Comparative studies of open qubit via Bloch equation and master equation of Redfield form*

Shen Yan(沈 燕), Lu Yu-Ping(卢俞平), and Liang Xian-Ting(梁先庭)†

Department of Physics and Institute of Modern Physics, Ningbo University, Ningbo 315211, *China*

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In this paper, we investigate the dynamics of an open qubit model by solving two sets of its reduced dynamical equations. One set of the equations is the well-known Bloch equations and the other is the widely investigated master equations of Redfield form. Both of them are obtained from the perturbation approximation which demands the system of interest weakly coupled to its environment. It is shown that the qubit has a longer decoherence and relaxiation time as the dynamics is described by the Redfield equantions.

Keywords: reduced dynamics, relaxation, decoherence

PACC: 0365, 0367, 3325B

1. Introduction

It is very important to investigate the reduced dynamics of open quantum systems because the evolutions of the quantum systems of interest are deduced from them. Many methods that focus on the reduced dynamics of quantum systems have been developed in the past several decades.[1,2] It has been shown that at low temperature non-Markov effects are important, and some non-Markov schemes for solving the dynamics of quantum systems with few levels have been developed[3−9] and applied,[10−13] recently. However, in general, when the cut-off frequency of the bath is large when compared to the characteristic frequency of the quantum system of interest, the Markov approximation is exact enough. Under the Markov approximation, the methods of the Bloch equations[14−16] and the master equations of Redfield form[17−21] are two important and straightforward schemes to obtain the dynamics of open quantum systems. The former, the method of Bloch equations, is developed from the studies of nuclear magnetic resonance (NMR) and has been widely used in magnetic resonance imaging (MRI), electron spin resonance (ESR), quantum optics, and many other fields of physics.[22] The latter, the method of the master equations of Redfield form, has also been successfully applied in many fields of physics[17,23,24] and chemistry,[25,26] etc. In particular, these two kinds of methods have been employed in the field of quantum communication and quantum calculation,[27−32] recently.

In fact, the two kinds of methods have the same origin. Both of them can be derived from the quantum Liouville equation based on the perturbation and Born–Markov approximations. But their final forms are always different. It is interesting to know what is the difference of the dynamics obtained from the different methods. In particular, we want to know that for the same few levels system, for example a qubit, what is the difference of the decoherence and relaxation calculated using the two methods. In this paper, we shall compare the two kinds of methods by investigating the dynamics of an open qubit model. In the following, we at first introduce a qubit model then investigate the dynamics of it using the Bloch and Redfield methods, finally some comparisons and a conclusion will be given based on the results.

2. Dynamics of a qubit model

We start with introducing the Hamiltonian of an open two-level system (TLS), namely an open qubit model as follows:

$$H = H_{\rm TLS} + H_{\rm B} + H_{\rm I}. \tag{1}$$

This Hamiltonian includes three terms, the Hamiltonian of the system of interest (qubit) $H_{\rm TLS}$, the Hamil-

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†Corresponding author. E-mail: xtliang@ustc.edu

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tonian of bath $H_{\rm B}$ which is supposed to be constructed with a set of harmonic oscillators, and the Hamiltonian of interaction between the TLS and bath, $H_{\rm I}$. We set an open qubit model as[14]

$$H_{\rm TLS} = \hbar\omega_0 \left|1\right\rangle \left\langle 1\right|,$$
$$H_{\rm B} = \sum_k \hbar\omega_k \left(b_k^\dagger b_k + \frac{1}{2}\right), \tag{2}$$
$$H_{\rm I} = \lambda[\hbar\Lambda \left|1\right\rangle \left\langle 0\right| + \hbar\Lambda^\dagger \left|0\right\rangle \left\langle 1\right|], \tag{3}$$

where

$$\Lambda = \sum_k c_k (b_k^\dagger + b_k). \tag{4}$$

Here, $b_k^\dagger$ and b_k are creation and annihilation operators of the bath modes, and λ is a controlling parameter for convenience of discussion.

For the investigating schemes that the total system is described with the microcosmic dynamical model plus its phenomenological environment, the spectral density function of bath is in fact a bridge to connect the microcosmic and macroscopical physical quantities of environment. According to Refs. [1] and [2], we introduce the spectral density as

$$J(\omega) = 2\pi \sum_k |c_k|^2 \, \delta(\omega - \omega_k), \tag{5}$$

which includes all the effects of the bath in the dynamical framework. To different environments, one always uses the macroscopical Ohmic, sub-Ohmic, and super-Ohmic spectral density functions to describe the bath. The spectral density function is defined as

$$J(\omega) = \frac{\pi}{2}\eta\omega \left(\frac{\omega}{\omega_{\rm c}}\right)^{s-1} {\rm e}^{-\omega/\omega_{\rm c}}, \tag{6}$$

where η is the dimensionless damping strength and $\omega_{\rm c}$ is the cut-off frequency of the bath. According to the different s, the spectral density functions belong to the Ohmic bath ($s = 1$), super-Ohmic bath ($s > 1$), and sub-Ohmic bath ($s < 1$).[1] In the following, we use the spectral density functions and the Bloch and Redfield equations to investigate the dynamics of the open qubit model of Eq. (1).

2.1. Bloch equation approach

In this subsection, we introduce the generalized Bloch equations as the interaction of system and bath up to the second-order perturbation. This derivation closely follows the general approach discussed by Budimir and Skinner.[14] The equation for the time evolution of the total density operator $\rho(t)$ of the total system can be generally described with the Liouville equation as

$$\frac{\partial\rho(t)}{\partial t} = -\frac{\rm i}{\hbar}[H, \rho(t)]. \tag{7}$$

Transforming it to the interaction picture, we have

$$\frac{\partial\tilde{\rho}(t)}{\partial t} = -\,{\rm i}\,\tilde{L}(t)\tilde{\rho}(t).$$

Here, $\tilde{\rho}(t)$ is defined by

$$\tilde{\rho}(t) \equiv {\rm e}^{{\rm i}\,H_0 t/\hbar}\rho(t){\rm e}^{-{\rm i}\,H_0 t/\hbar},$$

and the Liouville superoperator $\tilde{L}(t)$ is

$$\tilde{L}(t)\ldots \equiv \frac{1}{\hbar}[\tilde{H}_{\rm I}(t), \ldots],$$

where

$$\tilde{H}_{\rm I}(t) \equiv {\rm e}^{{\rm i}\,H_0 t/\hbar} H_{\rm I} {\rm e}^{-{\rm i}\,H_0 t/\hbar},$$

and H_0 is the Hamiltonian of the system of interest plus the Hamiltonian of the bath, namely,

$$H_0 = H_{\rm TLS} + H_{\rm B}.$$

From Eqs. (3) and (4), we have

$$\tilde{H}_{\rm I}(t) = \lambda[\hbar\tilde{\Lambda}(t) \left|1\right\rangle \left\langle 0\right| + \hbar\tilde{\Lambda}(t)^\dagger \left|0\right\rangle \left\langle 1\right|$$

with

$$\tilde{\Lambda}(t) = {\rm e}^{{\rm i}\,t\omega_0}\Lambda(t),$$

where

$$\Lambda(t) = \sum_k c_k (b_k^\dagger {\rm e}^{{\rm i}\,t\omega_k} + b_k {\rm e}^{-{\rm i}\,t\omega_k}). \tag{8}$$

The formal solution to the Liouville equation is

$$\tilde{\rho}(t) = \tilde{\rho}(0) - {\rm i}\int_0^t {\rm d}t_1 \tilde{L}(t_1)\tilde{\rho}(0) - \int_0^t {\rm d}t_1 \int_0^{t_1} {\rm d}t_2 \tilde{L}(t_1)\tilde{L}(t_2)\tilde{\rho}(0) + \cdots, \tag{9}$$

which has been iterated to give a perturbation series. Defining a reduced TLS density matrix

$$\sigma(t) \equiv {\rm Tr}_{\rm B}[\,\rho(t)],$$

and

$$\tilde{\sigma}(t) \equiv {\rm Tr}_{\rm B}[\,\tilde{\rho}(t)], \tag{10}$$

we have

$$\tilde{\sigma}(t) = \tilde{\sigma}(0) - {\rm i}\int_0^t {\rm d}t_1 \, {\rm Tr}_{\rm B}[\tilde{L}(t_1)\rho_{\rm B}] - \int_0^t {\rm d}t_1 \int_0^{t_1} {\rm d}t_2 \, {\rm Tr}_{\rm B}[\tilde{L}(t_1)\tilde{L}(t_2)\rho_{\rm B}] + \cdots.$$

Here, $\mathrm{Tr_B}[\cdots]$ denotes the trace over the bath modes, and we set the initial state of the total system be the product state of the system of interest and the bath, and the latter is set in the equilibrium state, namely, $\rho_{\mathrm{B}} = \mathrm{e}^{(-\beta H_{\mathrm{B}})}/\mathrm{tr_B}(\mathrm{e}^{\beta H_{\mathrm{B}}})$, where $\beta = 1/kT$. By defining

$$\lambda^n M^{(n)}(t) \equiv (-\mathrm{i})^n n! \int_0^t \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 \ldots \times \int_0^{t_{n-1}} \mathrm{d}t_n \,\mathrm{Tr}_b[\tilde{L}(t_1)\tilde{L}(t_2)\ldots\tilde{L}(t_n)\rho_b]$$

with $M^{(0)} \equiv 1$, and

$$\Phi(t) \equiv \sum_{n=0}^{\infty} \frac{\lambda^{2n}}{(2n)!} M^{(2n)}(t),$$

one has

$$\dot{\tilde{\sigma}}(t) = W(t)\tilde{\sigma}(t)$$

with

$$W(t) \equiv \dot{\Phi}(t)\Phi(t)^{-1}.$$

Defining

$$W(t) \equiv \sum_{n=1}^{\infty} \lambda^{2n} W^{(2n)}(t),$$

where $W^{(2n)}(t)$ denotes that obtained from the $2n$-order perturbation of the interaction Hamiltonian. For the second-order calculation, we have

$$\lambda^2 W^{(2)}(t) = -\int_0^t \mathrm{d}t_1 \,\mathrm{Tr}_b[\tilde{L}(t)\tilde{L}(t_1)\rho_b].$$

We can then obtain a set of equations in Schrödinger picture as (which uses the secular approximation as retaining terms $W_{\alpha\alpha'\beta\beta'}$ with $\alpha - \alpha' = \beta - \beta'$)[33,34]

$$\begin{aligned}
\dot{\sigma}_{00}(t) &= W_{0000}\sigma_{00}(t) + W_{0011}\sigma_{11}(t),\\
\dot{\sigma}_{11}(t) &= W_{1100}\sigma_{00}(t) + W_{1111}\sigma_{11}(t),\\
\dot{\sigma}_{10}(t) &= (-\mathrm{i}\,\omega_0 + W_{1010})\sigma_{10}(t) + W_{1001}\sigma_{01}(t),\\
\dot{\sigma}_{01}(t) &= (\mathrm{i}\,\omega_0 + W_{0101})\sigma_{01}(t) + W_{0110}\sigma_{10}(t), \quad (11)
\end{aligned}$$

where

$$W_{\alpha\alpha'\beta\beta'} = \lim_{t\to\infty} \mathrm{e}^{-\mathrm{i}(\alpha-\alpha'-\beta+\beta')\omega_0 t} \times \langle\alpha|\,\{W(t)\,|\beta\rangle\,\langle\beta'|\}\,|\alpha'\rangle\,.$$

From the probability conservation requirement of the state probabilities equations above, we know the relations

$$W_{0000} = -W_{1100}, \quad W_{1111} = -W_{0011},$$

Others, for the off-diagonal density matrix element pair, using $\sigma_{01}(t) = \sigma_{10}(t)^*$, the following must be true

$$W_{0101} = W_{1010}^*, \quad W_{0110} = W_{1001}^*.$$

For convenience, we define

$$\begin{aligned}
k_{10} &= -W_{0000}, \quad k_{01} = -W_{1111},\\
\frac{1}{T_2} &= -\,\mathrm{Re}(W_{1010}), \quad \Delta\omega = -\,\mathrm{Im}(W_{1010}). \quad (12)
\end{aligned}$$

In the weak-coupling limit, namely, $|W_{1001}| \ll \omega_0 - \mathrm{Im}\{W_{1010}\}$, setting $\Delta\omega = -\,\mathrm{Im}\{W_{1010}\}$, and $1/T_2 = -\,\mathrm{Re}\{W_{1010}\}$, one can obtain the Bloch equation

$$\begin{aligned}
\dot{\sigma}_{00}(t) &= -k_{10}\sigma_{00}(t) + k_{01}\sigma_{11}(t),\\
\dot{\sigma}_{11}(t) &= k_{10}\sigma_{00}(t) - k_{01}\sigma_{11}(t),\\
\dot{\sigma}_{10}(t) &= -\left[\mathrm{i}(\omega_0 + \Delta\omega) + \frac{1}{T_2}\right]\sigma_{10}(t) + k_{10}\sigma_{01}(t),\\
\dot{\sigma}_{01}(t) &= \left[\mathrm{i}(\omega_0 + \Delta\omega) - \frac{1}{T_2}\right]\sigma_{01}(t) + \frac{1}{T_2}\sigma_{10}(t). \quad (13)
\end{aligned}$$

For our model, by using the above definition, we have

$$k_{10}^{(2)} = \lim_{t\to\infty} \int_0^t \mathrm{d}t_1 \,\mathrm{Tr}_b[\tilde{\Lambda}^\dagger(t)\tilde{\Lambda}(t_1)\rho_b + \rho_b\tilde{\Lambda}^\dagger(t_1)\tilde{\Lambda}(t)]. \quad (14)$$

The bath correlation function is

$$C_1(t-t') = \mathrm{Tr_B}[\,\rho_{\mathrm{B}}\Lambda^\dagger(t)\Lambda(t')].$$

By making the variable change, $\tau = t - t_1$, and taking $t \to \infty$ limit, we have

$$k_{10}^{(2)} = 2\,\mathrm{Re}\left\{\int_0^\infty \mathrm{d}\tau \mathrm{e}^{-\mathrm{i}\,\omega_0\tau} C_1(\tau)\right\}.$$

The Fourier transformation of $C_1(\tau)$ is

$$C_1(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\,\omega\tau} C_1(\tau). \quad (15)$$

So we have

$$k_{10}^{(2)} = \lim_{\epsilon\to\infty} \frac{1}{\pi}\,\mathrm{Re}\left\{\int_{-\infty}^{\infty} \mathrm{d}\omega C_1(\omega) \times \int_0^\infty \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\,\tau(\omega+\omega_0)}\, \mathrm{e}^{-\varepsilon\tau}\right\},$$

where a convergence factor $\mathrm{e}^{-\varepsilon\tau}$ is added. By using the expressions

$$\begin{aligned}
&\int_0^\infty \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\,t(x-x_0)}\, \mathrm{e}^{-\varepsilon t} = \pi D(x-x_0) - \mathrm{i}\,P(x-x_0),\\
&\lim_{\epsilon\to\infty} D(x-x_0) = \delta(x-x_0),\\
&\lim_{\epsilon\to 0} \int_{-\infty}^{\infty} \mathrm{d}x f(x) P(x-x_0) = P\int_{-\infty}^{\infty} \mathrm{d}x \frac{f(x)}{x-x_0},
\end{aligned}$$

we can obtain

$$k_{10}^{(2)} = C_1(-\omega).$$

From Eqs. (8) and (14) above, we can explicitly write

$$C_1(t-t') = \sum_k |c_k|^2 \{[n(\omega_k)+1]\,\mathrm{e}^{-\mathrm{i}\,\omega_k(t-t')} + n(\omega_k)\,\mathrm{e}^{\mathrm{i}\,\omega_k(t-t')}\},$$

where $n(\omega) = 1/(\,\mathrm{e}^{\beta\hbar\omega}-1)$ is the distribution function for bosons. Using Eq. (15) and the integral representation of the delta function, we have

$$C_1(\omega) = 2\{\Gamma_1(\omega)[n(\omega)+1] + \Gamma_1(-\omega)n(-\omega)\},$$

where $\Gamma_1(\omega)$ is a weighted density of states, having dimensions of frequency, and is defined as follows:

$$\Gamma_1(\omega) = \pi \sum_k |c_k|^2\,\delta(\omega-\omega_k) = \frac{1}{2}J(\omega).$$

In the approximation of the second order,

$$k_{10} = k_{10}^{(2)}, \quad k_{01} = k_{01}^{(2)}, \quad W = W^{(2)},$$
$$\frac{1}{T_1} = \left(\frac{1}{T_1}\right)^{(2)}, \quad \frac{1}{T_2} = \left(\frac{1}{T_2}\right)^{(2)}.$$

Since all the ω_k are non-negative, ($\Gamma_1(\omega)$ is zero for $\omega<0$, and $\Gamma_1(-\omega)$ is zero for $\omega>0$), in the approximation we have

$$\begin{aligned} k_{10} &= C_1(-\omega_0) = 2\Gamma_1(\omega_0)n(\omega_0), \\ k_{01} &= C_1(\omega_0) = 2\Gamma_1(\omega_0)[n(\omega_0)+1], \\ \frac{1}{T_1} &= k_{10}+k_{01} = 2\Gamma_1(\omega_0)[2n(\omega_0)+1]. \end{aligned} \quad (16)$$

Using the same method, the second-order dephasing rate constant and frequency shift is given by

$$\begin{aligned} \frac{1}{T_2} &= -\,\mathrm{Re}\{W_{1010}\} = \frac{1}{2}(k_{01}+k_{10}), \\ \Delta\omega &= -\,\mathrm{Im}\{W_{1010}\} = \frac{\mathrm{i}}{2}[C_1(-\omega_0)-C_1(\omega_0)] \\ &= \frac{\mathrm{i}}{2}[k_{10}-k_{01}]. \end{aligned}$$

Others, if we ignore the coupling terms between the two off-diagonal elements of the reduced density matrix as in Ref. [14], we can obtain

$$\begin{aligned} \frac{\partial\sigma_{00}(t)}{\partial t} &= -k_{10}\sigma_{00}(t) + k_{01}\sigma_{11}(t), \\ \frac{\partial\sigma_{01}(t)}{\partial t} &= \left[\mathrm{i}(\omega_0+\Delta\omega) - \frac{1}{T_2}\right]\sigma_{01}(t), \\ \frac{\partial\sigma_{10}(t)}{\partial t} &= -\left[\mathrm{i}(\omega_0+\Delta\omega) + \frac{1}{T_2}\right]\sigma_{10}(t), \\ \frac{\partial\sigma_{11}(t)}{\partial t} &= k_{10}\sigma_{00}(t) - k_{01}\sigma_{11}(t), \end{aligned} \quad (17)$$

which may be exact as the ω_0 is big enough to compare to the couplings of the off-diagonal terms.

2.2. Master equation of Redfield form approach

In this subsection, we at first review the Redfield form of the master equation. From Eqs. (7) and (10), we can obtain the Nakajima–Zwanzig equation

$$\dot{\sigma}(t) = -\,\mathrm{i}[H_{\mathrm{TLS}}, \sigma(t)] - \int_0^t \mathrm{d}t'\,\mathrm{Tr}_{\mathrm{B}}[H_{\mathrm{I}}(t), [H_{\mathrm{I}}(t'), \sigma(t')\otimes\rho_{\mathrm{B}}]].$$

By implementing the perturbative[33] and the Born–Markov approximations, the master equation of the Redfield form reads[23]

$$\frac{\partial\sigma_{\mu\upsilon}(t)}{\partial t} = -\,\mathrm{i}\,\omega_{\mu\upsilon}\sigma_{\mu\upsilon} + \sum_{\kappa\lambda} R_{\mu\upsilon\kappa\lambda}\sigma_{\kappa\lambda}(t) \quad (18)$$

with

$$R_{\mu\upsilon\kappa\lambda} = -\int_0^\infty \mathrm{d}t\,\mathrm{Tr}_{\mathrm{B}}\,\langle n|\,[H_{\mathrm{I}}(t), [H_{\mathrm{I}}(0), |k(t)\rangle\,\langle l(t)|\,\rho_{\mathrm{B}}]]\,|m\rangle\,,$$

which takes the eigenstate representation of the system $H_{\mathrm{TLS}}\,|\mu\rangle = E_{\mathrm{TLS}}\,|\mu\rangle$. Here, $\omega_{\mu\upsilon} = (E_\mu - E_\upsilon)/\hbar$, and $R_{\mu\upsilon\kappa\lambda}$ is the relaxation or Redfield tensor. The first term on the right-hand side of Eq. (18) describes the isolated system evolution, while the second one represents the interaction of the system with the dissipative environment. Using the interaction Hamiltonian and system eigenstates

$$\begin{aligned} |k(t)\rangle &= \mathrm{e}^{\mathrm{i}\,H_S t}\,|k\rangle = \mathrm{e}^{\mathrm{i}\,\omega_k t}\,|k\rangle\,, \\ H_{\mathrm{I}}(t) &= \mathrm{e}^{\mathrm{i}(H_S+H_{\mathrm{B}})t}H_{\mathrm{I}}\,\mathrm{e}^{-\,\mathrm{i}(H_S+H_{\mathrm{B}})t}, \end{aligned}$$

we can obtain the Redfield tensor describing the system relaxation

$$R_{\mu\upsilon\kappa\lambda} = \Gamma^+_{\lambda\upsilon\mu\kappa} + \Gamma^-_{\lambda\upsilon\mu\kappa} - \delta_{\upsilon\lambda}\sum_\alpha \Gamma^+_{\lambda\alpha\alpha\kappa} - \delta_{\mu\kappa}\sum_\alpha \Gamma^-_{\lambda\alpha\alpha\upsilon}$$

with

$$\begin{aligned} \Gamma^+_{\lambda\upsilon\mu\kappa} &= \frac{1}{\hbar^2}\int_0^\infty \mathrm{d}t\langle\langle\lambda|\,H_{\mathrm{I}}(t)\,|\kappa\rangle\,\langle\lambda|\,H_{\mathrm{I}}\,|\kappa\rangle\rangle_{\mathrm{B}}\,\mathrm{e}^{-\,\mathrm{i}\,\omega_{\mu\upsilon}t}, \\ \Gamma^-_{\lambda\upsilon\mu\kappa} &= \frac{1}{\hbar^2}\int_0^\infty \mathrm{d}t\langle\langle\lambda|\,H_{\mathrm{I}}\,|\upsilon\rangle\,\langle\mu|\,H_{\mathrm{I}}(t)\,|\kappa\rangle\rangle_{\mathrm{B}}\,\mathrm{e}^{-\,\mathrm{i}\,\omega_{\mu\upsilon}t}, \\ H_{\mathrm{I}}(t) &= \mathrm{e}^{\mathrm{i}\,H_{\mathrm{B}}t/\hbar}H_{\mathrm{I}}\,\mathrm{e}^{-\,\mathrm{i}\,H_{\mathrm{B}}t/\hbar}, \end{aligned}$$

where $\langle\cdots\rangle_{\mathrm{B}}$ denotes the thermal average over the bath.

For the Hamiltonian defined as Eq. (1), the Redfield tensor components can be expressed explicitly as

$$\begin{aligned}
\Gamma^{+}_{\lambda\upsilon\mu\kappa} &= \frac{1}{4}\langle\lambda|\,\sigma_x\,|\kappa\rangle\,\langle\lambda|\,\sigma_x\,|\kappa\rangle\,J(\omega_{\mu\kappa})n(\omega_{\mu\kappa}), && \omega_{\mu\kappa}>0,\\
\Gamma^{+}_{\lambda\upsilon\mu\kappa} &= \frac{1}{4}\langle\lambda|\,\sigma_x\,|\kappa\rangle\,\langle\lambda|\,\sigma_x\,|\kappa\rangle\,J(\omega_{\kappa\mu})[n(\omega_{\kappa\mu})+1], && \omega_{\kappa\mu}>0,\\
\Gamma^{-}_{\lambda\upsilon\mu\kappa} &= \frac{1}{4}\langle\lambda|\,\sigma_x\,|\kappa\rangle\,\langle\lambda|\,\sigma_x\,|\kappa\rangle\,J(\omega_{\lambda\upsilon})n(\omega_{\lambda\upsilon}), && \omega_{\lambda\upsilon}>0,\\
\Gamma^{+}_{\lambda\upsilon\mu\kappa} &= \frac{1}{4}\langle\lambda|\,\sigma_x\,|\kappa\rangle\,\langle\lambda|\,\sigma_x\,|\kappa\rangle\,J(\omega_{\upsilon\lambda})[n(\omega_{\upsilon\lambda})+1], && \omega_{\upsilon\lambda}>0. \qquad (19)
\end{aligned}$$

So we can obtain the Redfield equation

$$\begin{aligned}
\frac{\partial\sigma_{00}(t)}{\partial t} &= R_{0000}\sigma_{00}(t)+R_{0001}\sigma_{01}(t)+R_{0010}\sigma_{10}(t)+R_{0011}\sigma_{11}(t),\\
\frac{\partial\sigma_{01}(t)}{\partial t} &= \mathrm{i}\,\omega_{10}\sigma_{01}(t)+R_{0100}\sigma_{00}(t)+R_{0101}\sigma_{01}(t)+R_{0110}\sigma_{10}(t)+R_{0111}\sigma_{11}(t),\\
\frac{\partial\sigma_{10}(t)}{\partial t} &= -\,\mathrm{i}\,\omega_{10}\sigma_{10}(t)+R_{1000}\sigma_{00}(t)+R_{1001}\sigma_{01}(t)+R_{1010}\sigma_{10}(t)+R_{1011}\sigma_{11}(t),\\
\frac{\partial\sigma_{11}(t)}{\partial t} &= R_{1100}\sigma_{00}(t)+R_{1101}\sigma_{01}(t)+R_{1110}\sigma_{10}(t)+R_{1111}\sigma_{11}(t). \qquad (20)
\end{aligned}$$

By using our model and Eq. (16), we finally attain the master equation of Redfield form for the open qubit model as

$$\begin{aligned}
\frac{\partial\sigma_{00}(t)}{\partial t} &= -\frac{1}{2}\frac{1}{T_2}\sigma_{00}(t)+\frac{1}{2}\frac{1}{T_2}\sigma_{11}(t),\\
\frac{\partial\sigma_{01}(t)}{\partial t} &= \left(\mathrm{i}\,\omega_{10}-\frac{1}{2}k_{10}\right)\sigma_{01}(t),\\
\frac{\partial\sigma_{10}(t)}{\partial t} &= -\left(\frac{1}{2}k_{01}+\mathrm{i}\,\omega_{10}\right)\sigma_{10}(t),\\
\frac{\partial\sigma_{11}(t)}{\partial t} &= \frac{1}{2}\frac{1}{T_2}\sigma_{00}(t)-\frac{1}{2}\frac{1}{T_2}\sigma_{11}(t). \qquad (21)
\end{aligned}$$

It is shown that the reduced dynamical equations of the open qubit can be described by the Bloch equations and the master equations of Redfield form. Although they are derived from the same Liouville equation, they have different forms. In the following we shall compare them with the concrete model Eq. (1).

3. Comparisons and Conclusions

In the last sections, we have obtained two kinds of reduced dynamical equations of the open qubit model Eq. (1). One is the Bloch equations and the other is the master equations of Redfield form. From the two sets of equations, we can plot the evolutions of the elements of the reduced density matrix. Here, we choose the parameters $\omega_0 = 0.001$ (ps)$^{-1}$, $\eta = 0.04$, and $\omega_c = 0.05$ (ps)$^{-1}$.

In Fig. 1 we plot the evolutions of the elements $\sigma_{00}(t)$ and $\sigma_{10}(t)$ of the reduced density matrix for the open qubit in the super-Ohmic ($s = 2$), Ohmic ($s = 1$), and sub-Ohmic ($s = 1/2$) baths. The upper three panels in Fig. 1 are the plots of $\sigma_{00}(t)$, which are obtained from initial state $\sigma(0) = |\Phi_1\rangle\langle\Phi_1|$, where $|\Phi_1\rangle = |0\rangle$. It is shown that the open qubit in a super-Ohmic bath has a longer relaxation time than that in the Ohmic bath, and the qubit has a longer relaxation time in the Ohmic bath than that in the sub-Ohmic bath. The lower three panels in Fig. 1 are the plots of $\sigma_{10}(t)$. Here, we set the system evolve from the initial state $\sigma(0) = |\Psi_1\rangle\langle\Psi_1|$, with $|\Psi_1\rangle = 1/\sqrt{2}(|0\rangle+|1\rangle)$. It is shown that the open qubit also has the longest decoherence time when it is embedded in the super-Ohmic bath, and has the shortest decoherence time when it is embedded in the sub-Ohmic bath. In each panel of Fig. 1, we plot the evolutions of the elements of reduced density matrix for the open qubit not only from master equations of Redfield form but also from the Bloch equations. It is shown that the Redfield equations predict longer decoherence and relaxation times than the Bloch equations, it does no matter which bath the qubit is embedded in. In Figs. 2 and 3 we consider the different initial states and different temperatures as the qubit in the Ohmic bath. In left panel of Fig. 2 we plot the evolutions of the $\sigma_{10}(t)$ as the initial states are $\sigma(0) = |\Psi_1\rangle\langle\Psi_1|$, $|\Psi_2\rangle\langle\Psi_2|$, and $|\Psi_3\rangle\langle\Psi_3|$, where $|\Psi_2\rangle = 1/2(\sqrt{3}|0\rangle + |1\rangle)$, and $|\Psi_3\rangle = 1/\sqrt{5}(2|0\rangle + |1\rangle)$. In the right panel of the Fig. 2 we plot the evolutions of the $\sigma_{00}(t)$ as the initial states are $\sigma(0) = |\Phi_1\rangle\langle\Phi_1|$, $|\Phi_2\rangle\langle\Phi_2|$, and $|\Phi_3\rangle\langle\Phi_3|$, where $|\Phi_2\rangle = |\Psi_2\rangle$, and $|\Phi_3\rangle = |\Psi_3\rangle$. It is shown that the Redfield equations predict longer decoherence and relaxation time than the Bloch equations do in different initial states. Similarly, the conclusion is also correct for different temperatures which can be seen in Fig. 3.

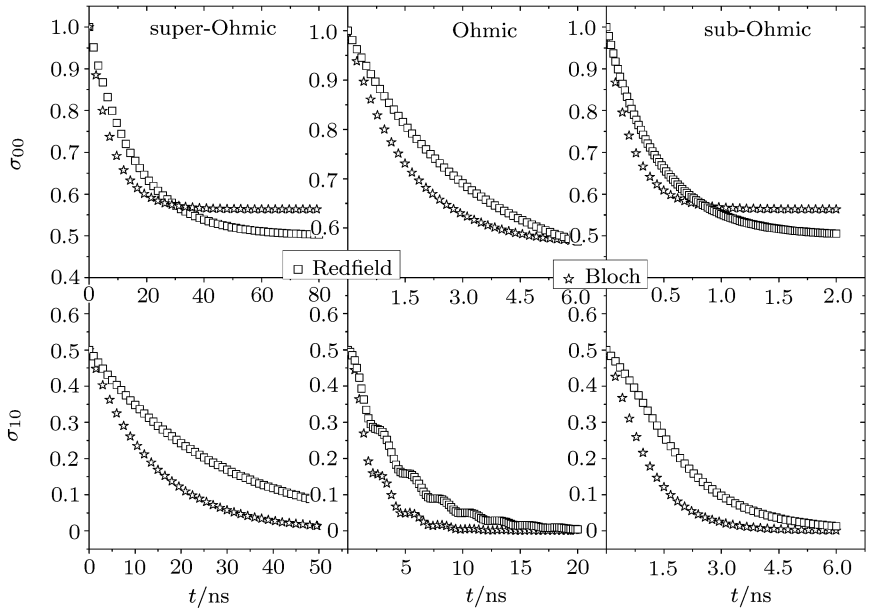


Fig. 1. The evolutions of $\sigma_{00}(t)$ [upper three panels, where the initial state of the qubit is set as $|\Psi_1\rangle$], and $\sigma_{10}(t)$ [lower three panels, where the initial state of the qubit is set at $|\Phi_1\rangle$], which are obtained from the master equations of Redfield form and the Bloch equations in super-Ohmic ($s = 2$), Ohmic ($s = 1$), and sub-Ohmic ($s = 1/2$) baths. Here, we set temperature $T = 30$ mK.

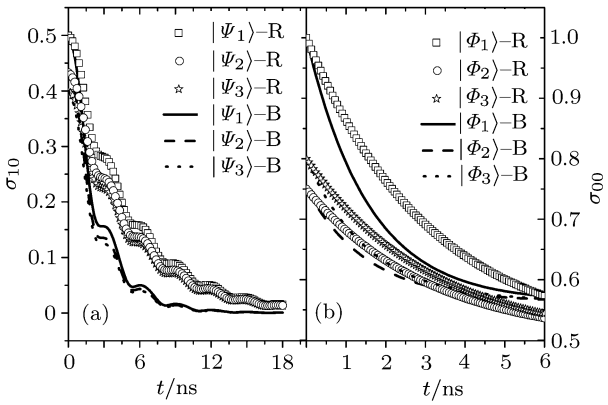


Fig. 2. The evolutions of the $\sigma_{10}(t)$ starting from initial state $|\Psi_1\rangle$, $|\Psi_2\rangle$, $|\Psi_3\rangle$, and $\sigma_{00}(t)$ starting from the initial state $|\Phi_1\rangle$, $|\Phi_2\rangle$, $|\Phi_3\rangle$ at temperature $T = 30$ mK. In the legends of Fig. 2 and the following Fig. 3, the letter "R" denotes the results calculated from solving the Redfield equations, and the letter "B" means that the results are obtained from solving the Bloch equations.

In this paper we set $\omega_c = 50\omega_0$. If the cut-off frequency is not much larger than the characteristic frequency of the qubit, for example, $\omega_c < 10\omega_0$, from the master equation of Redfield form and the Bloch equation we cannot obtain the practical dynamics of the system of interest seriously. In this case we should seek non-Markov methods to investigate the reduced dynamics of the qubit model.[35] However, as the cut-off frequency increases from $50\omega_0$, the dynamics will have no visible changes. It means that it is appropriate to use the cut-off frequency of the bath modes for solving the non-Markov dynamics.

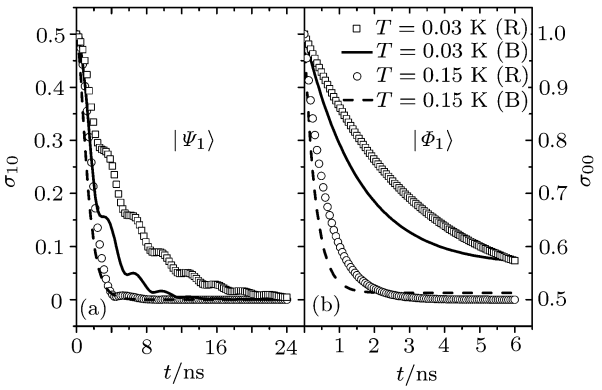


Fig. 3. The evolutions of $\sigma_{10}(t)$ start from initial state $|\Psi_1\rangle$, and $\sigma_{00}(t)$ from the initial state $|\Phi_1\rangle$ at different temperatures $T = 30$ mK and $T = 150$ mK.

In summary, in this paper we have investigated the dynamics of an open qubit model via solving the Bloch equations and master equations of the Redfield form. The two sets of equations are obtained from the perturbation and Markov approximations. It shows that the different approaches give different dynamics of the qubit, and the Redfield equations predict longer decoherence and relaxation times.

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