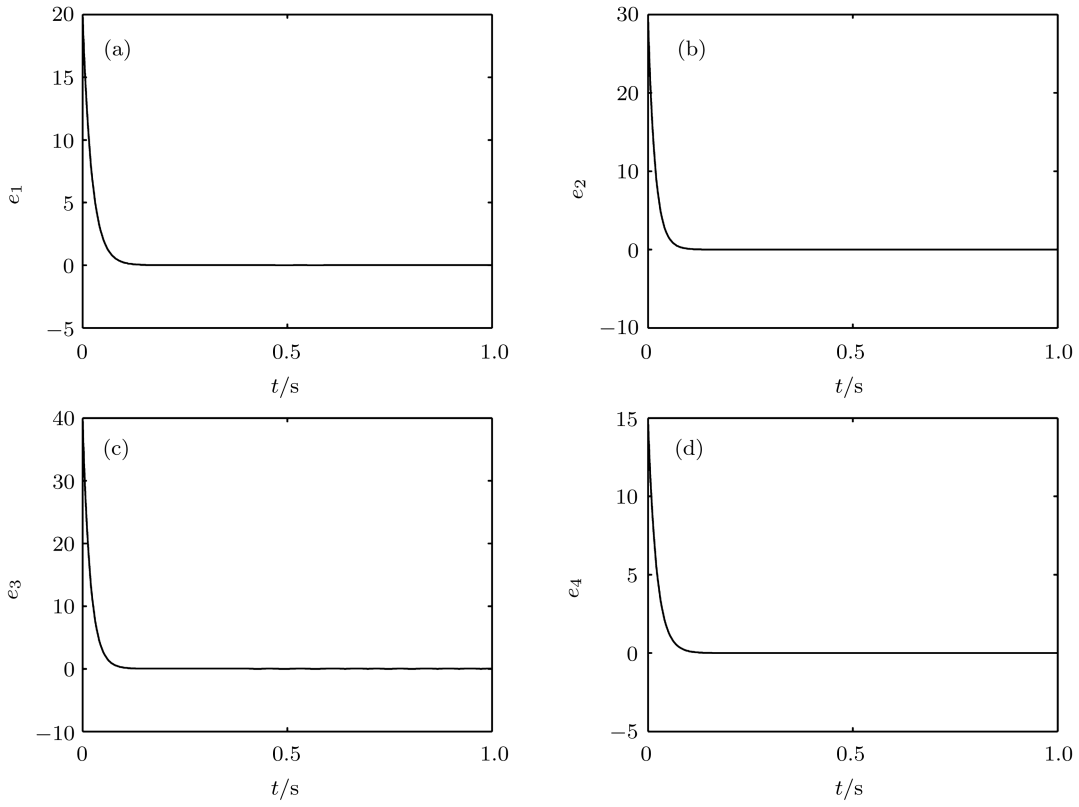


Fig. 2. Anti-synchronization errors of systems (12) and (13).

5. Anti-synchronization between the new hyperchaotic system and the new hyperchaotic Lorenz system

In order to achieve the behaviour of the anti-synchronization between the new hyperchaotic system and the new hyperchaotic Lorenz system by small-gain theorem, we assume that the new hyperchaotic system is the drive system whose variables are denoted by subscript 1 and the new hyperchaotic Lorenz system is the response system whose variables are denoted by subscript 2. The drive and the response systems are described, respectively, by the following equations:

$$\begin{aligned}
\dot{x}_1 &= -y_1 z_1 + a_1 x_1, \\
\dot{y}_1 &= x_1 z_1 + b_1 y_1, \\
\dot{z}_1 &= (1/3) x_1 y_1 + c_1 z_1 + 0.2 w_1, \\
\dot{w}_1 &= d_1 x_1 + 0.5 y_1 z_1 + 0.05 w_1,
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
\dot{x}_2 &= -a_2(x_2 - y_2) + w_2 + u_1, \\
\dot{y}_2 &= -x_2 z_2 + r_2 x_2 - y_2 + u_2, \\
\dot{z}_2 &= x_2 y_2 - b_2 z_2 + u_3, \\
\dot{w}_2 &= -x_2 z_2 + d_2 w_2 + u_4,
\end{aligned} \tag{23}$$

where $U = [u_1, u_2, u_3, u_4]^{\mathrm{T}}$ is the controller function. In the following, we design a nonlinear controller to achieve anti-synchronization between the new hyperchaotic system and the new hyperchaotic Lorenz system.

Define the error vector as

$$\begin{aligned}
e_1 &= x_1 + x_2, \quad e_2 = y_1 + y_2, \\
e_3 &= z_1 + z_2, \quad e_4 = w_1 + w_2.
\end{aligned} \tag{24}$$

The error dynamical system can be written as

$$\begin{aligned}
\dot{e}_1 &= a_1 e_1 - a_1 x_2 - a_2(x_2 - y_2) \\
&\quad - y_1 z_1 + w_2 + u_1, \\
\dot{e}_2 &= b_1 e_2 + x_1 z_1 - x_2 z_2 + r_2 x_2 \\
&\quad - (b_1 + 1) y_2 + u_2, \\
\dot{e}_3 &= c_1 e_3 + (1/3) x_1 y_1 + 0.2 w_1 + x_2 y_2 \\
&\quad - (c_1 + b_2) z_2 + u_3,
\end{aligned}$$

$$\begin{aligned}\dot{e}_4 &= d_2e_4 + (0.05 - d_2)w_1 + d_1x_1 + 0.5y_1z_1 \\ &\quad - x_2z_2 + u_4, \end{aligned} \tag{25}$$

we choose the following controllers

$$\begin{aligned}u_1 &= a_1x_2 + a_2(x_2 - y_2) + y_1z_1 - w_2 - P_1e_1, \\ u_2 &= -x_1z_1 + x_2z_2 - r_2x_2 + (b_1 + 1)y_2 \\ &\quad + e_3 - P_2e_2, \\ u_3 &= -(1/3)x_1y_1 - 0.2w_1 - x_2y_2 \\ &\quad +(c_1 + b_2)z_2 + e_1 - P_3e_3, \\ u_4 &= -(0.05 - d_2)w_1 - d_1x_1 - 0.5y_1z_1 \\ &\quad + x_2z_2 - P_4e_4, \end{aligned} \tag{26}$$

where P_i $(i = 1, 2, 3, 4)$ are control inputs.

Combining systems (25) and (26) leads to

$$\begin{aligned}\dot{e}_1 &= (a_1 - P_1)e_1, \\ \dot{e}_2 &= (b_1 - P_2)e_2 + e_3, \\ \dot{e}_3 &= (c_1 - P_3)e_3 + e_1, \\ \dot{e}_4 &= (d_2 - P_4)e_4. \end{aligned} \tag{27}$$

Similar to the discussion in Section 4, the drive system (22) and the response system (23) can approach anti-synchronization asymptotically with the controllers (26) by the following control law

$$2 + \max\{a_1 - P_1, b_1 - P_2\} < 0,$$

$$2 + \max\{c_1 - P_3, d_2 - P_4\} < 0. \tag{28}$$

In view of small-gain theorem, the origin of error system (25) is globally asymptotically stable if conditions (28) hold.

In this section, to verify and demonstrate the effectiveness of the proposed methods, we will display the numerical results for anti-synchronizing hyperchaotic systems (22) and (23) via small-gain theorem. In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with the time step size 0.001.

In the simulations, we set $P_1 = P_2 = P_3 = P_4 = 20$, which satisfy conditions (28). The initial values of the drive system (22) and the response system (23) are taken as $x_1(0) = 12$, $y_1(0) = 17$, $z_1(0) = 25$, $w_1(0) = 8$ and $x_2(0) = -32$, $y_2(0) = -7$, $z_2(0) = -35$, $w_2(0) = 7$, respectively. Thus, the initial errors are $e_1(0) = -20$, $e_2(0) = 10$, $e_3(0) = -10$, $e_4(0) = 15$. Figure 3 shows the time responses of anti-synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$, $e_4(t)$ which indicate that the anti-synchronization between systems (22) and (23) is achieved via small-gain theorem.

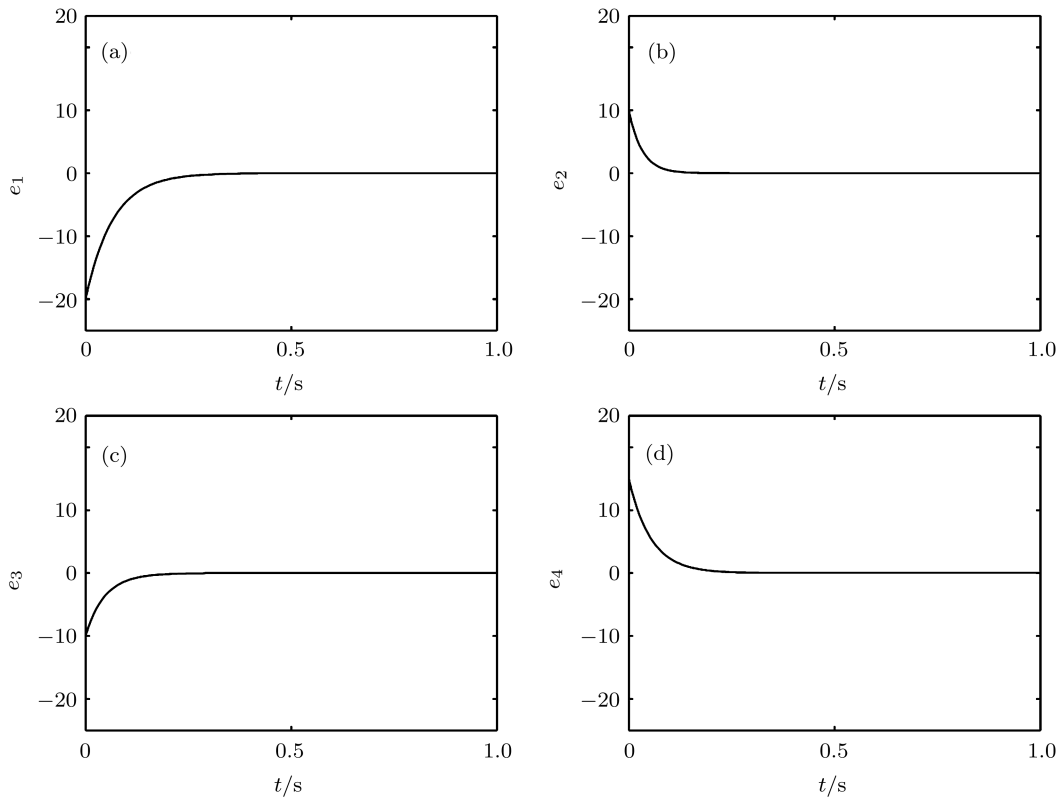


Fig. 3. The time responses of anti-synchronization errors.

6. Conclusion

Anti-synchronization of the new hyperchaotic system was studied by using small-gain theorem. The nonlinear controller was designed according to small-gain theorem to guarantee anti-synchronization, which includes two identical new hyperchaotic systems and two different hyperchaotic systems. Numerical simulation results illustrate the effectiveness of the proposed anti-synchronization method.

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