Solving (2 + 1)-dimensional sine-Poisson equation by a modified variable separated ordinary differential equation method*

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By introducing a more general auxiliary ordinary differential equation (ODE), a modified variable separated ordinary differential equation method is presented for solving the (2+1)-dimensional sine-Poisson equation. As a result, many explicit and exact solutions of the (2 + 1)-dimensional sine-Poisson equation are derived in a simple manner by this technique.

Keywords: modified variable separated ODE method, (2 + 1)-dimensional sine-Poisson equation, explicit and exact solution

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1. Introduction

Nonlinear evolution equations (NEEs) are frequently used to model a wide variety of phenomena in physics, chemistry, biology and even in social science field. Therefore, solving NEEs and seeking explicit and exact solutions of NEEs has become one of the most exciting and extremely active domains in nonlinear science. In recent years, a vast variety of powerful and efficient approaches have been developed to construct explicit and exact solutions to NEEs. They are the hyperbolic tangent function expansion method,[1−5] the homogeneous balance method,[6,7] the trial function method,[8−11] the Jacobi elliptic function expansion method,[12−16] the trial equation method,[17,18] the homotopy method,[19−23] the variable coefficient algebraic method,[24] the combination method,[25−27] the (ω/g)-expansion method,[28] the additional generating condition method,[29] the transplantation method of solution,[30] the (G'/G)-expansion method,[31−33] the extended symmetry method,[34] the direct method,[35] the Wronskian-form expansion method,[36] and so on. Recently, Sirendaoreji and Sun[37] developed a direct algebraic method named as the variable separated ordinary differential equation (ODE) method for solving sine-Gordon type equations, which has scientific value and reliability and which works effectively if the equation involves sine, cosine, hyperbolic sine and hyperbolic cosine functions. In the present paper, we would further modify the method of Sirendaoreji and Sun and present a modified variable separated ODE method to pick up rich types of solutions for the (2 + 1)-dimensional sine-Poisson equation by introducing a more general auxiliary ODE and with the aid of its manifold types of solutions.

2. Solutions to (2 + 1)-dimensional sine-Poisson equation

The celebrated (2 + 1)-dimensional sine-Poisson equation reads[38]

$$u_{tt} - u_{xx} - u_{yy} + m^2 \sin u = 0, \tag{1}$$

where the subscripts denote partial derives in terms of its argument. We look for its traveling wave solutions in the following form

$$u = u(\xi), \quad \xi = kx + ly - \omega t. \tag{2}$$

Substituting Eq. (2) into Eq. (1) yields the following ODE for ξ

$$(\omega^2 - k^2 - l^2)u'' + m^2 \sin u = 0, \tag{3}$$

where primes stand for the derivatives for ξ.

In order to facilitate the solution of Eq. (3), here we suppose that $u(\xi)$ in Eq. (3) satisfies the following

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additional ODE

$$u' = a_0 + \sum_{j=1}^{k} \left(a_j \cos \frac{1}{2}u + b_j \sin \frac{1}{2}u \right) \cos^{j-1} \frac{1}{2}u, \quad (4)$$

where a_j $(j = 0, 1, 2, \ldots)$ and b_j $(j = 1, 2, \ldots)$ are constants to be determined, and the positive integer k can be determined by the leading-order analysis method.

Making use of the following formulas

$$\sin u = \frac{\mathrm{e}^{\mathrm{i}u} - \mathrm{e}^{-\mathrm{i}u}}{2\mathrm{i}}, \quad \cos u = \frac{\mathrm{e}^{\mathrm{i}u} + \mathrm{e}^{-\mathrm{i}u}}{2}, \quad (5)$$

and inserting Eqs. (5) and (4) into Eq. (3) and employing the leading-order analysis method, we obtain $k = 1$, and hence

$$u' = a_0 + a_1 \cos \frac{1}{2}u + b_1 \sin \frac{1}{2}u. \quad (6)$$

For the sake of generality, in the following we first explore the solutions to Eq. (6). By the method of variable separation and with the aid of Mathematica, we can find the solutions of Eq. (6), which are listed as follows:

Case 1 When $a_1 = b_1 = 0,$ and $a_0 \neq 0,$

$$u(\xi) = a_0\xi + \xi_0. \quad (7)$$

Case 2 When $a_0 = b_1 = 0,$ and $a_1 \neq 0,$

$$u(\xi) = 4 \arctan \left[\exp \left(\frac{a_1}{2}\xi + \xi_0 \right) \right] - \pi, \quad (8)$$

$$u(\xi) = 4 \operatorname{arccot} \left[-\exp \left(\frac{a_1}{2}\xi + \xi_0 \right) \right] - \pi, \quad (9)$$

$$u(\xi) = \pm 4 \arccos \left\{ \sqrt{\frac{1}{2}\left[1 + \operatorname{sech} \left(\frac{a_1}{2}\xi + \xi_0 \right) \right]} \right\}, \quad (10)$$

$$u(\xi) = 2 \arcsin \left[\tanh \left(\frac{a_1}{2}\xi + \xi_0 \right) \right], \quad (11)$$

$$u(\xi) = 2 \arcsin \left[\coth \left(\frac{a_1}{2}\xi + \xi_0 \right) \right], \quad (12)$$

$$u(\xi) = 2 \arcsin \left[\tanh(a_1\xi + \xi_0) \pm \mathrm{i} \operatorname{sech}(a_1\xi + \xi_0) \right], \quad (13)$$

$$u(\xi) = 2 \arcsin \left[\coth(a_1\xi + \xi_0) \pm \operatorname{csch}(a_1\xi + \xi_0) \right], \quad (14)$$

$$u(\xi) = 2 \arcsin \left[\frac{\tanh(a_1\xi + \xi_0)}{1 \pm \operatorname{sech}(a_1\xi + \xi_0)} \right], \quad (15)$$

$$u(\xi) = 2 \arcsin \left[\frac{\coth(a_1\xi + \xi_0)}{1 \pm \mathrm{i} \operatorname{csch}(a_1\xi + \xi_0)} \right], \quad (16)$$

$$u(\xi) = 2 \arcsin \left[\frac{A \cosh(a_1\xi + \xi_0) \pm \sqrt{A^2 + B^2}}{A \sinh(a_1\xi + \xi_0) \pm B} \right], \quad (17)$$

$$u(\xi) = 2 \arcsin \left[\frac{A \sinh(a_1\xi + \xi_0) \pm \sqrt{B^2 - A^2}}{A \cosh(a_1\xi + \xi_0) \pm B} \right], \quad (18)$$

$$u(\xi) = 2 \arccos \left[\operatorname{sech} \left(\frac{a_1}{2}\xi + \xi_0 \right) \right], \quad (19)$$

$$u(\xi) = 2 \arccos \left[\mathrm{i} \operatorname{csch} \left(\frac{a_1}{2}\xi + \xi_0 \right) \right], \quad (20)$$

$$u(\xi) = 4 \arctan \left[\tanh \left(\frac{a_1}{4}\xi + \xi_0 \right) \right], \quad (21)$$

$$u(\xi) = 4 \arctan \left[\coth \left(\frac{a_1}{4}\xi + \xi_0 \right) \right], \quad (22)$$

$$u(\xi) = 4 \operatorname{arccot} \left[\tanh \left(\frac{a_1}{4}\xi + \xi_0 \right) \right], \quad (23)$$

$$u(\xi) = 4 \operatorname{arccot} \left[\coth \left(\frac{a_1}{4}\xi + \xi_0 \right) \right]. \quad (24)$$

In the above equations, A and B are two nonzero real constants and satisfy the constraint condition $B^2 > A^2$.

Case 3 When $a_0 = a_1 = 0,\ b_1 \neq 0$

$$u(\xi) = 4 \arctan \left[\exp \left(\frac{b_1}{2}\xi + \xi_0 \right) \right], \quad (25)$$

$$u(\xi) = 4 \operatorname{arccot} \left[-\exp \left(\frac{b_1}{2}\xi + \xi_0 \right) \right], \quad (26)$$

$$u(\xi) = \pm 4 \arccos \left\{ \sqrt{\frac{1}{2}\left[1 + \operatorname{sech} \left(\frac{b_1}{2}\xi + \xi_0 \right) \right]} \right\} - \pi, \quad (27)$$

$$u(\xi) = 2 \arcsin \left[\operatorname{sech} \left(\frac{b_1}{2}\xi + \xi_0 \right) \right], \quad (28)$$

$$u(\xi) = 2 \arcsin \left[\mathrm{i} \operatorname{csch} \left(\frac{b_1}{2}\xi + \xi_0 \right) \right], \quad (29)$$

$$u(\xi) = 2 \arccos \left[\tanh \left(\frac{b_1}{2}\xi + \xi_0 \right) \right], \quad (30)$$

$$u(\xi) = 2 \arccos \left[\coth \left(\frac{b_1}{2}\xi + \xi_0 \right) \right], \quad (31)$$

$$u(\xi) = 2 \arccos \left[\tanh(b_1\xi + \xi_0) \pm \mathrm{i} \operatorname{sech}(b_1\xi + \xi_0) \right], \quad (32)$$

$$u(\xi) = 2 \arccos \left[\coth(b_1\xi + \xi_0) \pm \operatorname{csch}(b_1\xi + \xi_0) \right], \quad (33)$$

$$u(\xi) = 2 \arccos \left[\frac{\tanh(b_1\xi + \xi_0)}{1 \pm \operatorname{sech}(b_1\xi + \xi_0)} \right], \quad (34)$$

$$u(\xi) = 2 \arccos \left[\frac{\coth(b_1\xi + \xi_0)}{1 \pm \mathrm{i} \operatorname{csch}(b_1\xi + \xi_0)} \right], \quad (35)$$

$$u(\xi) = 2 \arccos \left[\frac{A \cosh(b_1\xi + \xi_0) \pm \sqrt{A^2 + B^2}}{A \sinh(b_1\xi + \xi_0) \pm B} \right], \quad (36)$$

$$u(\xi) = 2 \arccos \left[\frac{A \sinh(b_1\xi + \xi_0) \pm \sqrt{B^2 - A^2}}{A \cosh(b_1\xi + \xi_0) \pm B} \right]. \quad (37)$$

In the above equations, A and B are two non-zero real constants and satisfy the constraint condition $B^2 > A^2$.

Case 4 When $a_1 = 0,\ a_0 \neq 0,\ b_1 \neq 0,$

Case 4.1 For $a_0^2 > b_1^2,$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\tan\Big(\frac{\sqrt{a_0^2-b_1^2}}{4}\xi+\xi_0\Big)\right]+\pi, \tag{38}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\cot\frac{\sqrt{a_0^2-b_1^2}}{4}(\xi+\xi_0)\right]+\pi, \tag{39}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\Big(\tan\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\sec\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\Big)\right]+\pi, \tag{40}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\Big(\cot\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\csc\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\Big)\right]+\pi, \tag{41}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\frac{\tan\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)}{1\pm\sec\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)}\right]+\pi, \tag{42}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\frac{\cot\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)}{1\pm\csc\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)}\right]+\pi, \tag{43}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\frac{A\cos\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\sqrt{A^2-B^2}}{A\sin\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\pm B}\right]+\pi, \tag{44}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+b_1}{a_0-b_1}}\frac{A\sin\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\sqrt{A^2-B^2}}{A\cos\Big(\frac{\sqrt{a_0^2-b_1^2}}{2}\xi+\xi_0\Big)\pm B}\right]+\pi\,. \tag{45}$$

In the above equations, A and B are two non-zero real constants and satisfy the constraint condition $A^2>B^2$.

Case 4.2 for $a_0^2<b_1^2$,

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\tanh\Big(\frac{\sqrt{b_1^2-a_0^2}}{4}\xi+\xi_0\Big)\right]+\pi, \tag{46}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\coth\Big(\frac{\sqrt{b_1^2-a_0^2}}{4}\xi+\xi_0\Big)\right]+\pi, \tag{47}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\Big(\tanh\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\pm\mathrm{i}\,\mathrm{sech}\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\Big)\right]+\pi, \tag{48}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\Big(\coth\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\pm\mathrm{csch}\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\Big)\right]+\pi, \tag{49}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\frac{\tanh\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)}{1\pm\mathrm{sech}\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)}\right]+\pi, \tag{50}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\frac{\coth\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)}{1\pm\mathrm{i}\,\mathrm{csch}\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)}\right]+\pi, \tag{51}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\frac{A\cosh\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\pm\sqrt{A^2+B^2}}{A\sinh\Big(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\pm B}\right]+\pi, \tag{52}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{b_1+a_0}{b_1-a_0}}\frac{A\sinh\left(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm\sqrt{B^2-A^2}}{A\cosh\left(\frac{\sqrt{b_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm B}\right]+\pi. \tag{53}$$

In the above equations, A and B are two non-zero real constants and satisfy the constraint condition $B^2>A^2$.

Case 4.3 For $b_1=a_0$,

$$u(\xi)=4\arctan\left(\frac{a_0}{2}\xi+\xi_0\right)+\pi. \tag{54}$$

Case 4.4 For $b_1=-a_0$,

$$u(\xi)=-4\operatorname{arccot}\left(\frac{a_0}{2}\xi+\xi_0\right)+\pi. \tag{55}$$

Case 5 When $b_1=0$, $a_0\neq 0$, $a_1\neq 0$,

Case 5.1 For $a_0^2>a_1^2$,

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\tan\left(\frac{\sqrt{a_0^2-a_1^2}}{4}\xi+\xi_0\right)\right], \tag{56}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\cot\left(\frac{\sqrt{a_0^2-a_1^2}}{4}\xi+\xi_0\right)\right], \tag{57}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\left(\tan\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\pm\sec\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\right)\right], \tag{58}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\left(\cot\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\pm\csc\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\right)\right], \tag{59}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\frac{\tan\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)}{1\pm\sec\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)}\right], \tag{60}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\frac{\cot\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)}{1\pm\csc\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)}\right], \tag{61}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\frac{A\cos\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\pm\sqrt{A^2-B^2}}{A\sin\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\pm B}\right], \tag{62}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_0+a_1}{a_0-a_1}}\frac{A\sin\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\pm\sqrt{A^2-B^2}}{A\cos\left(\frac{\sqrt{a_0^2-a_1^2}}{2}\xi+\xi_0\right)\pm B}\right]. \tag{63}$$

In the above equations, A and B are two non-zero real constants and satisfy the constraint condition $A^2>B^2$.

Case 5.2 For $a_0^2<a_1^2$,

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\tanh\left(\frac{\sqrt{a_1^2-a_0^2}}{4}\xi+\xi_0\right)\right], \tag{64}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\coth\left(\frac{\sqrt{a_1^2-a_0^2}}{4}\xi+\xi_0\right)\right], \tag{65}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\left(\tanh\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm\mathrm{i}\operatorname{sech}\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\right)\right], \tag{66}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\left(\coth\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm\operatorname{csch}\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\right)\right], \tag{67}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\frac{\tanh\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)}{1\pm\operatorname{sech}\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)}\right], \tag{68}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\frac{\coth\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)}{1\pm \mathrm{i}\operatorname{csch}\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)}\right], \tag{69}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\frac{A\cosh\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm\sqrt{A^2+B^2}}{A\sinh\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm B}\right], \tag{70}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{a_1+a_0}{a_1-a_0}}\frac{A\sinh\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm\sqrt{B^2-A^2}}{A\cosh\left(\frac{\sqrt{a_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm B}\right]. \tag{71}$$

In the equations, A and B are two non-zero real constants and satisfy the constraint condition $B^2>A^2$.

Case 5.3 For $a_1=a_0$,

$$u(\xi)=4\arctan\left(\frac{a_0}{2}\xi+\xi_0\right). \tag{72}$$

Case 5.4 For $a_1=-a_0$,

$$u(\xi)=-4\operatorname{arccot}\left(\frac{a_0}{2}\xi+\xi_0\right). \tag{73}$$

Case 6 When $a_0=0$, $a_1\neq 0$, $b_1\neq 0$,

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\tanh\left(\frac{\sqrt{a_1^2+b_1^2}}{4}\xi+\xi_0\right)+\frac{b_1}{a_1}\right], \tag{74}$$

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\coth\left(\frac{\sqrt{a_1^2+b_1^2}}{4}\xi+\xi_0\right)+\frac{b_1}{a_1}\right], \tag{75}$$

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\left(\tanh\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\pm \mathrm{i}\operatorname{sech}\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\right)+\frac{b_1}{a_1}\right], \tag{76}$$

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\left(\coth\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\pm\operatorname{csch}\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\right)+\frac{b_1}{a_1}\right], \tag{77}$$

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\frac{\tanh\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)}{1\pm\operatorname{sech}\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)}+\frac{b_1}{a_1}\right], \tag{78}$$

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\frac{\coth\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)}{1\pm \mathrm{i}\operatorname{csch}\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)}+\frac{b_1}{a_1}\right], \tag{79}$$

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\frac{A\cosh\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\pm\sqrt{A^2+B^2}}{A\sinh\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\pm B}+\frac{b_1}{a_1}\right], \tag{80}$$

$$u(\xi)=4\arctan\left[\frac{\sqrt{a_1^2+b_1^2}}{a_1}\frac{A\sinh\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\pm\sqrt{B^2-A^2}}{A\cosh\left(\frac{\sqrt{a_1^2+b_1^2}}{2}\xi+\xi_0\right)\pm B}+\frac{b_1}{a_1}\right]. \tag{81}$$

In the above equations, A and B are two non-zero real constants and satisfy the constraint condition $B^2>A^2$.

Case 7 When $a_0 \neq 0$, $a_1 \neq 0$, $b_1 \neq 0$,

Case 7.1 For $a_0^2 > a_1^2 + b_1^2$,

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\tan\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{4}\xi+\xi_0\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{82}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\cot\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{4}\xi+\xi_0\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{83}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\Big(\tan\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\sec\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{84}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\Big(\cot\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\csc\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{85}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\frac{\tan\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)}{1\pm\sec\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)}\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{86}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\frac{\cot\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)}{1\pm\csc\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)}\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{87}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\frac{A\cos\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\sqrt{A^2-B^2}}{A\sin\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\pm B}\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{88}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{a_0+\sqrt{a_1^2+b_1^2}}{a_0-\sqrt{a_1^2+b_1^2}}}\frac{A\sin\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\pm\sqrt{A^2-B^2}}{A\cos\Big(\frac{\sqrt{a_0^2-a_1^2-b_1^2}}{2}\xi+\xi_0\Big)\pm B}\right] - 2\arctan\frac{a_1}{b_1}+\pi. \tag{89}$$

In the above equations, A and B are two non-zero real constants and satisfy the constraint condition $A^2 > B^2$.

Case 7.2 For $a_0^2 < a_1^2 + b_1^2$,

$$u(\xi) = 4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\tanh\Big(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{4}\xi+\xi_0\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{90}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\coth\Big(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{4}\xi+\xi_0\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{91}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\Big(\tanh\Big(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\pm\mathrm{i}\,\mathrm{sech}\Big(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{92}$$

$$u(\xi) = 4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\Big(\coth\Big(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\pm\mathrm{csch}\Big(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\Big)\Big)\right] - 2\arctan\frac{a_1}{b_1}+\pi, \tag{93}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\frac{\tanh\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)}{1\pm\operatorname{sech}\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)}\right]-2\arctan\frac{a_1}{b_1}+\pi, \tag{94}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\frac{\coth\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)}{1\pm\mathrm{i}\operatorname{csch}\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)}\right]-2\arctan\frac{a_1}{b_1}+\pi, \tag{95}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\frac{A\cosh\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm\sqrt{A^2+B^2}}{A\sinh\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm B}\right]-2\arctan\frac{a_1}{b_1}+\pi, \tag{96}$$

$$u(\xi)=4\arctan\left[\sqrt{\frac{\sqrt{a_1^2+b_1^2}+a_0}{\sqrt{a_1^2+b_1^2}-a_0}}\frac{A\sinh\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm\sqrt{B^2-A^2}}{A\cosh\left(\frac{\sqrt{a_1^2+b_1^2-a_0^2}}{2}\xi+\xi_0\right)\pm B}\right]-2\arctan\frac{a_1}{b_1}+\pi. \tag{97}$$

In the above equations, A and B are two non-zero real constants and satisfy the constraint condition $B^2>A^2$.

Case 7.3 For $\sqrt{a_1^2+b_1^2}=a_0$,

$$u(\xi)=4\arctan\left(\frac{a_0}{2}\xi+\xi_0\right)-2\arctan\frac{a_1}{b_1}+\pi. \tag{98}$$

Case 7.4 For $\sqrt{a_1^2+b_1^2}=-a_0$,

$$u(\xi)=-4\operatorname{arccot}\left(\frac{a_0}{2}\xi+\xi_0\right)-2\arctan\frac{a_1}{b_1}+\pi. \tag{99}$$

Substituting Eq. (6) into Eq. (3) and equating the coefficients of those sine and cosine functions yield a system of algebraic equations for the undetermined constants a_0, a_1, b_1, k, l and ω as follows:

$$\begin{aligned}&a_0b_1(k^2+l^2-\omega^2)=0,\\&a_0a_1(k^2+l^2-\omega^2)=0,\\&a_1b_1(k^2+l^2-\omega^2)=0,\\&(k^2+l^2-\omega^2)(a_1^2-b_1^2)+4m^2=0.\end{aligned} \tag{100}$$

Solving the above set of algebraic equations, we obtain the following results:

$$a_0=a_1=0,\ b_1\neq0,\ \omega=\pm\sqrt{k^2+l^2-\frac{4m^2}{b_1^2}}, \tag{101}$$

$$a_0=b_1=0,\ a_1\neq0,\ \omega=\pm\sqrt{k^2+l^2+\frac{4m^2}{a_1^2}}. \tag{102}$$

Utilizing Eqs. (101) and (102) and considering Case 2 and Case 3, we obtain the following solutions to the (2+1)-dimensional sine-Poisson equation (1)

$$u(x,t)=4\arctan\left[\exp\left(\frac{b_1}{2}\xi+\xi_0\right)\right], \tag{103}$$

$$u(x,t)=4\operatorname{arccot}\left[-\exp\left(\frac{b_1}{2}\xi+\xi_0\right)\right], \tag{104}$$

$$u(x,t)=\pm4\arccos\left\{\sqrt{\frac{1}{2}\left[1+\operatorname{sech}\left(\frac{b_1}{2}\xi+\xi_0\right)\right]}\right\}-\pi, \tag{105}$$

$$u(x,t)=2\arcsin\left[\operatorname{sech}\left(\frac{b_1}{2}\xi+\xi_0\right)\right], \tag{106}$$

$$u(x,t)=2\arcsin\left[\mathrm{i}\operatorname{csch}\left(\frac{b_1}{2}\xi+\xi_0\right)\right], \tag{107}$$

$$u(x,t)=2\arccos\left[\tanh\left(\frac{b_1}{2}\xi+\xi_0\right)\right], \tag{108}$$

$$u(x,t)=2\arccos\left[\coth\left(\frac{b_1}{2}\xi+\xi_0\right)\right], \tag{109}$$

$$u(x,t)=2\arccos\left[\tanh(b_1\xi+\xi_0)\pm\mathrm{i}\operatorname{sech}(b_1\xi+\xi_0)\right], \tag{110}$$

$$u(x,t)=2\arccos\left[\coth(b_1\xi+\xi_0)\pm\operatorname{csch}(b_1\xi+\xi_0)\right], \tag{111}$$

$$u(x,t)=2\arccos\left[\frac{\tanh(b_1\xi+\xi_0)}{1\pm\operatorname{sech}(b_1\xi+\xi_0)}\right], \tag{112}$$

$$u(x,t)=2\arccos\left[\frac{\coth(b_1\xi+\xi_0)}{1\pm\mathrm{i}\operatorname{csch}(b_1\xi+\xi_0)}\right], \tag{113}$$

$$u(x,t)=2\arccos\left[\frac{A\cosh(b_1\xi+\xi_0)\pm\sqrt{A^2+B^2}}{A\sinh(b_1\xi+\xi_0)\pm B}\right], \tag{114}$$

$$u(x,t)=2\arccos\left[\frac{A\sinh(b_1\xi+\xi_0)\pm\sqrt{B^2-A^2}}{A\cosh(b_1\xi+\xi_0)\pm B}\right]. \tag{115}$$

In the above equations,

$$\xi = kx + ly \mp \sqrt{k^2 + l^2 - \frac{4m^2}{b_1^2}}t\,,$$

and b_1, k, l and ξ_0 are arbitrary constants with $b_1 \neq 0$.

$$u(x,t) = 4\arctan\left[\exp\left(\frac{a_1}{2}\xi + \xi_0\right)\right] - \pi, \tag{116}$$

$$u(x,t) = 4\,\mathrm{arccot}\left[-\exp\left(\frac{a_1}{2}\xi + \xi_0\right)\right] - \pi, \tag{117}$$

$$u(x,t) = \pm 4\arccos\left\{\sqrt{\frac{1}{2}\left[1 + \mathrm{sech}\left(\frac{a_1}{2}\xi + \xi_0\right)\right]}\right\}, \tag{118}$$

$$u(x,t) = 2\arcsin\left[\tanh\left(\frac{a_1}{2}\xi + \xi_0\right)\right], \tag{119}$$

$$u(x,t) = 2\arcsin\left[\coth\left(\frac{a_1}{2}\xi + \xi_0\right)\right], \tag{120}$$

$$u(x,t) = 2\arcsin\left[\tanh(a_1\xi + \xi_0) \pm \mathrm{i}\,\mathrm{sech}(a_1\xi + \xi_0)\right], \tag{121}$$

$$u(x,t) = 2\arcsin\left[\coth(a_1\xi + \xi_0) \pm \mathrm{csch}(a_1\xi + \xi_0)\right], \tag{122}$$

$$u(x,t) = 2\arcsin\left[\frac{\tanh(a_1\xi + \xi_0)}{1 \pm \mathrm{sech}(a_1\xi + \xi_0)}\right], \tag{123}$$

$$u(x,t) = 2\arcsin\left[\frac{\coth(a_1\xi + \xi_0)}{1 \pm \mathrm{i}\,\mathrm{csch}(a_1\xi + \xi_0)}\right], \tag{124}$$

$$u(x,t) = 2\arcsin\left[\frac{A\cosh(a_1\xi + \xi_0) \pm \sqrt{A^2 + B^2}}{A\sinh(a_1\xi + \xi_0) \pm B}\right], \tag{125}$$

$$u(x,t) = 2\arcsin\left[\frac{A\sinh(a_1\xi + \xi_0) \pm \sqrt{B^2 - A^2}}{A\cosh(a_1\xi + \xi_0) \pm B}\right], \tag{126}$$

$$u(x,t) = 2\arccos\left[\mathrm{sech}\left(\frac{a_1}{2}\xi + \xi_0\right)\right], \tag{127}$$

$$u(x,t) = 2\arccos\left[\mathrm{i}\,\mathrm{csch}\left(\frac{a_1}{2}\xi + \xi_0\right)\right], \tag{128}$$

$$u(x,t) = 4\arctan\left[\tanh\left(\frac{a_1}{4}\xi + \xi_0\right)\right], \tag{129}$$

$$u(x,t) = 4\arctan\left[\coth\left(\frac{a_1}{4}\xi + \xi_0\right)\right], \tag{130}$$

$$u(x,t) = 4\,\mathrm{arccot}\left[\tanh\left(\frac{a_1}{4}\xi + \xi_0\right)\right], \tag{131}$$

$$u(x,t) = 4\,\mathrm{arccot}\left[\coth\left(\frac{a_1}{4}\xi + \xi_0\right)\right]. \tag{132}$$

In the above equations, $\xi = kx + ly \mp (k^2 + l^2 + 4m^2t/a_1^2)^{1/2}$, and a_1, k, l, and ξ_0 are arbitrary constants with $a_1 \neq 0$.

3. Conclusions

To sum up, we have modified the method of Sirendaoreji and Sun by introducing a more general ODE, and with the aid of its rich types of solutions, we pick up many explicit and exact solutions for the (2+1)-dimensional sine-Poisson equation in a concise way. The main idea of our approach is to convert a nonlinear partial differential equation in question into an auxiliary ODE and to solve the resulting equation by separation of variables. Thus, the problem for solving nonlinear partial differential equations is significantly simplified. The topic we discussed may be interesting in solution theory of nonlinear partial differential equations, and the results obtained could be a good supplement to the existing literature as a new example of exact solutions for the (2+1)-dimensional sine-Poisson equation. How to extend this approach to solve other NEEs in mathematical physics is worth while to further study.

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