Sliding mode control of a new chaotic system

Li Ming(李 铭)† and Liu Chong-Xin(刘崇新)‡

College of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, *China*

(Received 7 March 2010; revised manuscript received 9 April 2010)

In this paper, a sliding mode controller (SMC) is designed to control a new dynamical system with canonical chaos characters. With the proposed SMC, the new chaotic system can be regulated to a fixed point in the state space. Simulation shows that the chattering phenomenon is greatly alleviated.

Keywords: chaos, sliding mode control, chattering

PACC: 0545

1. Introduction

In recent years the theory and methods of controlling the chaotic system have been tremendously developed. A lot of researches have been done to different kinds of chaotic systems.[1−7]

Recently, sliding-mode control systems have been studied due to its robustness and simplicity. In Ref. [8] Yu *et al.* proposed a sliding mode nonlinear PI control scheme. In Ref. [9] Guo *et al.* used sliding mode controller (SMC) to achieve chaos synchronization. And much work has been done to solve the problems in implementation of SMCs.[10−14]

The sliding mode control employs discontinuous control to drive the system state trajectories onto some prescribed sliding surface. This discontinuous control and non-ideal switching often result in an undesirable phenomenon called chattering. In this paper, with the control law, the dynamical system responses rapidly and the chattering phenomenon is led to a very low level while the state trajectories of the controlled system being regulated to a desired point.

2. Brief analysis of the new chaotic system

In Ref. [15] Liu proposed a new three-dimensional autonomous system

$$\begin{cases} \dot{x} = a(y - x + yz), \\ \dot{y} = by - hxz, \\ \dot{z} = ky - gz, \end{cases} \tag{1}$$

where parameters $a = 1$, $b = 2.5$, $k = 1$, $g = 4$, $h = 1$.

Reference [15] also analysed that system (1) had 3 equilibrium points $O(0, 0, 0)$, $E_1(10, -8.633, -2.158)$, $E_2(10, 4.633, 1.158)$, and found that all of them were unstable.

Figure 1 shows the state-plane behaviour of x, y and z under the initial value $[x,\ y,\ z]^{\mathrm{T}} = [10,\ 10,\ 1.6]^{\mathrm{T}}$.

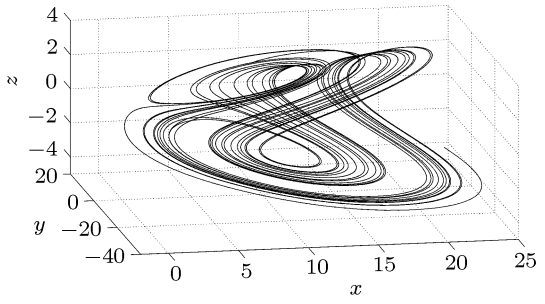


Fig. 1. Three-dimensional view of the state-plane behaviour.

3. Sliding surface design

Let the fixed point be $(x_d,\ y_d,\ z_d)$, then the trajectory error states could be defined as $e_1 = x - x_d$, $e_2 = y - y_d$, $e_3 = z - z_d$. Considering execute control on the second and third equations of system (1), we find that the error state dynamic equations can be obtained as

$$\begin{cases} \dot{e}_1 = -ae_1 + a(1 + z_d)e_2 + ay_d e_3 + ae_2 e_3 \\ \qquad - ax_d + ay_d + ay_d z_d, \\ \dot{e}_2 = -hz_d e_1 + be_2 - hx_d e_3 - he_1 e_3 \\ \qquad + by_d - hx_d z_d + u, \\ \dot{e}_3 = ke_2 - ge_3 + ky_d - gz_d + w, \end{cases} \tag{2}$$

†Corresponding author. E-mail: liming.0.0.9@stu.xjtu.edu.cn

‡E-mail: liucx@mail.xjtu.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

where u and w are control input.

When x_d, y_d and z_d satisfy the following two equations:

$$y_d = \frac{g}{k} z_d, \tag{3}$$

$$x_d = \frac{g}{k} z_d (1 + z_d), \tag{4}$$

we can calculate that $-ax_d + ay_d + ay_d z_d = 0$ and $ky_d - gz_d = 0$, therefore the error state dynamic equations can be simplified as

$$\begin{cases} \dot{e}_1 = -ae_1 + a(1+z_d)e_2 + ay_d e_3 + ae_2 e_3, \\ \dot{e}_2 = -hz_d e_1 + be_2 - hx_d e_3 - he_1 e_3 \\ \qquad + by_d - hx_d z_d + u, \\ \dot{e}_3 = ke_2 - ge_3 + w. \end{cases} \tag{5}$$

When the control input w satisfies the following equation:

$$w = -ke_2, \tag{6}$$

we can obtain $\dot{e}_3 = -ge_3$. As $g = 4$ is given, so when $t \to \infty$, e_3 converges to zero.

From Eq. (5), we can see that when e_2 and e_3 all become zero, then $\dot{e}_1 = -ae_1$, i.e. when $t \to \infty$, e_1 would converge to zero. So the proposed sliding surface should guarantee that e_2 could converge to zero, thus e_1, e_2 and e_3 would all converge to zero. For this purpose, the sliding surface is designed as

$$s = e_2 + k_1 e_3, \tag{7}$$

where k_1 is a positive real parameter.

4. Sliding mode control law

If $s\dot{s} < 0$, then there exists sliding mode. The sliding mode control law is designed as

$$\begin{aligned} u = {} & hz_d e_1 - be_2 + (hx_d + gk_1)e_3 + he_1 e_3 - by_d \\ & + hx_d z_d - k_2 s - k_3 \operatorname{sign}(s), \end{aligned} \tag{8}$$

when $z_d \neq 0$, or

$$\begin{aligned} u = {} & hz_d e_1 - be_2 + (hx_d + gk_1)e_3 + he_1 e_3 \\ & - by_d + hx_d z_d - k_2 s, \quad \text{when} \quad z_d = 0, \end{aligned} \tag{9}$$

where k_2 and k_3 are positive real parameters. The $\operatorname{sign}(\cdot)$ is sign function, i.e. when $s > 0$, $\operatorname{sign}(s) = 1$; when $s = 0$, $\operatorname{sign}(s) = 0$; when $s < 0$, $\operatorname{sign}(s) = -1$. The proposed input (8) or (9) will drive the system (1) with nonlinear input onto the sliding mode $s = 0$.

Differentiating the sliding surface, we can obtain

$$\dot{s} = -k_2 s - k_3 \operatorname{sign}(s), \quad \text{or} \quad \dot{s} = -k_2 s. \tag{10}$$

Letting the Lyapunov function of the system be $V = \frac{1}{2}s^2$, we obtain its derivative with respect to time t as

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= \begin{cases} -k_2 s^2 - (s)k_3 \operatorname{sign}(s), & (z_d \neq 0), \\ -k_2 s^2, & (z_d = 0). \end{cases} \end{aligned}$$

As $k_2, k_3 > 0$, $s^2 \geq 0$ and $(s)k_3 \operatorname{sign}(s) = k_3|s| \geq 0$, thus $\dot{V} \leq 0$. This condition makes sure that the designed sliding surface s would converge to zero.

5. Simulation results

Simulation results are presented in this part to verify and demonstrate the effectiveness of the proposed control scheme. The simulation results are carried out by using the MATLAB software. The fourth and fifth order Runge–Kutta integration algorithm was performed to solve the system differential equations. A time step size 0.01 was employed in this part.

Let the desired point be $[x_d,\ y_d,\ z_d]^{\mathrm{T}} = [0,\ 0,\ 0]^{\mathrm{T}}$ (x_d, y_d and z_d are under constraints of Eqs. (3) and (4)). When the control input was carried out at $t = 40$ s, as shown in Fig. 2, the behaviour of the system changes from unstable (chaos) to stable state.

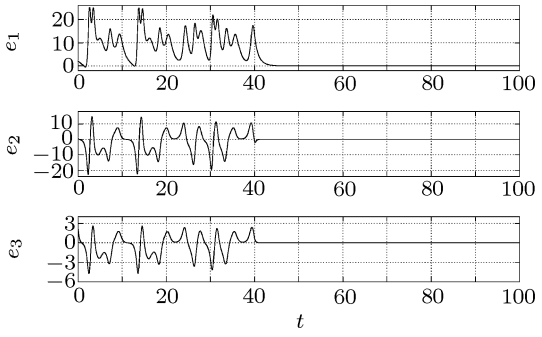


Fig. 2. Simulation results with $k_1 = 8$, $k_2 = 10$.

Let the desired point be $[x_d,\ y_d,\ z_d]^{\mathrm{T}} = [8,\ 4,\ 1]^{\mathrm{T}}$ (x_d, y_d and z_d are under constraints of Eqs. (3) and (4)). When the control input was carried out at $t = 40$ s, figure 3 shows the evolution of the dynamical system.

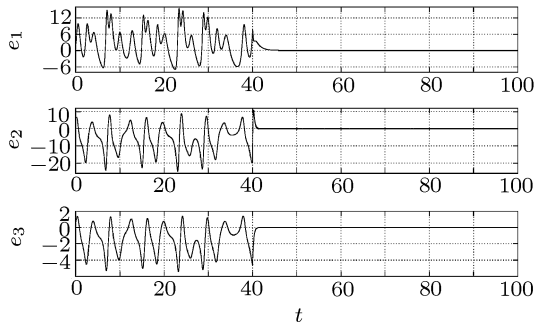


Fig. 3. Simulation results with $k_1 = 8$, $k_2 = 10$, $k_3 = 20$.

It can be seen from Figs. 2 and 3 that chattering does not appear and the error attenuates exponentially.

6. Conclusion

The nonlinear sliding mode control theory is used to design the control signal to eliminate chaos in a new dynamical system. The theory and simulation study show the effectiveness of the designed sliding mode control and the chattering phenomenon is successfully restrained.

References

[1] Li X C, Xiao Y Z and Xu W 2008 *Acta Phys. Sin.* **57** 4721 (in Chinese)

[2] Qiao X H and Bao B C 2009 *Acta Phys. Sin.* **58** 8152 (in Chinese)

[3] Han X J, Jiang B and Bi Q S 2009 *Acta Phys. Sin.* **58** 6006 (in Chinese)

[4] Wang Y N, Tan W and Duan F 2006 *Chin. Phys.* **15** 89

[5] Wang X Y and Wang M J 2008 *Acta Phys. Sin.* **57** 731 (in Chinese)

[6] Wang X Y, Nian F Z and Guo G 2009 *Phys. Lett.* A **373** 1754

[7] Liu C X, Liu T, Liu L and Liu K 2004 *Chaos, Solitons and Fractals* **22** 1031

[8] Yu D C, Wu A G and Yang C P 2005 *Chin. Phys.* **14** 914

[9] Guo H J, Yin Y W and Wang H M 2008 *Chin. Phys.* B **17** 1652

[10] Chen S H and Kong C C 2009 *Chin. Phys.* B **18** 91

[11] Chang W D and Yan J J 2005 *Chaos, Solitons and Fractals* **26** 167

[12] Wang X Y, Lin D and Wang Z J 2009 *Mod. Phys. Lett.* B **23** 2021

[13] Wang X Y, Liu M, Wang M J and He Y J 2008 *Int. J. Mod. Phys.* B **22** 2187

[14] Wang M J and Wang X Y 2009 *Mod. Phys. Lett.* B **23** 1805

[15] Liu C X 2009 *Chaos, Solitons and Fractals* **39** 1037