A new four-dimensional hyperchaotic Chen system and its generalized synchronization

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Based on the Chen chaotic system, a new four-dimensional hyperchaotic Chen system is constructed, and the basic dynamic behaviours of the system were studied, and the generalized synchronization has been observed in the coupled four-dimensional hyperchaotic Chen system with unknown parameters. The Routh–Hurwitz theorem is used to derive the conditions of stability of this system. Furthermore based on Lyapunov stability theory, the control laws and adaptive laws of parameters are obtained to make generalized synchronization of the coupled new four-dimensional hyperchaotic Chen systems. Numerical simulation results are presented to illustrate the effectiveness of this method.

Keywords: hyperchaotic Chen system, generalized synchronization, Lyapunov stability theory

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1. Introduction

Since Lorenz found the first chaotic attractor in a three-dimensional (3D) autonomous chaotic system in 1963,[1] chaos has been intensively studied in the last three decades.[2−4] Based on the Lorenz system, other chaotic systems were proposed in succession: the Chen and Lü chaotic system[5] and Liu chaotic system.[6] These chaotic systems are 3D chaotic systems, each one has only one positive Lyapunov exponent. Hyperchaotic system has two or more than two positive Lyapunov exponents, and dynamic behaviours of hyperchaotic system are more complex and difficult to predict. In recent years, four-dimensional (4D) chaotic systems[7−12] have been studied by many scholars, and the hyperchaotic system synchronization also have been concerned. In this paper, based on the Chen chaotic system, a new 4D hyperchaotic system is constructed, and the basic dynamic behaviours were studied. Furthermore the drive–response method[13] is used to achieve the generalized synchronization[14] in the coupled 4D hyperchaotic Chen system with unknown parameters. By improving the adaptive laws of parameters in Ref. [15], and selecting the appropriate Lyapunov function, the synchronization errors converge faster and the effect of synchronization is better in comparison with the method in Ref. [15].

2. A new 4D hyperchaotic Chen system

In Ref. [5], the Chen chaotic system is given by

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = (c - a)x - xz + cy, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

where x, y and z are state variables and $a = 35$, $b = 3$ and $c = 28$ are parameters. Chen chaotic attractor and phase diagrams are shown in Figs. 1(a) and 1(b).

The hyperchaotic system must satisfy the two necessary conditions: 1) for autonomous system, four dimensions are required at least; 2) two or more than two positive Lyapunov exponents are needed and the sum of all the Lyapunov exponents is less than 0. Based on the above two necessary conditions, a new 4D hyperchaotic system is constructed on the basis of the Chen system. By increasing the fourth state variable w in the second equation of state of Chen system and changing nonlinear term and the coefficients of some terms of Chen system, equations of state are expressed as:

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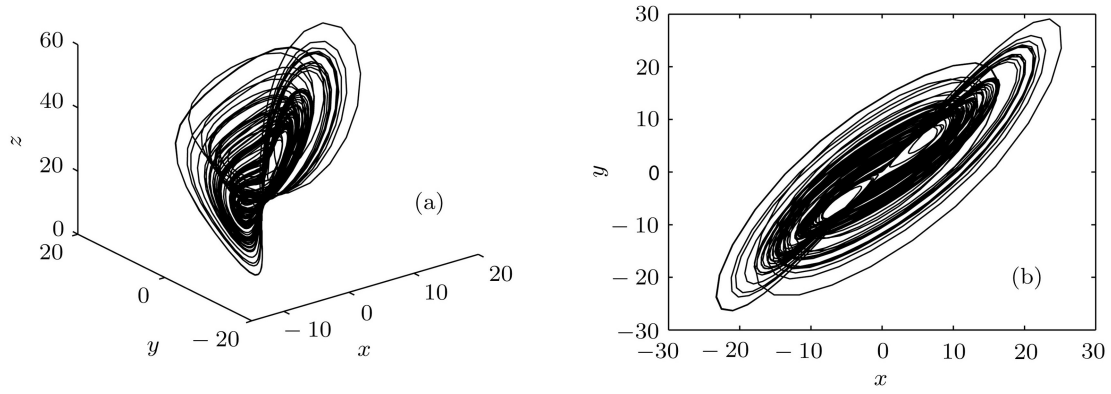


Fig. 1. Chaotic attractor: (a) Chen chaotic attractor (3D), (b) phase diagram (x–y).

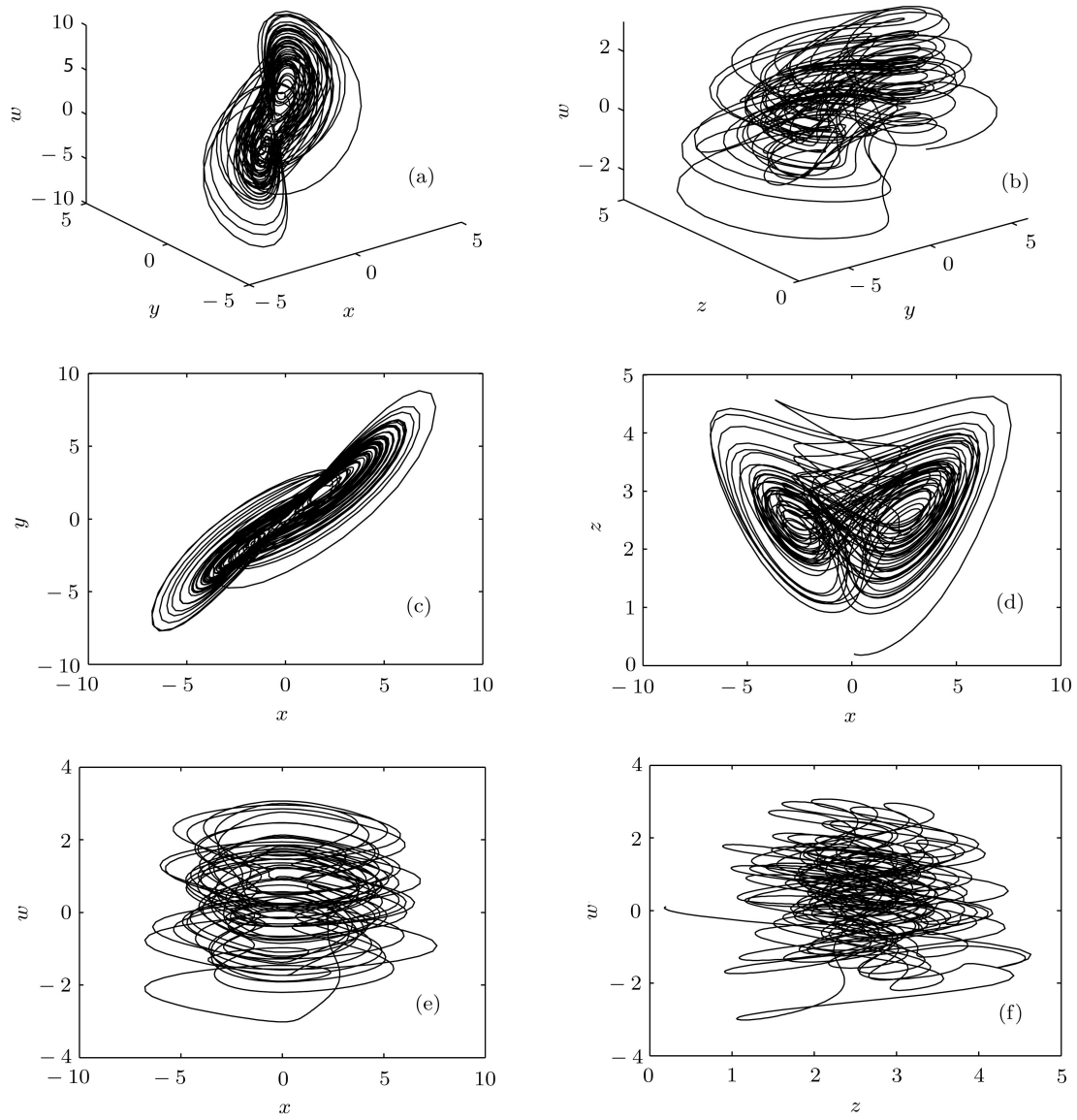


Fig. 2. Hyperchaotic attractor: (a) hyperchaotic attractor in (x, y, w) space, (b) hyperchaotic attractor in (y, z, w) space, (c) phase diagram (x–y), (d) phase diagram (x–z), (e) phase diagram (x–w), and (f) phase diagram (z–w).

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = 4x - 10xz + cy + 4w, \\ \dot{z} = y^2 - bz, \\ \dot{w} = -dx. \end{cases} \tag{2}$$

Four Lyapunov exponents are $\lambda_1 = 1.360$, $\lambda_2 = 0.258$, $\lambda_3 = 0$ and $\lambda_4 = -20.851$. Where x, y, z and w are state variables and $a = 35$, $b = 3$, $c = 21$ and $d = 2$ are parameters. Hyperchaotic Chen attractors and phase diagrams are shown in Figs. 2(a)–2(f); and bifurcation diagram of state variable x with the parameter c is shown in Fig. 3. Only the local maximum and minimum of the state variable x are drawn in Fig. 3.

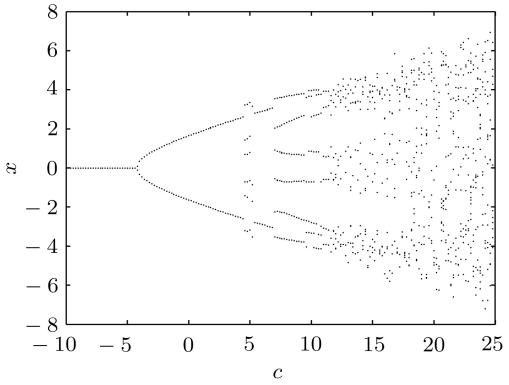


Fig. 3. Bifurcation diagram of state variable x with the parameter c.

3. Basic dynamic behaviour analysis

3.1. Symmetry

Applying the coordinate transformation to the original system: $(x, y, z, w) \to (-x, -y, z, -w)$, if the equations of state remain unchanged, it means that the system's map is symmetric about the z axis.

3.2. Dissipation and attractor existence

The divergence of the system (2) is defined by

$$\begin{aligned} \nabla V &= \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} \\ &= -a + c - b \\ &= -17. \end{aligned} \tag{3}$$

Because of $\nabla V < 0$, the system (2) is a dissipative system, and converges with an index rate of e^{-17t}. Volume element V_0 shrinks to $V_0\mathrm{e}^{-17t}$ at time t. When $t \to \infty$, volume element V_0 shrinks to 0. Therefore, all trajectories of the system will be confined to a congregation, whose volume is 0. And the gradual movement behaviours are fixed in an attractor.

3.3. Equilibrium and stability

Let the right-hand side of Eq. (2) equal 0,

$$\begin{cases} a(y - x) = 0, \\ 4x - 10xz + cy + 4w = 0, \\ y^2 - bz = 0, \\ -dx = 0, \end{cases} \tag{4}$$

this means that the only equilibrium $O(0, 0, 0, 0)$ is obtained.

For the given hyperchaotic Chen system (2), the Jacobian matrix at the equilibrium $O(0, 0, 0, 0)$ is

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ 4 & c & 0 & 4 \\ 0 & 0 & -b & 0 \\ -d & 0 & 0 & 0 \end{bmatrix}. \tag{5}$$

Thus the corresponding characteristic equation can be obtained:

$$f(\lambda) = (\lambda + b)(\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3) = 0, \tag{6}$$

where

$$\begin{cases} a_1 = a - c, \\ a_2 = -ac - 4a, \\ a_3 = 4ad, \end{cases} \tag{7}$$

we then obtain

$$\lambda_1 = -b, \tag{8}$$

and

$$f_1(\lambda) = \lambda^3 + (a - c)\lambda^2 + (-ac - 4a)\lambda + 4ad = 0. \tag{9}$$

When $b > 0$, then $\lambda_1 < 0$. If we make the system (2) stable at the equilibrium $O(0, 0, 0, 0)$ and $f_1(\lambda) = 0$, then λ_2, λ_3 and λ_4 must be less than 0. According to Routh–Hurwitz theorem, the system (2) being in stable equilibrium $O(0, 0, 0, 0)$ must satisfy the following conditions:

$$\begin{cases} b > 0, \\ a_1 = a - c > 0, \\ a_3 = 4ad > 0, \\ a_1a_2 - a_3 = (a - c)(-ac - 4a) - 4ad > 0. \end{cases} \tag{10}$$

When $a > 0$, then from the formula (10): $d > 0$, $c < a$ and $c < c_1 < 0$ or $0 < c_2 < c$, where

$$c_1 = \{(a - 4) - \sqrt{(a - 4)^2 + 16(a + d)}\}/2,$$

and

$$c_2 = \{(a-4) + \sqrt{(a-4)^2 + 16(a+d)}\}/2.$$

Because of $a < c_2$ and $c < a$, so we choose

$$c < c_1 = \{(a-4) - \sqrt{(a-4)^2 + 16(a+d)}\}/2,$$

this means that the system (2) stabilizing to the equilibrium $O(0,0,0,0)$ must satisfy the following conditions:

$$\begin{cases} a, b, d > 0, \\ c < \dfrac{(a-4) - \sqrt{(a-4)^2 + 16(a+d)}}{2}. \end{cases} \tag{11}$$

In this paper, we choose parameters $a = 35$, $b = 3$ and $d = 2$, then $c < -4.2041$.

When $a < 0$, from the formula (10), we have $d < 0$ and $c < a$.

(i) When $-4 < a < 0$, we have $-(a+4)^2/16 < d < 0$ and $c_1 < c < c_2$, where

$$c_1 = \{(a-4) - \sqrt{(a-4)^2 + 16(a+d)}\}/2,$$

and

$$c_2 = \{(a-4) + \sqrt{(a-4)^2 + 16(a+d)}\}/2.$$

Because $a > c_2$ and $c < a$, so we choose $c_1 < c < c_2$, this means that the system (2) stabilizing to the equilibrium $O(0,0,0,0)$ must satisfy the following conditions:

$$\begin{cases} -4 < a < 0, \\ b > 0, \\ c_1 < c < c_2, \\ -\dfrac{(a+4)^2}{16} < d < 0, \end{cases} \tag{12}$$

where

$$c_1 = \{(a-4) - \sqrt{(a-4)^2 + 16(a+d)}\}/2,$$

and

$$c_2 = \{(a-4) + \sqrt{(a-4)^2 + 16(a+d)}\}/2.$$

(ii) When $a < -4$, then we have $-(a+4)^2/16 < d < 0$ and $c_1 < c < c_2$, where

$$c_1 = \{(a-4) - \sqrt{(a-4)^2 + 16(a+d)}\}/2,$$

and

$$c_2 = \{(a-4) + \sqrt{(a-4)^2 + 16(a+d)}\}/2.$$

Because $c_1 < a < c_2$ and $c < a$, so we choose $c_1 < c < a$, this means that the system (2) stabilizing to the equilibrium $O(0,0,0,0)$ must satisfy the following conditions:

$$\begin{cases} a < -4, \\ b > 0, \\ c_1 < c < a, \\ -\dfrac{(a+4)^2}{16} < d < 0, \end{cases} \tag{13}$$

where

$$c_1 = \{(a-4) - \sqrt{(a-4)^2 + 16(a+d)}\}/2.$$

3.4. The impact of system parameters

In this paper, we choose $a = 35$, $b = 3$ and $d = 2$, and change the value of c, the system (2) shows different attractors and different forms of movement as follows:

(I) When $c = -5$, the system (2) stabilizes to the equilibrium $O(0,0,0,0)$ in Figs. 4(a) and 4(b).

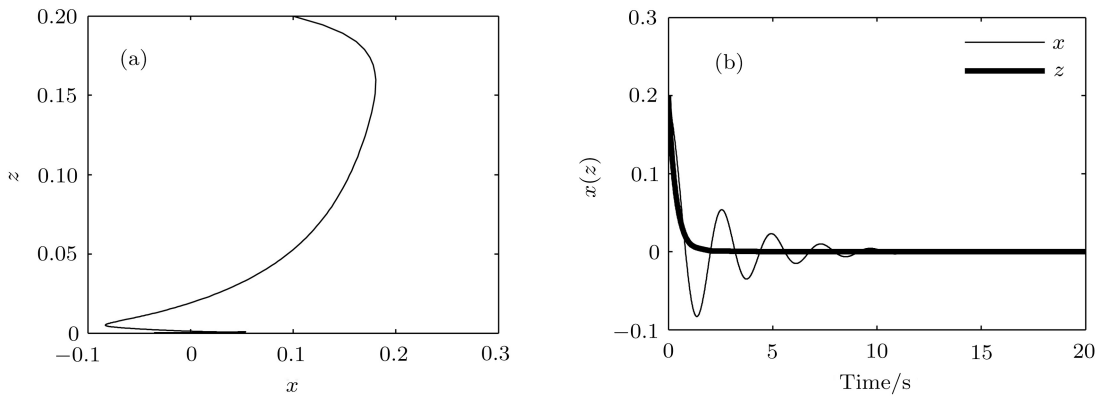


Fig. 4. Phase diagram and state trajectories: (a) phase diagram (x–z), (b) state trajectories of x and z.

(II) When $c = 0$, the system (2) exhibits the periodic motion, and results in the limit cycle as shown in Figs. 5(a) and 5(b).

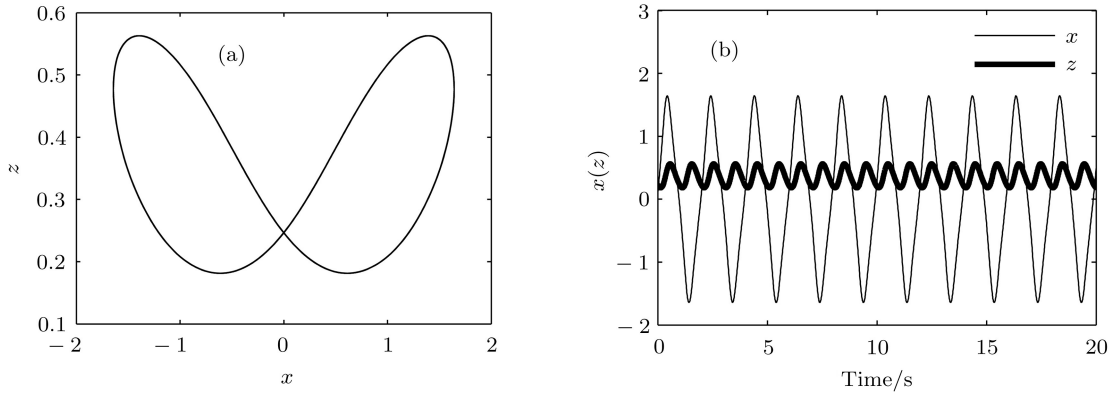


Fig. 5. Phase diagram and state trajectories: (a) phase diagram (x–z), (b) state trajectories of x and z.

(III) When $c = 5$, the system (2) exhibits quasi-periodic motion, and results in the 2D torus as shown in Figs. 6(a) and 6(b).

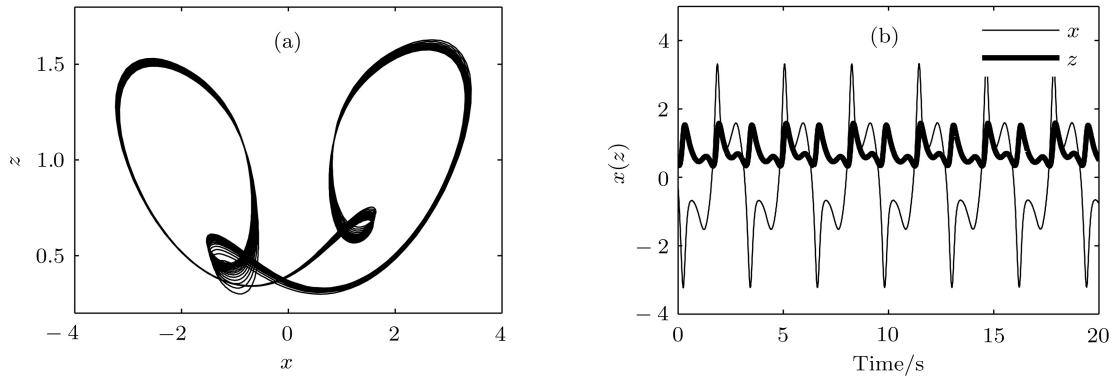


Fig. 6. Phase diagram and state trajectories: (a) phase diagram (x–z), (b) state trajectories of x and z.

(IV) When $c = 10$, the system (2) exhibits periodic motion, and results in the complex limit cycle as shown in Figs. 7(a) and 7(b).

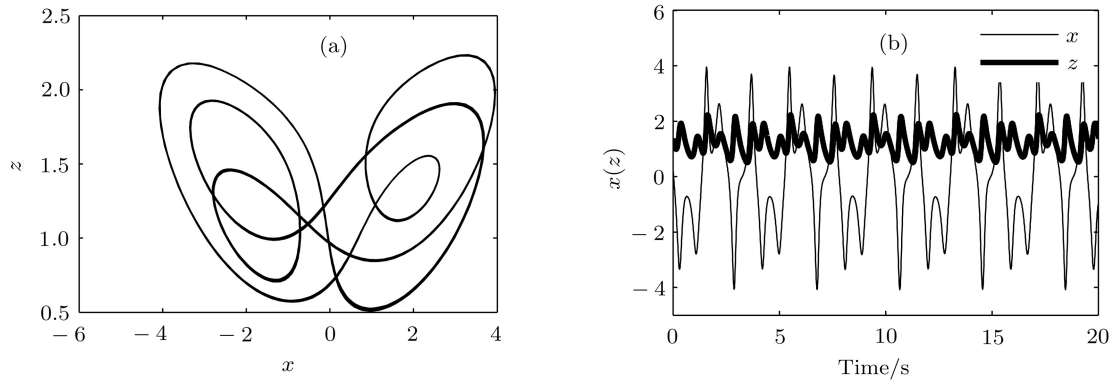


Fig. 7. Phase diagram and state trajectories: (a) phase diagram (x–z), (b) state trajectories of x and z.

(V) When $c = 13$, the system (2) exhibits chaotic motion, and results in the chaotic attractor as shown in Figs. 8(a) and 8(b).

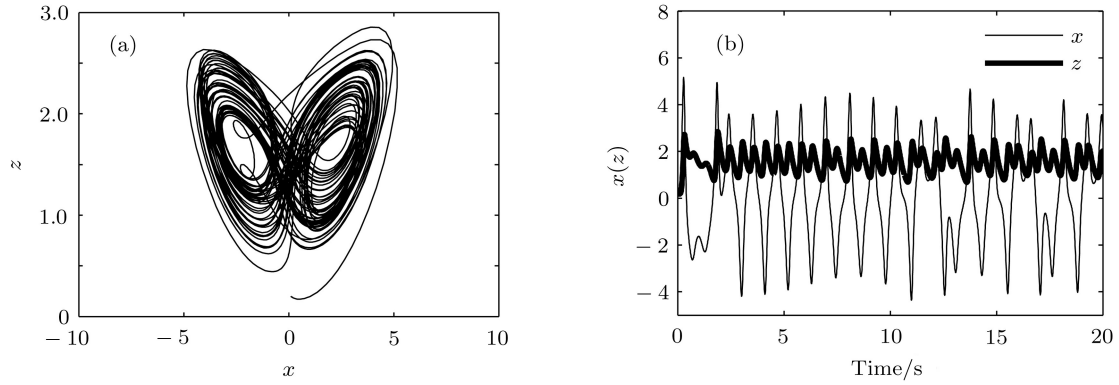


Fig. 8. Phase diagram and state trajectories: (a) phase diagram (x–z), (b) state trajectories of x and z.

(VI) When $c = 21$, the system (2) exhibits hyperchaotic motion, and results in the hyperchaotic attractor as shown in Figs. 9(a) and 9(b).

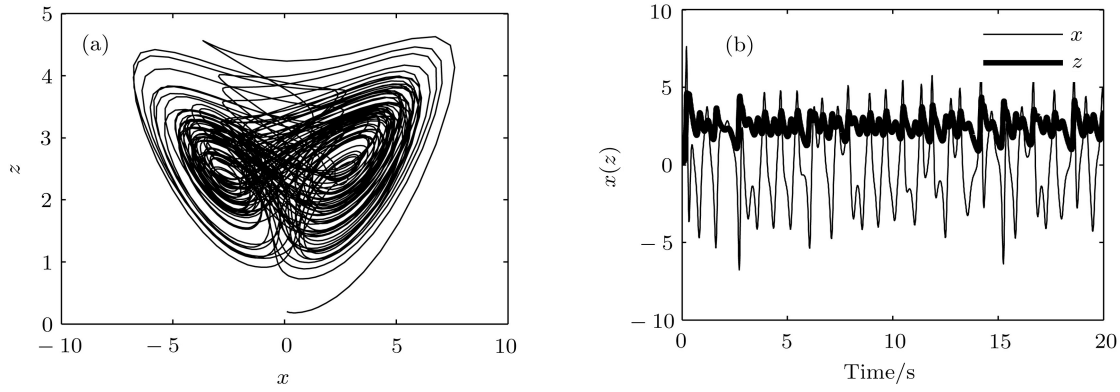


Fig. 9. Phase diagram and state trajectories: (a) phase diagram (x–z), (b) state trajectories of x and z.

4. Generalized synchronization in coupled 4D hyperchaotic Chen system with unknown parameters

Consider the following nonlinear chaotic systems:

$$\dot{x} = F(x), \tag{14}$$

$$\dot{y} = F(y) + u, \tag{15}$$

where $x, y \in R^n$ are state variables; F is a differentiable function of $R^n \to R^n$ and u is a controller.

The system (14) is a driving system and the system (15) is a response system. We define the generalized synchronization error as

$$e = y - \alpha x, \tag{16}$$

where $e = (e_1, e_2, e_3, ..., e_n)^{\mathrm{T}}$.

If there is a constant $\alpha \neq 0$, so that

$$\lim_{t\to\infty} e = \lim_{t\to\infty} [\, y(t) - \alpha x(t) \,] = 0, \tag{17}$$

systems (14) and (15) achieve the generalized synchronization, where $\alpha \neq 0$ is a scale factor.

In this paper, we choose a new 4D hyperchaotic Chen system as a driving system:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1), \\ \dot{x}_2 &= 4x_1 - 10x_1x_3 + cx_2 + 4x_4, \\ \dot{x}_3 &= x_2^2 - bx_3, \end{aligned}$$

$$\dot{x}_4 = -dx_1. \tag{18}$$

The response system is:

$$\begin{aligned}
\dot{y}_1 &= a_1(y_2 - y_1) + u_1,\\
\dot{y}_2 &= 4y_1 - 10y_1y_3 + c_1y_2 + 4y_4 + u_2,\\
\dot{y}_3 &= y_2^2 - b_1y_3 + u_3,\\
\dot{y}_4 &= -d_1y_1 + u_4,
\end{aligned} \tag{19}$$

where $x = (x_1, x_2, x_3, x_4)^{\mathrm{T}}$ and $y = (y_1, y_2, y_3, y_4)^{\mathrm{T}}$ are state variables; a, b, c, d and a_1, b_1, c_1, d_1 are unknown parameters of the driving system and the response system respectively; $u = (u_1, u_2, u_3, u_4)^{\mathrm{T}}$ is a controller.

Referring to Ref. [15], the driving system and the response system can be written in the following form:

$$\begin{bmatrix} \dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0\\ 4x_1 - 10x_1x_3 + 4x_4\\ x_2^2\\ 0 \end{bmatrix} + \begin{bmatrix} x_2 - x_1 & 0 & 0 & 0\\ 0 & 0 & x_2 & 0\\ 0 & -x_3 & 0 & 0\\ 0 & 0 & 0 & -x_1 \end{bmatrix} \begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}, \tag{20}$$

$$\begin{bmatrix} \dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} 0\\ 4y_1 - 10y_1y_3 + 4y_4\\ y_2^2\\ 0 \end{bmatrix} + \begin{bmatrix} y_2 - y_1 & 0 & 0 & 0\\ 0 & 0 & y_2 & 0\\ 0 & -y_3 & 0 & 0\\ 0 & 0 & 0 & -y_1 \end{bmatrix} \begin{bmatrix} a_1\\ b_1\\ c_1\\ d_1 \end{bmatrix} + \begin{bmatrix} u_1\\ u_2\\ u_3\\ u_4 \end{bmatrix}, \tag{21}$$

and let

$$g(x) = \begin{bmatrix} 0\\ 4x_1 - 10x_1x_3 + 4x_4\\ x_2^2\\ 0 \end{bmatrix}, \qquad g(y) = \begin{bmatrix} 0\\ 4y_1 - 10y_1y_3 + 4y_4\\ y_2^2\\ 0 \end{bmatrix},$$

$$G(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 & 0\\ 0 & 0 & x_2 & 0\\ 0 & -x_3 & 0 & 0\\ 0 & 0 & 0 & -x_1 \end{bmatrix}, \qquad G(y) = \begin{bmatrix} y_2 - y_1 & 0 & 0 & 0\\ 0 & 0 & y_2 & 0\\ 0 & -y_3 & 0 & 0\\ 0 & 0 & 0 & -y_1 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}, \qquad \Phi_1 = \begin{bmatrix} a_1\\ b_1\\ c_1\\ d_1 \end{bmatrix} \qquad \text{and} \qquad u = \begin{bmatrix} u_1\\ u_2\\ u_3\\ u_4 \end{bmatrix}.$$

The error system between the driving system (20) and the response system (21) can be expressed by

$$\begin{aligned}
\dot{e} = \dot{y} - \alpha\dot{x} &= g(y) + G(y)\Phi_1 - \alpha g(x)\\
&\quad - \alpha G(x)\Phi + u.
\end{aligned} \tag{22}$$

The problem of generalized synchronization of the original system can be transformed into the stability of the error system.

In this paper, we choose the controller in the following form:

$$\begin{aligned}
u = &- g(y) + \alpha g(x) - G(y)\Phi\\
&+ \alpha G(x)\Phi - ke,
\end{aligned} \tag{23}$$

where k is a constant and $k > 0$.

Substituting $g(x)$, $G(x)$, $g(y)$, $G(y)$, and Φ into formula (23), then the control laws can be expressed

by the following formula:

$$\begin{cases} u_1 = -a(e_2 - e_1) - ke_1, \\ u_2 = -4(e_1 + e_4) + 10[e_1e_3 + \alpha(e_1x_3 + e_3x_1) \\ \qquad + \alpha(\alpha - 1)x_1x_3] - ce_2 - ke_2, \\ u_3 = -e_2^2 - 2\alpha e_2x_2 - \alpha(\alpha - 1)x_2^2 + be_3 - ke_3, \\ u_4 = de_1 - ke_4. \end{cases} \tag{24}$$

The adaptive law of unknown parameter Φ_1 is chosen as the following form:

$$\dot{\Phi}_1 = -N[G(y)]^{\mathrm{T}}e, \tag{25}$$

where N is a constant and $N > 0$, N controls the speed of convergence of parameters Φ_1.

For the error system (22), the Lyapunov function is chosen as follows:

$$V = \frac{N}{2}e^{\mathrm{T}}e + \frac{1}{2}\Psi^{\mathrm{T}}\Psi > 0, \tag{26}$$

where $\Psi = \Phi_1 - \Phi$ and $\dot{\Psi} = \dot{\Phi}_1$.

Substituting the control law (23) into the formula (22), then the error system can be expressed by the following formula:

$$\dot{e} = G(y)\Psi - ke. \tag{27}$$

In this paper, by substituting $G(y)$ and Ψ into formula (27), the error system can be expressed by the following formula:

$$\begin{cases} \dot{e}_1 = (a_1 - a)(y_2 - y_1) - ke_1, \\ \dot{e}_2 = (c_1 - c)y_2 - ke_2, \\ \dot{e}_3 = -(b_1 - b)y_3 - ke_3, \\ \dot{e}_4 = -(d_1 - d)y_1 - ke_4. \end{cases} \tag{28}$$

Then we have

$$\begin{aligned} \dot{V} &= \frac{N}{2}(\dot{e}^{\mathrm{T}}e + e^{\mathrm{T}}\dot{e}) + \frac{1}{2}(\dot{\Psi}^{\mathrm{T}}\Psi + \Psi^{\mathrm{T}}\dot{\Psi}) \\ &= \frac{N}{2}[(\Psi^{\mathrm{T}}[G(y)]^{\mathrm{T}} - ke^{\mathrm{T}})e + e^{\mathrm{T}}(G(y)\Psi - ke)] \\ &\quad + \frac{1}{2}[-Ne^{\mathrm{T}}G(y)\Psi + \Psi^{\mathrm{T}}(-N[G(y)]^{\mathrm{T}}e)] \\ &= -Nke^{\mathrm{T}}e < 0. \end{aligned} \tag{29}$$

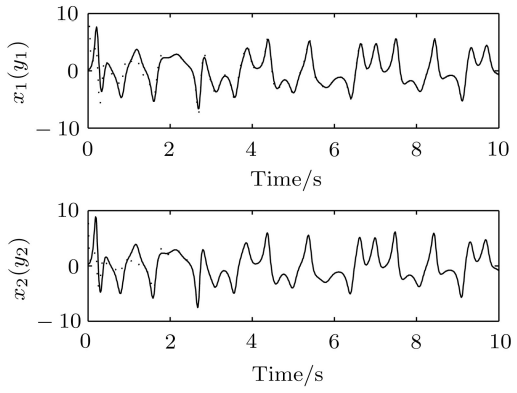
Because $N > 0$ and $k > 0$, then $V'(t) < 0$, and $\lim_{t\to\infty} e(t) = 0$. Finally the generalized synchronization will be accomplished.

5. Simulation results

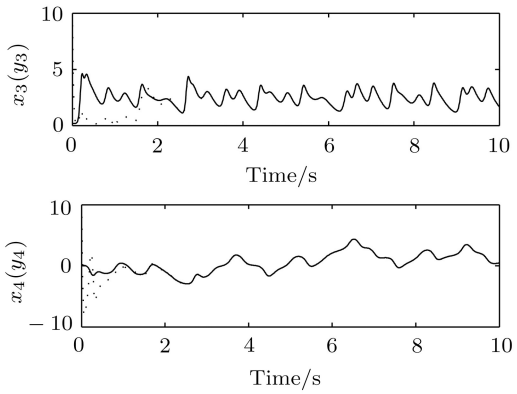
In what follows, through calculating synchronization error e, numerical simulation results have illustrated the effectiveness of this method to accomplish generalized synchronization in the coupled 4D hyperchaotic Chen system with unknown parameters.

In this section, the synchronization between the driving system (20) and the response system (21) is demonstrated. The values of initial conditions of the driving system and the response system are given below: $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.1, 0.2, 0.2, 0.1)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (10, 10, 10, 10)$. Choose the unknown parameters of the driving system $\Phi = (a, b, c, d)^{\mathrm{T}} = (35, 3, 21, 2)^{\mathrm{T}}$ and the value of initial conditions of unknown parameters of the response system $\Phi_1(0) = (a_1(0), b_1(0), c_1(0), d_1(0))^{\mathrm{T}} = (25, 10, 15, 10)^{\mathrm{T}}$, then choose the control parameters $k = 5$ and $N = 30$.

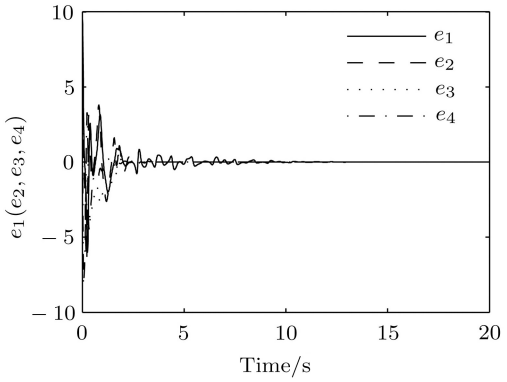
Case 1 By choosing $\alpha = 1$, a complete synchronization is achieved in the coupled 4D hyperchaotic Chen system with unknown parameters. Numerical simulation results are shown in Figs. 10(a)–10(d).



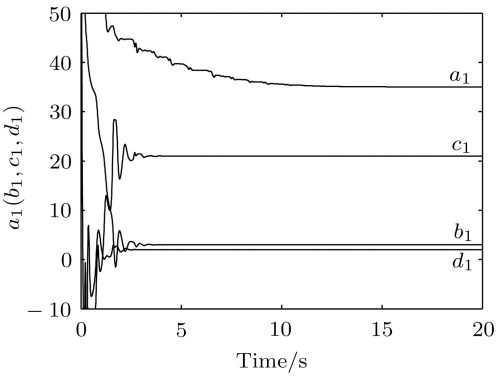
(a) State trajectories (x_1, x_2, y_1, y_2)



(b) State trajectories (x_3, x_4, y_3, y_4)



(c) Synchronization errors with time t



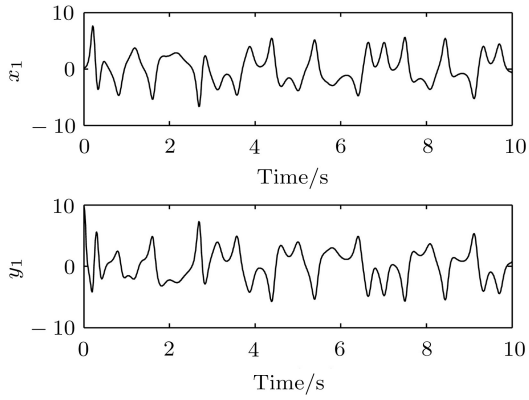
(d) Unknown parameters Φ_1 with time t

Fig. 10. (a) Time waveforms of state trajectories x_1 (solid), x_2 (solid) and y_1 (dashed), y_2 (dashed); (b) time waveforms of state trajectories x_3 (solid), x_4 (solid) and y_3 (dashed), y_4 (dashed); (c) time waveforms of synchronization errors e; and (d) time waveforms of unknown parameters Φ_1.

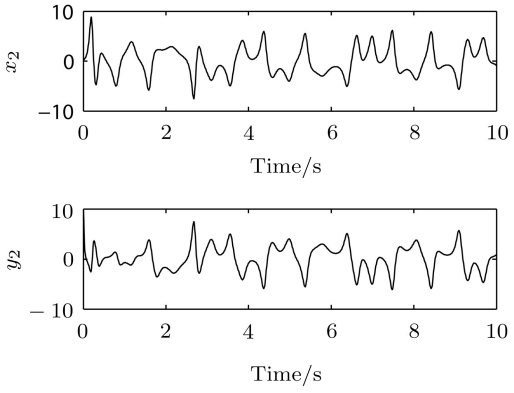
Case 2 By choosing $\alpha = -1$, an anti-synchronization is achieved in the coupled 4D hyperchaotic Chen system with unknown parameters. Numerical simulation results are shown in Figs. 11(a)–11(f).

Case 3 By choosing $\alpha = -2$, a generalized anti-synchronization is gained in the coupled 4D hyperchaotic Chen system with unknown parameters. Numerical simulation results are shown in Figs. 12(a)–12(d).

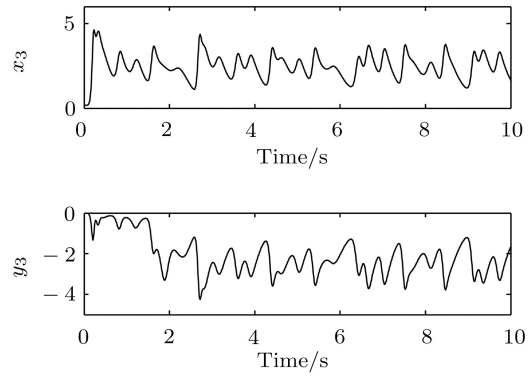
From the above numerical simulation results, the limit of the synchronization error vector $e(t) = (e_1, e_2, e_3, e_4)^{\mathrm{T}}$ asymptotically approaches zero. So, by making use of the drive-response method, the generalized synchronization in the coupled 4D hyperchaotic Chen system with unknown parameters can be accomplished and the unknown parameters $\Phi_1 = (a_1, b_1, c_1, d_1)^{\mathrm{T}}$ asymptotically approach to $\Phi = (a, b, c, d)^{\mathrm{T}}$.



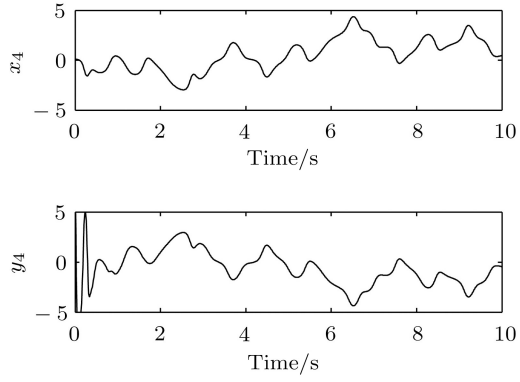
(a) State trajectories (x_1, y_1)



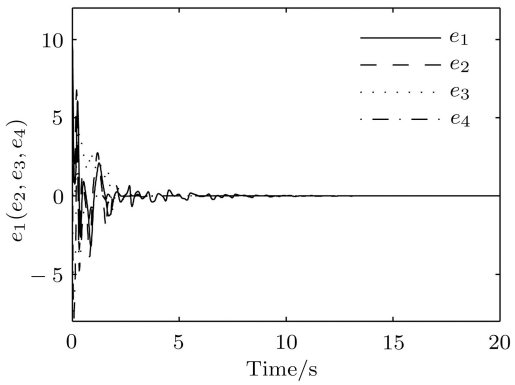
(b) State trajectories (x_2, y_2)



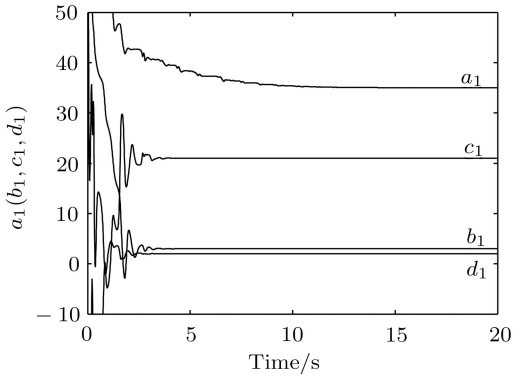
(c) State trajectories (x_3, y_3)



(d) State trajectories (x_4, y_4)

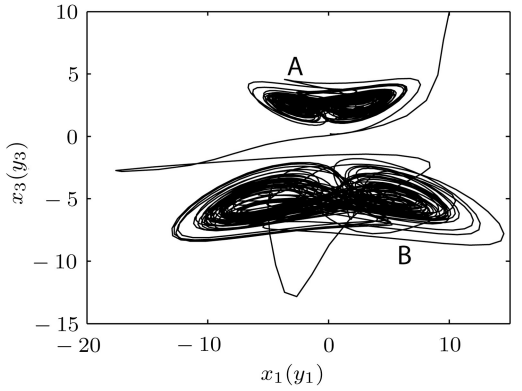


(e) Synchronization errors with time t

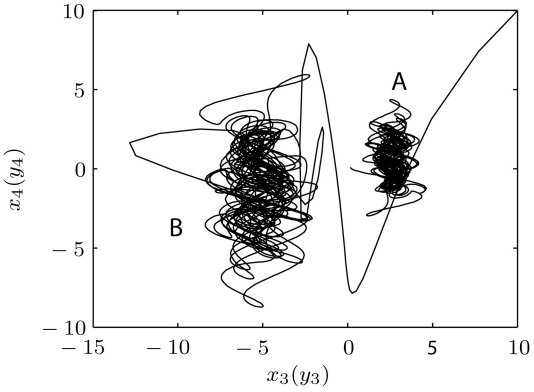


(f) Unknown parameters Φ_1 with time t

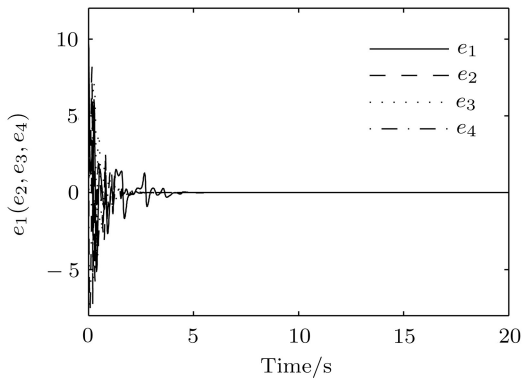
Fig. 11. (a) Time waveforms of state trajectories x_1 and y_1; (b) time waveforms of state trajectories x_2 and y_2; (c) time waveforms of state trajectories x_3 and y_3; (d) time waveforms of state trajectories x_4 and y_4; (e) time waveforms of synchronization errors e; and (f) time waveforms of unknown parameters Φ_1.



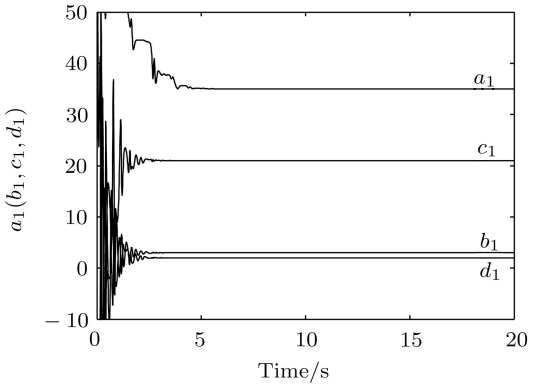
(a) Phase diagram (x_1–x_3 and y_1–y_3)



(b) Phase diagram (x_3–x_4 and y_3–y_4)



(c) Synchronization errors with time t



(d) Unknown parameters Φ_1 with time t

Fig. 12. (a) Phase diagrams of the driving system (A) and response system (B): x_1–x_3 and y_1–y_3; (b) phase diagrams of the driving system (A) and response system (B): x_3–x_4 and y_3–y_4; (c) time waveforms of synchronization errors e; and (d) time waveforms of unknown parameters Φ_1.

6. Conclusions

First, based on the Chen system and other 4D Chen system, and by increasing the state variables and changing nonlinear terms and parameters of the Chen system, a new 4D hyperchaotic Chen system is constructed. Research on the basic dynamic behaviours of the new 4D hyperchaotic Chen system shows that the

new 4D hyperchaotic Chen system has abundant dynamic behaviours. Secondly, Lyapunov stability theory has been used, and the control laws $u(t)$ and adaptive laws of parameters are designed to accomplish the generalized synchronization in the coupled 4D hyperchaotic Chen system with unknown parameters. And thirdly, by improving the adaptive laws of parameters in Ref. [15], and selecting the appropriate Lyapunov function, the synchronization errors converge faster and the effect of synchronization is better in comparison with the method in Ref. [15].

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