Adaptive generalized synchronization between Chen system and a multi-scroll chaotic system*

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Based on Lyapunov theory, the adaptive generalized synchronization between Chen system and a multi-scroll chaotic system is investigated. According to the form of target function a proper adaptive controller is designed, by which the controlled Chen system can be synchronized with a multi-scroll chaotic system including unknown parameters. The Lyapunov direct method is exploited to prove that the synchronization error and parameter identification error both converge to zero. Numerical simulation results verify the feasibility of the proposed method further.

Keywords: Chen system, multi-scroll chaotic system, generalized synchronization

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1. Introduction

In 1990, Pecora[1] realized chaos synchronization (CS) theoretically and experimentally. After that CS has been applied widely to vibration control, secure communication and biomedicine, and various synchronization methods were investigated in this period.[2−8] Initial CS methods were mainly concentrated on synchronization of two identical chaotic systems, and then two different chaotic systems can also be synchronized with some other methods, which extended the application of CS greatly.

Generalized synchronization[9−18] (GS) is a special kind of CS developed in recent years, by which a functional relation can be realized between the variables of two chaotic systems. In fact, most of existing CS methods can be regarded as the special cases of GS, such as projection synchronization, phase synchronization and so on. In Ref. [11], GS via active control was investigated. GS of continuous dynamical systems is studied in Ref. [14], and a universal adaptive GS approach is proposed in Ref. [17]. The feasibilities of GS are all proved in the above references, but the convergences of parameter estimation errors are not analysed and proved when adaptive control methods are adopted. In addition, GS of multi-scroll chaotic systems is seldom studied.

In this paper, the adaptive GS (AGS) between Chen system and a multi-scroll chaotic system is investigated, and the Lyapunov direct method is exploited to prove that the synchronization error and parameter identification error both converge to zero. Numerical simulation results verify the feasibility of the proposed method.

2. Description of the GS scheme

Take a multi-scroll chaotic system[19] as the drive system, which has a simple structure as follows:

$$\begin{cases} \dot{x}_1 = px_2, \\ \dot{x}_2 = x_1 - f(x_1 + x_3) + qx_2, \\ \dot{x}_3 = r[x_3 - f(x_1 + x_3)], \end{cases} \tag{1}$$

where the piecewise linear function

$$f(v) = \frac{1}{2}\sum_{k=1}^{N}\{\mathrm{sgn}[v + (2k-1)] + \mathrm{sgn}[v - (2k-1)]\},$$

when parameters $p = -4$, $q = 0.2$, $r = -5$, there exists a multi-scroll chaotic attractor, as shown in Fig. 1.

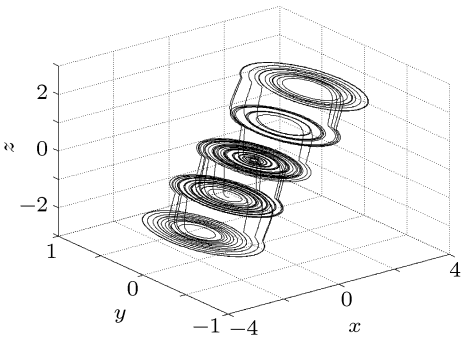


Fig. 1. The multi-scroll chaotic attractor of system (1).

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Take the chaotic Chen system as the response system, and the controlled system can be presented as follows:

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + u_1, \\ \dot{y}_2 = (c-a)y_1 - y_1y_3 + cy_2 + u_2, \\ \dot{y}_3 = y_1y_2 - by_3 + u_3. \end{cases} \tag{2}$$

The target function of AGS is the combination of variables in system (1). For simplicity, it is selected as

$$\varphi = (2x_3, x_1 + x_2, x_1 - x_2)^{\mathrm{T}}, \tag{3}$$

thus the error variables of AGS are

$$e_1 = y_1 - \varphi_1, \quad e_2 = y_2 - \varphi_2, \quad e_3 = y_3 - \varphi_3. \tag{4}$$

In addition, we define the whole error variable of AGS as $e = (e_1^2 + e_2^2 + e_3^2)^{1/2}$. If $\lim\limits_{t\to+\infty} e(t) = 0$, AGS related to function (3) is achieved between system (1) and system (2).

3. Design and proof of the controller

According to the form of target function (3), a nonlinear adaptive controller is designed as follows:

$$\begin{aligned} \dot{u}_1 &= (a-1)y_1 - ay_2 + 2x_3(1+\hat{r}) - 2\hat{r}f(x_1+x_3), \\ \dot{u}_2 &= (a-c)y_1 - (c+1)y_2 + y_1y_3 + 2x_1 \\ &\quad + (\hat{p}+\hat{q}+1)x_2 - f(x_1+x_3), \\ \dot{u}_3 &= (b-1)y_3 - y_1y_2 + (\hat{p}-\hat{q}-1)x_2 \\ &\quad + f(x_1+x_3), \end{aligned} \tag{5}$$

where $\hat{p}, \hat{q}, \hat{r}$ are the estimations of unknown parameters p, q, r. Denote the estimation errors as

$$\bar{p} = \hat{p} - p, \quad \bar{q} = \hat{q} - q, \quad \bar{r} = \hat{r} - r. \tag{6}$$

The adaptive law of parameter estimation is taken as

$$\begin{aligned} \dot{\hat{p}} &= -kx_2(e_2 + e_3), \\ \dot{\hat{q}} &= -kx_2(e_2 - e_3), \\ \dot{\hat{r}} &= -2ke_1[x_3 - f(x_1+x_3)], \end{aligned} \tag{7}$$

where parameter k is the control coefficient used to adjust converging velocity.

Theorem For the multi-scroll chaotic system (1) and controlled Chen system (2), AGS can be realized with respect to target function (3) when the controller (5) and adaptive law (7) are adopted, and the parameter estimation errors converge to zero.

Proof Take the Lyapunov function of the controlled Chen system (2) as

$$V = (e_1^2 + e_2^2 + e_3^2)/2 + (\bar{p}^2 + \bar{q}^2 + \bar{r}^2)/2k, \tag{8}$$

by differentiating and substituting Eqs. (3)–(7) into Eq. (8) we get

$$\begin{aligned} \dot{V} &= e_1(\dot{y}_1 - \dot{\varphi}_1) + e_2(\dot{y}_2 - \dot{\varphi}_2) + e_3(\dot{y}_3 - \dot{\varphi}_3) \\ &\quad + (\bar{p}\dot{\hat{p}} + \bar{q}\dot{\hat{q}} + \bar{r}\dot{\hat{r}})/k \\ &= e_1(-y_1 + \varphi_1 + 2\bar{r}[x_3 - f(x_1+x_3)]) \\ &\quad + e_2[-y_2 + \varphi_2 + (\bar{p}+\bar{q})x_2] \\ &\quad + e_3\{-y_3 + \varphi_3 + (\bar{p}-\bar{q})x_2\} - \bar{p}x_2(e_2+e_3) \\ &\quad - \bar{q}x_2(e_2-e_3) - 2\bar{r}e_1[x_3 - f(x_1+x_3)] \\ &= -(e_1^2 + e_2^2 + e_3^2) \\ &= -e^2 \le 0, \end{aligned}$$

because $\dot{V}$ is positively semi-definite, one cannot determine whether the error variable of AGS is asymptotically stable at zero just by the Lyapunov stability theory. Considering $\dot{V} \le 0$, we get e, $\bar{p}$, $\bar{q}$, $\bar{r} \in L_\infty$, and

$$\int_0^t e^2 \mathrm{d}t = \int_0^t -\dot{V}\mathrm{d}t = V(0) - V(t) \le V(0),$$

so $e(t) \in L_2$. According to the Barbalat lemma we obtain $\lim\limits_{t\to\infty} e(t) = 0$, namely the AGS between systems (1) and (2) is achieved.

Following is the proof about parameter estimation errors. According to the system equation and controller we obtain

$$\begin{aligned} \dot{e}_1 &= \dot{y}_1 - \dot{\varphi}_1 \\ &= a(y_2 - y_1) + u_1 - 2\dot{x}_3 \\ &= -(y_1 - 2x_3) + 2(\hat{r} - r)[x_3 - f(x_1+x_3)] \\ &= -e_1 + 2\bar{r}[x_3 - f(x_1+x_3)], \end{aligned} \tag{9}$$

when $t \to \infty$, considering the result $e(t) \to 0$, it is obvious that $e_1(t) \to 0$. Since $e_1(t)$ is a piecewise smooth function, we obtain $\dot{e}_1(t) \to 0$. Because $[x_3 - f(x_1+x_3)]$ is the combination of nonzero variables of system (1), if equation (9) converges to zero there must be $\bar{r} \to 0$, namely $\hat{r} \to r$, thus the unknown parameter is estimated exactly. Similarly, there exist

$$\begin{aligned} \dot{e}_2 &= -e_2 + (\bar{p} + \bar{q})x_2, \\ \dot{e}_3 &= -e_3 + (\bar{p} - \bar{q})x_2. \end{aligned}$$

By the same way it can be proved that $(\bar{p} + \bar{q}) \to 0$ and $(\bar{p} - \bar{q}) \to 0$, thus $\bar{p} \to 0$ and $\bar{q} \to 0$, namely $\hat{p} \to p$ and $\hat{q} \to q$.

4. Numerical simulation results

In the simulations we choose the control coefficient $k = 20$ in Eq. (7), and the initial values of controlled Chen system (2) $[y_1(0),\ y_2(0),\ y_3(0)] = [10, 11, 23]$, parameters $a = 35$, $b = 3$ and $c = 28$. The initial values of the multi-scroll system (1) are given by $[1, 1, 1]$. Take practical parameters $p = -4$, $q = 0.2$, $r = -5$, and the corresponding initial values of parameter estimation $[\hat{p}(0), \hat{q}(0), \hat{r}(0)] = [-3, 0.5, -7]$. Numerical simulation results are shown in Fig. 2. The error variables converge at zero at $t = 20$, thus AGS of two systems is realized.

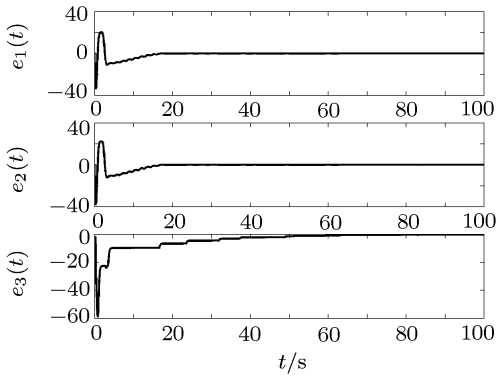


Fig. 2. The error variables of AGS.

During the course of AGS, the parameter estimations $\hat{p}$, $\hat{q}$, $\hat{r}$ in controller $u(t)$ vary according to adaptive law (7), and all of them converge to the real values respectively, as shown in Fig. 3.

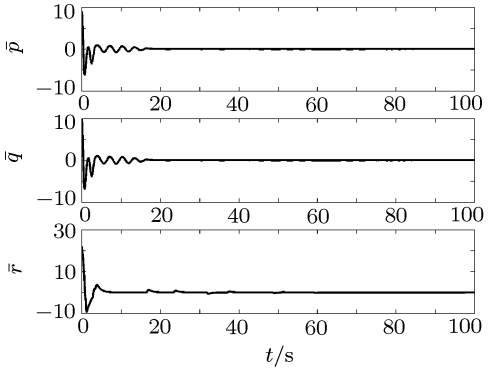


Fig. 3. The parameter estimation errors.

5. Conclusion

In this paper, an adaptive control method of GS between Chen system and a multi-scroll chaotic system was proposed, by which AGS is realized according to a given object function. Based on Lyapunov stability theory we proved that the AGS errors and parameter estimation errors both converge to zero. Numerical simulation results verify the effectiveness of the control method further. This adaptive control method can also be applied to GS of other chaotic systems, and the convergence of corresponding synchronization errors and parameter estimation errors can be proved similarly.

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