Pairwise thermal entanglement in a three-qubit Heisenberg XX model with a nonuniform magnetic field and Dzyaloshinski–Moriya interaction

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Pairwise thermal entanglement in a three-qubit Heisenberg XX model is investigated when a nonuniform magnetic field and the Dzyaloshinski–Moriya interaction are included. We find that the nonuniform magnetic field and Dzyaloshinski–Moriya interaction are the more efficient control parameters for the increase of entanglement and critical temperature. For both the nearest neighbour sites and the next nearest neighbour sites, the magnetic field can induce entanglement to a certain extent and the Dzyaloshinski–Moriya interaction can enhance the entanglement to a stable value. The steady value of the nearest neighbour site entanglement C_{12} is larger than the next nearest neighbour site entanglement C_{13}. An interesting phenomenon is that the entanglement curve of C_{12} appears a peak value when the Dzyaloshinski–Moriya interaction is considered in a nonuniform magnetic field.

Keywords: thermal entanglement, nonuniform magnetic field, Dzyaloshinski–Moriya interaction, Heisenberg model

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1. Introduction

Entanglement is one of the fascinating features in quantum mechanics that makes fundamental distinctions between quantum and classical physics. It is of crucial importance in the realization of many quantum information protocols, such as quantum teleportation,[1] superdense coding,[2] quantum cryptography,[3] quantum secure direct communication,[4] and so on. In particular, the thermal entanglement, which is a natural type of entanglement, was introduced and analysed in various types of quantum spin systems.[5−8] Thermal entanglement refers to quantum entanglement of thermal states, for the physically realizable quantum mechanisms are not always at zero temperature but often in a thermal equilibrium state, and it has been investigated extensively. In recent years, there has been an outstanding effort to characterize qualitatively and quantitatively the entanglement properties of Heisenberg model, including XY model, XXX model, XXZ model and XYZ model.

One of the interesting problems concerning thermal entanglement is the effect of magnetic field on thermal entanglement. In Ref. [9], the authors analysed the effect of a nonuniform magnetic field on thermal entanglement of a three-qubit XXX model. The authors of Ref. [10] studied the pairwise thermal entanglement between alternate qubits of a four-qubit XX model in a homogenous magnetic field. These studies illuminated that, in contrast to the uniform magnetic field, the nonuniform magnetic field could not only enhance the entanglement, but also improve the critical temperature.

Dzyaloshinski–Moriya (DM) interaction, introduced by Dzyaloshinski and Moriya, is a combination of superexchange and spin–orbital interactions.[11−13] In fact, a lot of spin models have manifested the existence of such an interaction.[14,15] Therefore, it is worth while to consider DM interaction in the studies of spin-chain entanglement. Recently, the effect of DM interaction on thermal entanglement of the Heisenberg

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model has been intensively studied. Li *et al.* investigated thermal entanglement of a two-qubit Heisenberg XXZ model in the presence of DM interaction. It was found that the DM interaction was an efficient control parameter of entanglement, controlling it could enhance the entanglement or prevent the decrease of the entanglement.[16] Xie *et al.* studied the influence of DM interaction parameter on the thermal entanglement of a three-qubit Heisenberg XXZ model. They showed that the DM interaction might excite the entanglement effectively.[17] Most of the previous studies were done under the periodic boundary condition, so those models were called Heisenberg rings. In this paper, we investigate the Heisenberg model without the restriction. That is to say, our model is an open chain, which means that there is no direct interaction between the first and the end qubit.

In order to provide a detailed theoretical and numerical analysis, we take concurrence C as a measure of entanglement. The concurrence introduced by Hill *et al.* is a physical quantity that describes the entanglement of qubits.[18,19] Let ρ_{12} be the density matrix of the pair and it can be either pure or mixed. The concurrence of the density matrix is defined as[19]

$$C_{12} = \max\left\{0, 2\max(\lambda_i) - \sum_{i=1}^{4}\lambda_i\right\}, \tag{1}$$

where the quantities λ_i $(i = 1, 2, 3, 4)$ are the square roots of the eigenvalues of the operator $R = \rho_{12}(\sigma_1^y \otimes \sigma_2^y)\rho_{12}^*(\sigma_1^y \otimes \sigma_2^y)$, the asterisk stands for the complex conjugate. The values of the concurrence range from zero to one when quantum states are changed for unentangled to maximally entangled states.

The state of the system at thermal equilibrium is described by the density matrix

$$\rho(T) = \frac{1}{Z}\exp(-\beta H) = \frac{1}{Z}\sum_{i=1}^{N}\exp(-\beta E_i)|\psi_i\rangle\langle\psi_i|, \tag{2}$$

where $Z = \mathrm{Tr}\left[\exp(-\beta H)\right]$ is the partition function, H is the system Hamiltonian, β is the inverse of the product of temperature T and k, k is the Boltzman constant which is assumed to be one throughout the paper. Note that all the parameters are dimensionless.

2. Model and solution

We consider the three-qubit Heisenberg XX model with a nonuniform external magnetic field and DM interaction. The Hamiltonian of the system is defined as

$$\begin{aligned} H =\ & J\sum_{i=1}^{3}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) \\ & + (B+b)\sigma_1^z + B\sigma_2^z + (B-b)\sigma_3^z \\ & + \sum_{i=1}^{3} D(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x), \end{aligned} \tag{3}$$

where σ_i^α $(\alpha = x, y, z; i = 1, 2, 3)$ are the Pauli operators. The first sum denotes the spin interaction between the nearest neighbour sites, J is the strength of spin interaction. The values of positive and negative J correspond to the antiferromagnetic and ferromagnetic cases, respectively. The three middle terms denote the interaction of the external magnetic fields and qubits. The magnetic fields are assumed to be along the z direction, B describes the uniformity of the field while b measures the degree of the inhomogeneity of the field. The external magnetic field is a uniform magnetic field when $b = 0$, and is an inhomogeneous magnetic field when $b \neq 0$. The last sum denotes the interaction of the DM interaction and qubits; and D is the z component of DM coupling parameter.

In the standard basis $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$, the eigenvalues are given by

$$\begin{aligned} & E_1 = 3B, \quad E_2 = -B, \quad E_3 = B, \quad E_4 = -B - 2\eta, \quad E_5 = B - 2\eta, \\ & E_6 = -B + 2\eta, \quad E_7 = B + 2\eta, \quad E_8 = -3B, \end{aligned} \tag{4}$$

and the corresponding eigenstates can be explicitly expressed as

$$|\psi_1\rangle = |000\rangle, \tag{5a}$$

$$|\psi_2\rangle = A[-(J+Di)^2|011\rangle + b(J+Di)|101\rangle + (J^2+D^2)|110\rangle], \tag{5b}$$

$$|\psi_3\rangle = A[-(J+Di)^2|001\rangle + b(J+Di)|010\rangle + (J^2+D^2)|100\rangle], \tag{5c}$$

$$|\psi_4\rangle = B\left[\frac{b^2+D^2+J^2-b\eta}{(J^2+D^2)^2}(J+Di)^2|011\rangle + \frac{b-\eta}{J^2+D^2}(J+Di)|101\rangle + |110\rangle\right], \tag{5d}$$

$$|\psi_5\rangle = B\left[\frac{b^2+D^2+J^2-b\eta}{(J^2+D^2)^2}(J+Di)^2|001\rangle + \frac{b-\eta}{J^2+D^2}(J+Di)|010\rangle + |100\rangle\right], \tag{5e}$$

$$|\psi_6\rangle = C\left[\frac{b^2+D^2+J^2+b\eta}{(J^2+D^2)^2}(J+Di)^2|011\rangle + \frac{b+\eta}{J^2+D^2}(J+Di)|101\rangle + |110\rangle\right], \tag{5f}$$

$$|\psi_7\rangle = C\left[\frac{b^2+D^2+J^2+b\eta}{(J^2+D^2)^2}(J+Di)^2|001\rangle + \frac{b+\eta}{J^2+D^2}(J+Di)|010\rangle + |100\rangle\right], \tag{5g}$$

$$|\psi_8\rangle = |111\rangle, \tag{5h}$$

where $\eta = \sqrt{b^2+2D^2+2J^2}$, A, B, and C are the normalization coefficients for quantum states of $|\psi_i\rangle$ ($i = 2,\ldots,7$), respectively. For the composite system in equilibrium at temperature T, the density operator is

$$\begin{aligned}\rho_{123} = \frac{1}{Z}[&\mathrm{e}^{-3\beta B}|\psi_1\rangle\langle\psi_1| + \mathrm{e}^{\beta B}|\psi_2\rangle\langle\psi_2| + \mathrm{e}^{-\beta B}|\psi_3\rangle\langle\psi_3| + \mathrm{e}^{\beta(B+2\eta)}|\psi_4\rangle\langle\psi_4| \\ &+ \mathrm{e}^{\beta(2\eta-B)}|\psi_5\rangle\langle\psi_5| + \mathrm{e}^{\beta(B-2\eta)}|\psi_6\rangle\langle\psi_6| + \mathrm{e}^{-\beta(B+2\eta)}|\psi_7\rangle\langle\psi_7| + \mathrm{e}^{3\beta B}|\psi_8\rangle\langle\psi_8|],\end{aligned} \tag{6}$$

where the partition function of the system is

$$Z = \mathrm{e}^{3\beta B} + \mathrm{e}^{\beta B} + \mathrm{e}^{-\beta B} + \mathrm{e}^{-3\beta B} + \mathrm{e}^{\beta(B+2\eta)} + \mathrm{e}^{\beta(2\eta-B)} + \mathrm{e}^{\beta(B-2\eta)} + \mathrm{e}^{-\beta(B+2\eta)}. \tag{7}$$

3. Pairwise thermal entanglement

If the concurrence of two arbitrary qubits is considered, the density matrix for the different pair is obtained by tracing over the third qubit. Due to the fact that the entanglement between the nearest neighbour sites 1 and 2 is similar to that between sites 2 and 3, we only need to consider the entanglement between sites 1 and 2, and the next nearest neighbour sites 1 and 3 in the following discussion. We first consider the entanglement of sites 1 and 2, the reduced density matrix $\rho_{12}(T) = \mathrm{Tr}_3\rho_{123}$ can be given by

$$\rho_{12}(T) = \frac{1}{Z}\begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & a_5 & 0 \\ 0 & a_6 & a_3 & 0 \\ 0 & 0 & 0 & a_4 \end{pmatrix} \tag{8}$$

in the standard bases $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$, where

$$a_1 = \mathrm{e}^{-3\beta B} + \frac{J^2+D^2}{\eta^2}\mathrm{e}^{-\beta B} + \frac{(b-\eta)^2}{4\eta^2}\mathrm{e}^{\beta(2\eta-B)} + \frac{(b+\eta)^2}{4\eta^2}\mathrm{e}^{-\beta(2\eta+B)}, \tag{9a}$$

$$a_2 = \frac{b^2}{\eta^2}\mathrm{e}^{-\beta B} + \frac{(J^2+D^2)}{\eta^2}[\mathrm{e}^{\beta B} + \mathrm{e}^{\beta(2\eta-B)} + \mathrm{e}^{-\beta(2\eta+B)}] + \frac{(b-\eta)^2}{4\eta^2}\mathrm{e}^{\beta(B+2\eta)} + \frac{(b+\eta)^2}{4\eta^2}\mathrm{e}^{\beta(B-2\eta)}, \tag{9b}$$

$$a_3 = \frac{b^2}{\eta^2}\mathrm{e}^{\beta B} + \frac{J^2+D^2}{\eta^2}[\mathrm{e}^{-\beta B} + \mathrm{e}^{\beta(B+2\eta)} + \mathrm{e}^{\beta(B-2\eta)}] + \frac{(J^2+D^2)^2}{(\eta^2-b\eta)^2}\mathrm{e}^{\beta(2\eta-B)} + \frac{(J^2+D^2)^2}{(\eta^2+b\eta)^2}\mathrm{e}^{-\beta(2\eta+B)}, \tag{9c}$$

$$a_4 = \mathrm{e}^{3\beta B} + \frac{J^2+D^2}{\eta^2}\mathrm{e}^{\beta B} + \frac{(J^2+D^2)^2}{(\eta^2-b\eta)^2}\mathrm{e}^{\beta(2\eta+B)} + \frac{(J^2+D^2)^2}{(\eta^2+b\eta)^2}\mathrm{e}^{\beta(B-2\eta)}, \tag{9d}$$

$$a_5 = \left[\frac{b}{\eta^2}(\mathrm{e}^{-\beta B} - \mathrm{e}^{\beta B}) + \frac{b-\eta}{2\eta^2}\mathrm{e}^{\beta(B+2\eta)} + \frac{J^2+D^2}{\eta^2(b-\eta)}\mathrm{e}^{\beta(2\eta-B)} + \frac{b+\eta}{2\eta^2}\mathrm{e}^{\beta(B-2\eta)} + \frac{J^2+D^2}{\eta^2(b+\eta)}\mathrm{e}^{-\beta(2\eta+B)}\right](J-Di), \tag{9e}$$

$$a_6 = a_5^*. \tag{9f}$$

The square roots of the four eigenvalues of the product matrix R are

$$\lambda_1 = \lambda_2 = \frac{\sqrt{a_1a_4}}{Z}, \quad \lambda_3 = \frac{\sqrt{a_2a_3} - |a_5|}{Z}, \quad \lambda_4 = \frac{\sqrt{a_2a_3} + |a_5|}{Z}. \tag{10}$$

From Eqs. (1), (9), and (10), the concurrence can be obtained

$$C_{12} = \frac{2}{Z}\max\{0, |a_5| - \sqrt{a_1a_4}\}. \tag{11}$$

From Eqs. (9) and (11), we can see that the concurrence C_{12} is determined by the temperature, the nonuniform external magnetic field and the DM interaction. The change of these parameters can induce the change of the entanglement. So, we discuss the effects of the temperature, the nonuniform external magnetic field and the DM interaction on the thermal entanglement in our model. Also we can see that the concurrence is invariant under the substitution $J \to -J$ and $D \to -D$. We thus restrict our considerations to the case of $D > 0$ and the antiferromagnetic $J = 1$. Next, we analyse Eq. (11) to see the change of the concurrence in detail. By fixing some parameters, we can know the roles of the other parameters and the variation of the entanglement.

First, we investigate the effect of the temperature T on the thermal entanglement for different external parameters. There are three cases for $D = b = 0$, $D = B = 0$, and $B = b = 0$. The corresponding concurrence C_{12}^B, C_{12}^b, C_{12}^D will be

$$C_{12}^B = \frac{\operatorname{sech}\left(\frac{B}{T}\right)\left[4\sqrt{\cosh^2\left(\frac{B}{T}\right)\sinh^2\left(\frac{2\sqrt{2}}{T}\right)} - \sqrt{11+\cosh\left(\frac{4\sqrt{2}}{T}\right)+8\cosh\left(\frac{2B}{T}\right)+\cosh\left(\frac{2\sqrt{2}}{T}\right)\left[4+8\cosh\left(\frac{2B}{T}\right)\right]}\right]}{4\sqrt{2}\left[\cosh\left(\frac{2\sqrt{2}}{T}\right)+\cosh\left(\frac{2B}{T}\right)\right]}, \tag{12a}$$

$$C_{12}^b = \frac{\left[2\sqrt{\frac{\cosh\left(\frac{4\sqrt{2+b^2}}{T}\right)-1}{2+b^2}} - \sqrt{\frac{19+4b^2(4+b^2)4(3+4b^2+b^4)\cosh\left(\frac{2\sqrt{2+b^2}}{T}\right)+\cosh\left(\frac{4\sqrt{2+b^2}}{T}\right)}{(2+b^2)^2}}\right]\operatorname{sech}^2\left(\frac{\sqrt{2+b^2}}{T}\right)}{4\sqrt{2}}, \tag{12b}$$

$$C_{12}^D = \frac{\left[2\sqrt{\cosh\left(\frac{4\sqrt{2+2D^2}}{T}\right)-1}-\cosh\left(\frac{2\sqrt{2+2D^2}}{T}\right)-3\right]\operatorname{sech}^2\left(\frac{\sqrt{2+2D^2}}{T}\right)}{8}. \tag{12c}$$

We give the plots of the different cases in Fig. 1. We can see that the entanglement will reach the maximum when the temperature is close to zero. However, by increasing the temperature, the entanglement will decrease to zero. The decrease in entanglement is due to the mixing of the maximally entangled state with other states. There exists a critical temperature T_c, above which the entanglement disappears. It is interesting to find that the critical temperature T_c is dependent on the magnetic field and the DM interaction. The critical temperature will increase with the increase of the external conditions. Comparing the three figures, it is natural to find that if there is only B, the maximal value of the entanglement is larger than the other cases. But the critical temperature is not enhanced obviously. For both B and b, they can enhance the entanglement and the critical temperature only in a small range, beyond which the entanglement will decrease. However, the entanglement will become steady with the increase of D and the critical temperature T_c increases gradually.

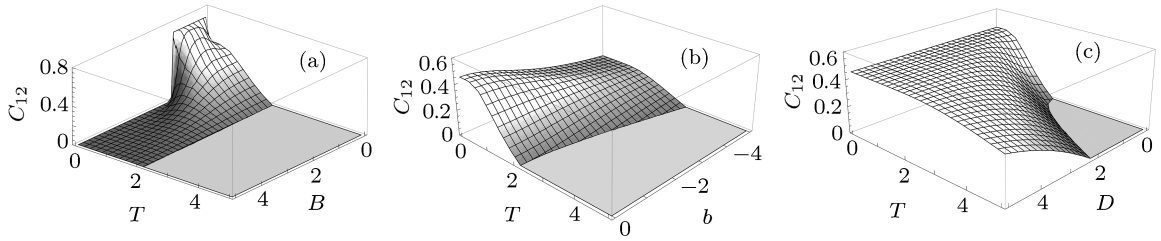


Fig. 1. The concurrence C_{12} versus temperature T and the different external parameters: (a) $D = b = 0$, (b) $D = B = 0$, (c) $B = b = 0$.

Secondly, we plot the concurrence C_{12} versus magnetic field B and b for the DM interaction $D = 0$ and finite temperature $T = 0.1$ in Fig. 2. It can be seen that the concurrence is centrosymmetric about the point $B = b = 0$ from the contour map Fig. 2(b). From Fig. 2(a), it is clear that the entanglement drops to zero rapidly when B and b are in the same direction. However, if B and b are in the opposite directions, the entanglement will keep nonzero for a larger scale. In addition, there are two peaks in this scale and the maximal entanglement value can reach 0.92, in terms of a straightforward calculation. Therefore, we can obtain a strong entanglement between sites 1 and 2 by adjusting the external uniform and inhomogeneous magnetic fields.

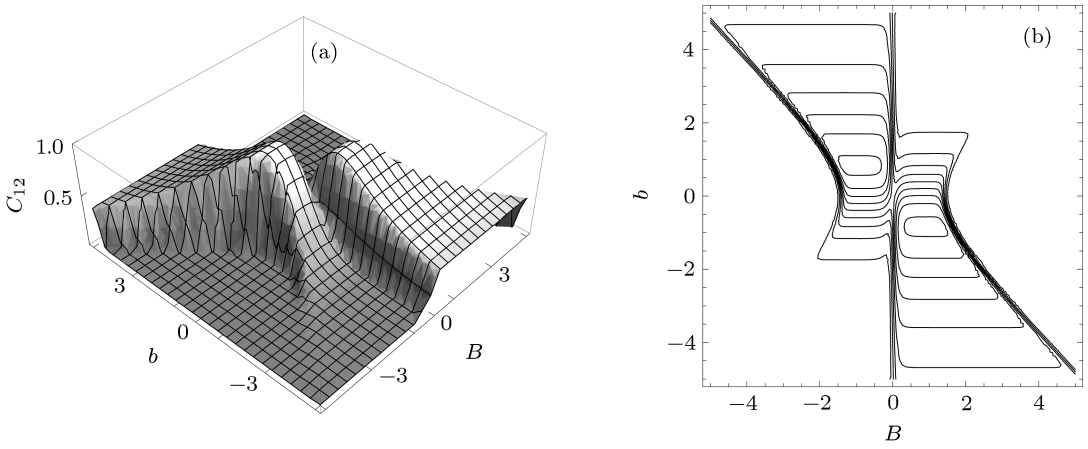


Fig. 2. The curve and contour map of the concurrence C_{12} plotted versus B and b with $D = 0$, here $T = 0.1$.

Then, we investigate the dependence of thermal entanglement on the magnetic field B and D. Here $b = 0$ means that our model is in a uniform magnetic field. At a finite temperature $T = 0.1$, figure 3(a) shows the entanglement property in terms of the DM interaction D for different values of B. It is clear that the concurrence is small when there is no magnetic field and it can hardly change with the increase of D. When B is nonzero, the thermal entanglement C_{12} will be enhanced to a stable value rapidly with the increase of D. When D reaches a certain value D_0, C_{12} will become steady. It is obvious that the value of D_0 depends on the external conditions of the model.

In Fig. 3(b), we plot the thermal entanglement of the model as a function of D in an inhomogeneous magnetic field. We find that the thermal entanglement C_{12} increases quickly at first, and then gradually decreases to a stable value with the increase of D. There is a peak in the curve in the case of $b \neq 0$, and the maximum of the peak value is much larger than that in the case $b = 0$ (uniform magnetic field). The peak value is enhanced with the increase of the absolute value of b in a finite range. In addition, we can see that the stable value in Fig. 3(b) is larger than that of C_{12} presented in Fig. 3(a). Therefore, for our model in a uniform magnetic field, introducing b is preferable for obtaining stationary entanglement.

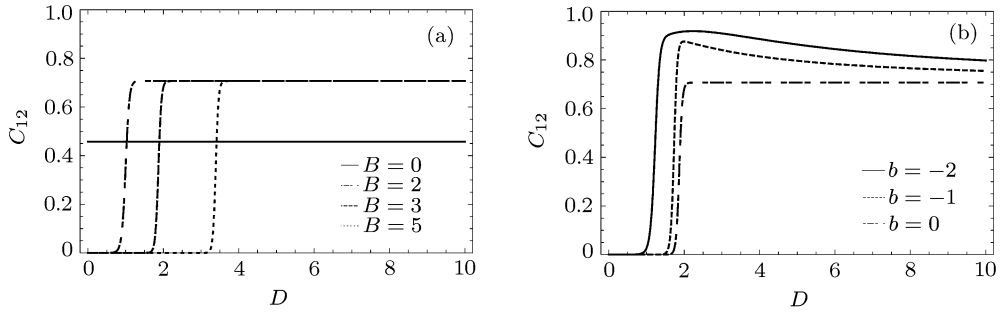


Fig. 3. The concurrence C_{12} versus D when $T = 0.1$: (a) the cases of uniform magnetic field when $b = 0$ for different B, (b) the cases of inhomogeneous magnetic field when $B = 3$ for different b.

Next, we turn to the entanglement between the next nearest neighbour sites 1 and 3. Following the above-mentioned method, we obtain $\rho_{13}(T) = \mathrm{Tr}_2\rho_{123}$. The reduced density matrix $\rho_{13}(T)$ is given by

$$\rho_{13}(T) = \frac{1}{Z}\begin{pmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & b_5 & 0 \\ 0 & b_6 & b_3 & 0 \\ 0 & 0 & 0 & b_4 \end{pmatrix}, \tag{13}$$

where

$$b_1 = \mathrm{e}^{-3\beta B} + \frac{b^2}{\eta^2}\mathrm{e}^{-\beta B} + \frac{J^2+D^2}{\eta^2}[\mathrm{e}^{-\beta(B+2\eta)} + \mathrm{e}^{\beta(2\eta-B)}], \tag{14a}$$

$$b_2 = \frac{J^2+D^2}{\eta^2}(\mathrm{e}^{-\beta B} + \mathrm{e}^{\beta B}) + \frac{(b-\eta)^2}{4\eta^2}[\mathrm{e}^{\beta(B+2\eta)} + \mathrm{e}^{\beta(2\eta-B)}] + \frac{(b+\eta)^2}{4\eta^2}[\mathrm{e}^{\beta(B-2\eta)} + \mathrm{e}^{-\beta(2\eta+B)}], \tag{14b}$$

$$b_3 = \frac{J^2+D^2}{\eta^2}(\mathrm{e}^{-\beta B} + \mathrm{e}^{\beta B}) + \frac{(J^2+D^2)^2}{(\eta^2-b\eta)^2}[\mathrm{e}^{\beta(B+2\eta)} + \mathrm{e}^{\beta(2\eta-B)}] + \frac{(J^2+D^2)^2}{(\eta^2+b\eta)^2}[\mathrm{e}^{\beta(B-2\eta)} + \mathrm{e}^{-\beta(2\eta+B)}], \tag{14c}$$

$$b_4 = \mathrm{e}^{3\beta B} + \frac{b^2}{\eta^2}\mathrm{e}^{\beta B} + \frac{J^2+D^2}{\eta^2}[\mathrm{e}^{\beta(B+2\eta)} + \mathrm{e}^{\beta(B-2\eta)}], \tag{14d}$$

$$b_5 = -\frac{(J-Di)^2}{\eta^2}(\mathrm{e}^{-\beta B} + \mathrm{e}^{\beta B}) + \frac{(J-Di)^2}{2\eta^2}[\mathrm{e}^{-\beta(B+2\eta)} + \mathrm{e}^{\beta(2\eta-B)} + \mathrm{e}^{\beta(B+2\eta)} + \mathrm{e}^{\beta(B-2\eta)}], \tag{14e}$$

$$b_6 = b_5^*. \tag{14f}$$

From Eq. (1), the concurrence is obtained

$$C_{13} = \frac{2}{Z}\max\{0, |b_5| - \sqrt{b_1 b_4}\}. \tag{15}$$

Figure 4 shows the dependence of thermal entanglement C_{13} on T and external conditions. From Fig. 4(a) we can see that there is hardly entanglement when $B = 0$, for convenience, we take $B = 1$ when studying the other cases. These figures show that the concurrence decreases with increasing temperature T, and increases with the increase of external parameters at a certain temperature. The critical temperature T_c is dependent on the external parameters, so the increase of them can increase T_c. These results are similar to the case of C_{12} in Fig. 1.

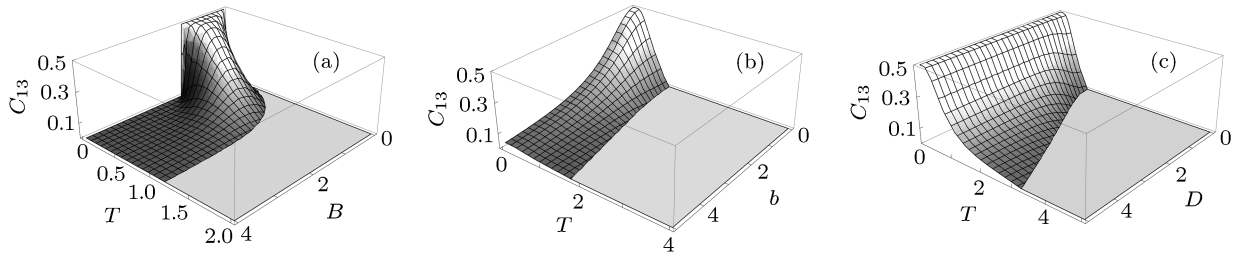


Fig. 4. The concurrence C_{13} versus temperature T and the different external parameters: (a) $D = b = 0$, (b) $B = 1, D = 0$, (c) $B = 1, b = 0$.

The next nearest neighbour concurrence C_{13}, as a function of the magnetic fields B and b, is plotted in Fig. 5 for $D = 0$, which is similar to Fig. 2. However, the maximal entanglement value is smaller and there is also a large scale of entanglement when B and b are in the opposite directions.

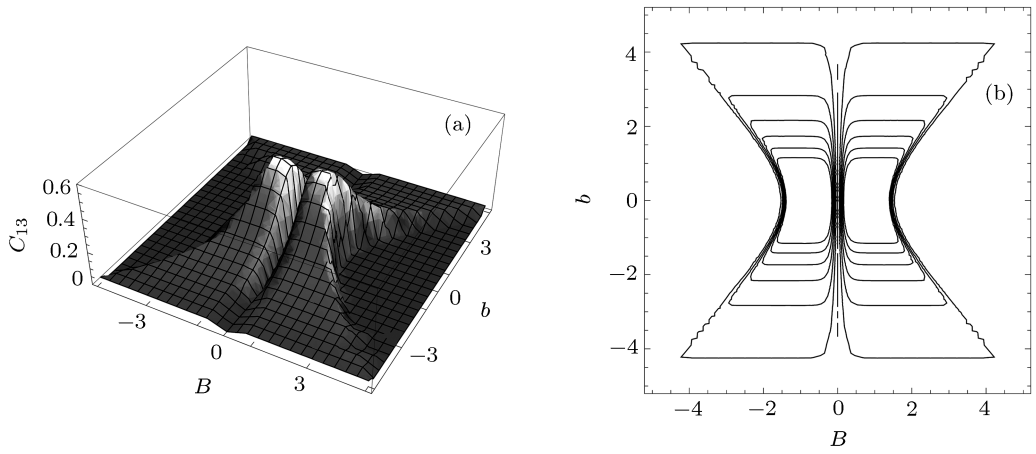


Fig. 5. The curve and contour map of the concurrence C_{13} plotted versus B and b with $D = 0$, here $T = 0.1$.

Then, we investigate the dependence of thermal entanglement on the DM interaction D and the magnetic field B or b. It is obvious that the steady entanglement in Fig. 6 is smaller compared to that in Fig. 3. It is interesting to note that, when $B = b = 0$, there is no entanglement between the next nearest neighbour sites 1 and 3. Moreover, for the case of inhomogeneous magnetic field, we also find that there is no peak in the curve of Fig. 6(b) and the maximal entanglement value is equal to that in Fig. 6(a). These indicate that the inhomogeneous magnetic field b cannot enhance the stationary entanglement value of the next nearest neighbour sites 1 and 3.

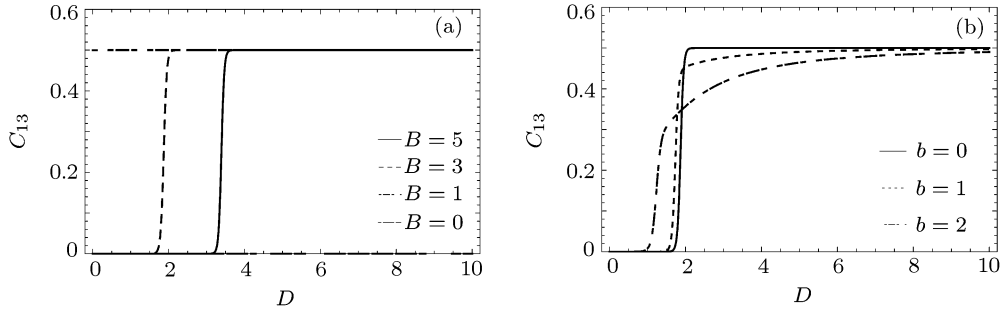


Fig. 6. The concurrence C_{13} versus D when $T = 0.1$: (a) the cases of uniform magnetic field when $b = 0$ for different B, (b) the cases of inhomogeneous magnetic field when $B = 3$ for different b.

4. Conclusion

We have investigated the pairwise thermal entanglement of a three-qubit Heisenberg XX model under inhomogeneous magnetic field in the presence of DM interaction. We showed that the magnetic field and DM interaction can enhance the pairwise entanglement and the critical temperature in a certain range. Furthermore, we found that the entanglement between the nearest neighbour sites 1 and 2 is larger than the next nearest neighbour sites 1 and 3. In a uniform magnetic field, for either C_{12} or C_{13}, D will enhance the entanglement to a stable value rapidly. But in the case of inhomogeneous magnetic field, there appears a peak value in C_{12} and then gradually decreases to a stable value with the increase of D. The stable value is larger than that in the uniform magnetic field. But the performance does not appear in C_{13}.

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