Interaction behaviours of solitoffs: fission, fusion, reconnection and annihilation*

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The interactions between solitoffs are extensively investigated. Besides the known solitoff fission and fusion interactions, two new types of solitoff interactions are discovered, named the solitoff reconnection and the solitoff annihilation. Taking the asymmetric Nizhnik–Novikov–Veselov equation as an illustrative system, five types of solitoff interactions are graphically revealed on the basis of the analytical solution obtained by the modified tanh function expansion method.

Keywords: fission, fusion, reconnection, annihilation
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1. Introduction

There are two main important tasks in soliton theory. One is to find exact solutions of the nonlinear integrable systems, and the other is to construct nonlinear coherent structures in high dimensions (at least two space variables) and to study their interactions.

Up to now, a large number of studies have been devoted to the interaction properties of solitons and solitary wave solutions (see Refs. [1]–[12] and references therein). Generally speaking, the interaction properties of nonlinear coherent structures can be divided into two categories: elastic and non-elastic. In detail, a nonlinear structure possesses an elastic interaction when it maintains its shape and velocity with or without phase shift during the interaction. A nonlinear structure will have a non-elastic interaction if it suffers any of the following situations during the interaction: (i) either partially or totally exchanging its shape; (ii) changing its velocity; (iii) having a fission interaction, namely, a nonlinear structure splitting into several structures of the same type; (iv) having a fusion phenomenon, say, several similar structures forming one structure. Nonetheless, the interaction behaviours are still far beyond being fully understood.

It is noted that a type of reconnection interaction, first discovered for multiple "Y" shape solitons more than one decade ago,[13] has recently been revealed for dromions. The dromion reconnection interaction is a scenario in which the bounded dromions can be opened and reconnected during the interaction.[14] In addition, the dromion reflection interaction was also discovered first in Ref. [14]. A dromion may change its moving direction just like a ball bouncing against an "invisible" wall and then reflected. Essentially, it is a kind of interaction between dromion and a ghost (invisible) wall caused by a ghost line soliton. These interaction behaviours have not yet been discovered for other excitation modes.

The goal of this paper is to report the interaction behaviours of solitoffs. Taking the asymmetric Nizhnik–Novikov–Veselov (ANNV) equation as an illustrative system, we first obtain its special exact solution with arbitrary lower-dimensional functions by a modified function expansion method. The solitoffs can be readily constructed via fixing the arbitrary functions as some particular concrete expressions with several extra constant parameters. Finally, through adjusting the parameters, we obtain five types of interaction behaviours of solitoffs including the solitoff reconnection and reflection interactions. All the inter-

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esting interactions are graphically revealed.

2. Exact solution of the ANNV system

The hyperbolic function expansion method together with various extensions[15−21] has been widely used to obtain exact solutions of nonlinear partial differential equations (NPDEs) due to the simple principle and easy implementation with the help of computers. Here, we briefly introduce a modified function expansion method. The basic idea of the method is to search for the solution that can be expressed as a finite series of some special functions.

Considering a given NPDE in three variables

$$F(U, U_t, U_x, U_y, \ldots) = 0, \qquad U \equiv U(x, y, t), \quad (1)$$

we write down its solution in the form of

$$U = \sum_{i=0}^{n} \alpha_i \phi^i, \quad (2)$$

where $\phi = Z(f(x, y, t)) \equiv Z(f)$, $\alpha_i \equiv \alpha_i(x, y, t)$, and n is a positive integer determined by balancing the highest nonlinear term with the highest dispersive term.

It is noted that if $Z(f)$ is taken as $\tanh(f)$, or $\mathrm{sech}(f)$, or $\mathrm{sn}(f)$ ($\mathrm{cn}(f)$ or $\mathrm{dn}(f)$) and $f = kx+ly-\omega t$ (k, l, ω are constants), then we have the tanh, or the sech, or the Jacobi elliptic function expansion method, respectively. If $Z(f)$ is assumed to be in the form of some combinations of the functions mentioned above, then we have the relevant extended function expansion methods. Furthermore, one may require that $Z(f)$ should satisfy some special differential equations, and as a consequence, many different function expansion methods are unified.[22]

For seeking for solutions with arbitrary functions, f can no longer be taken as a function of a travelling wave variable, i.e., $f \neq f(kx+ly-\omega t)$. The implementation of the method is as follows. Substituting Eq. (2) with particular selections or constraints of $Z(f)$ into Eq. (1) and letting all the coefficients of different powers of $Z(f)$ and/or different orders of $Z(f)$ be equal to zero yield a set of differential equation, α_i and f are solved from the set of differential equations. Finally, substituting the known n, α_i and f into Eq. (2) gives the exact solution of Eq. (1). A similar method has been considered recently in Refs. [23] and [24].

In principle, it is possible to allow infinite sums in formula (2), and seeking for solutions of the type $U = F(\tanh(f))$. One example where this more general ansatz proves useful is the case of the Camassa–Holm (CH) equation,[25]

$$U_t - U_{txx} + 3UU_x = 2U_xU_{xx} + UU_{xxx},$$

which is an integrable model for shallow water waves.[26] It has the explicit solution[27,28]

$$U(x,t) = \frac{\cosh(x - t - [x - t] - \frac{1}{2})}{\sinh(\frac{1}{2})}, \qquad t \geq 0,\ x \in R,$$

where $[y]$ is the integer part of $y \in R$: $[y] = k$ if $k \leq y < k + 1$. Since

$$\cosh(y) = \frac{1}{\sqrt{1 - \tanh^2(y)}},$$

this periodic travelling wave of the CH equation is one of the mentioned types with

$$F(y) = \frac{1}{\sinh(\frac{1}{2})\sqrt{1 - y^2}}$$

for $y \in (-1, 1)$, and $f(x, t) = x - t - [x - t] - \frac{1}{2}$ for $t \geq 0$, $x \in R$. Notice that it is a peaked travelling wave (not continuously differentiable), which is not a classical solution but a weak solution.[27,29]

In the following, we assume that $Z(f) = \tanh(f)$ and apply the above method to the ANNV system,[7,30−38] thus we have

$$U_t + U_{xxx} - 3U_xV - 3UV_x = 0, \quad (3)$$

$$U_x - V_y = 0. \quad (4)$$

Based on the previous descriptions, the exact solution of ANNV equations (3) and(4) can be written as

$$U = d_0 + d_1 \tanh(f) + d_2 \tanh^2(f), \quad \text{and} \quad V = e_0 + e_1 \tanh(f) + e_2 \tanh^2(f), \quad (5)$$

where d_i, e_i ($i = 0, 1, 2$) and f are all functions of $\{x, y, t\}$. Substituting Eq. (5) into Eqs. (3) and (4), and then requiring all the coefficients of the different powers of tanh to equal zero, we obtain

$$e_2 - 2f_x^2 = 0, \quad d_2f_x - e_2f_y = 0, \quad (6)$$

$$e_1 f_y - d_1 f_x - e_{2y} + d_{2x} = 0, \qquad d_{1x} - e_{1y} - 2e_2 f_y + 2d_2 f_x = 0, \tag{7}$$

$$-2d_1 f_x^3 - (d_2 e_2)_x + 3(e_1 d_2 + d_1 e_2) f_x + 6d_2 f_x f_{xx} + 6d_{2x} f_x^2 = 0, \tag{8}$$

$$\begin{aligned} 6f_x(e_2 e_0 &- d_{2xx} - 2d_2 e_2 + d_0 e_2 + d_1 f_{xx} + e_1 d_1) - 3(d_1 e_2 + e_1 d_2)_x + 40 d_2 f_x^3 \\ &- 2d_2 f_{xxx} + 6d_{1x} f_x^2 - 6d_{2x} f_{xx} - 2d_2 f_t = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} -3(d_1 e_1 &+ d_2 e_0 + d_0 e_2 + d_{1x} f_x)_x + 8d_1 f_x^3 - 24 d_{2x} f_x^2 - d_1 f_{xxx} + d_{2xxx} \\ &+ 3f_x(d_1 e_0 + d_0 e_1 - 3d_2 e_1 - 3d_1 e_2 - 8d_2 f_{xx}) + d_{2t} - d_1 f_t = 0, \end{aligned} \tag{10}$$

$$\begin{aligned} 3(2d_{2x} f_x &- d_1 e_0 - d_0 e_1)_x - 6f_x(d_2 e_0 + d_1 e_1 + d_0 e_2) + d_{1t} + d_{1xxx} + 2d_2 f_{xxx} \\ &- 6f_x^2(d_{1x} + d_1 f_x) - 16 d_2 f_x^3 + 2d_2 f_t = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} 3(d_{1x} f_x &- d_0 e_0)_x + 3f_x(2d_2 f_{xx} - d_1 e_0 - d_0 e_1) - 2d_1 f_x^3 + 6d_{2x} f_x^2 \\ &+ d_1 f_t + d_{0t} + d_{0xxx} + d_1 f_{xxx} = 0, \end{aligned} \tag{12}$$

$$d_{0x} - e_1 f_y + d_1 f_x - e_{0y} = 0. \tag{13}$$

Solving Eqs. (6)–(11) gives

$$e_1 = -2f_{xx}, \quad e_2 = 2{f_x}^2,$$

$$d_1 = -2f_{xy}, \quad d_2 = 2f_x f_y, \tag{14}$$

$$\begin{aligned} e_0 = {} & [3f_x f_{xx} f_y - 3f_x^2 f_{xy}]^{-1}\big[f_{xx} f_y (f_t + f_{xxx} - 2f_x^3) \\ & + 3f_x f_{xx} f_{xxy} + 6f_x^4 f_{xy} - f_x^2 (f_{xxxy} + f_{yt}) \\ & - 3f_{xx}^2 f_{xy}\big], \end{aligned} \tag{15}$$

$$\begin{aligned} d_0 = {} & [3f_x f_{xx} f_y - 3f_x^2 f_{xy}]^{-1}\big[f_x f_y (f_{yt} + f_{xxxy}) \\ & + f_y f_{xy}(2f_x^3 - f_{xxx}) + 3f_{xy}^2 f_{xx} \\ & - f_{xy}(f_y f_t + 3f_x f_{xxy}) - 6f_x^2 f_y^2 f_{xx}\big]. \end{aligned} \tag{16}$$

It can be easily verified that f in a variable separated form

$$f = p(x,t) + q(y) \equiv p + q \tag{17}$$

with p and q being arbitrary functions of their arguments, can solve the remaining equations (12) and (13) with Eqs. (14)–(16). Thus, all the coefficients in solution (5) are identified as

$$d_0 = -2p_x q_y, \quad d_1 = 0, \quad d_2 = 2p_x q_y, \tag{18}$$

$$e_0 = \frac{p_t - 2p_x^3 + p_{xxx}}{3p_x},$$

$$e_1 = -2p_{xx}, \quad e_2 = 2p_x^3, \tag{19}$$

and the exact solution of the ANNV system is

$$U = -2p_x q_y \operatorname{sech}(p+q)^2, \tag{20}$$

$$\begin{aligned} V = {} & \frac{p_t - 2p_x^3 + p_{xxx}}{3p_x} - 2p_{xx}\tanh(p+q) \\ & + 2p_x^2 \tanh(p+q)^2, \end{aligned} \tag{21}$$

where $p \equiv p(x,t)$ and $q \equiv q(y)$ are arbitrary functions of their arguments.

If we write p and q as

$$p = \ln P', \quad q = \ln \frac{1}{Q'}, \tag{22}$$

or

$$p = \ln P', \quad q = \ln Q', \tag{23}$$

where $P' \equiv P'(x,t)$ and $Q' \equiv Q'(y)$ are two arbitrary functions of their indicated arguments, then equations (20) becomes

$$U = \frac{2(P'^2)_x (Q'^2)_y}{(P'^2 + Q'^2)^2}, \tag{24}$$

or

$$U = -\frac{2(P'^2)_x (Q'^2)_y}{(1 + P'^2 Q'^2)^2}, \tag{25}$$

respectively. Further, assuming $P'^2 = P$ and $Q'^2 = Q$, then equations (24) and (25) become

$$U = \frac{2P_x Q_y}{(P+Q)^2}, \tag{26}$$

and

$$U = -\frac{2P_x Q_y}{(1+PQ)^2}, \tag{27}$$

respectively.

It is noted that in the study of integrable systems, by means of the multi-linear variable separation approach (MLVSA),[7,39,40] a quite "universal" formula

$$U \equiv \frac{2(a_1 a_2 - a_0 a_3) p_x q_y}{(a_0 + a_1 p + a_2 q + a_3 pq)^2} \tag{28}$$

is proved valid for the suitable fields of a diversity of (2+1)-dimensional physically interesting integrable

models. In Eq. (28), a_0, a_1, a_2 and a_3 are arbitrary constants, $p \equiv p(x,t)$ is an arbitrary function, and $q \equiv q(y,t)$ may be either an arbitrary function for some models or an arbitrary solution of a simpler (1+1)-dimensional equation like the Riccati equation for other systems.

Evidently, if $a_0 = a_3 = 0$, $a_1 = a_2 = 1$, or contrarily, $a_0 = a_3 = 1$, $a_1 = a_2 = 0$, equation (28) becomes Eq. (26) or Eq. (27) respectively with a distinct difference that q is an arbitrary function of y only.

3. Interaction phenomena of solitoffs

The arbitrary functions in the solution contribute much to the construction of nonlinear coherent structures.[7] In order to investigate the interaction behaviours of solitoffs, we simply take the arbitrary functions P and Q in Eqs. (26) and (27) as

$$P = \exp(k_1 x + \omega_1 t) + \exp(k_2 x + \omega_2 t),$$
$$Q = \exp(K_1 y) + \exp(K_2 y) + C \tag{29}$$

with seven constant parameters k_i, K_i, ω_i $(i = 1, 2)$ and C. With different values of the parameters, solutions (26) and (27) with Eq. (29) can be solitoffs and possess various interaction behaviours.

3.1. Interaction among three solitoffs

If we fix the parameters to be

$$k_1 = K_1 = 1, \quad k_2 = K_2 = 2,$$
$$-\omega_1 = \omega_2 = 3, \quad C = 0, \tag{30}$$

solution expressed by Eq. (26) with Eqs. (29) and (30) may display an interaction of three solitoffs (Fig. 1).

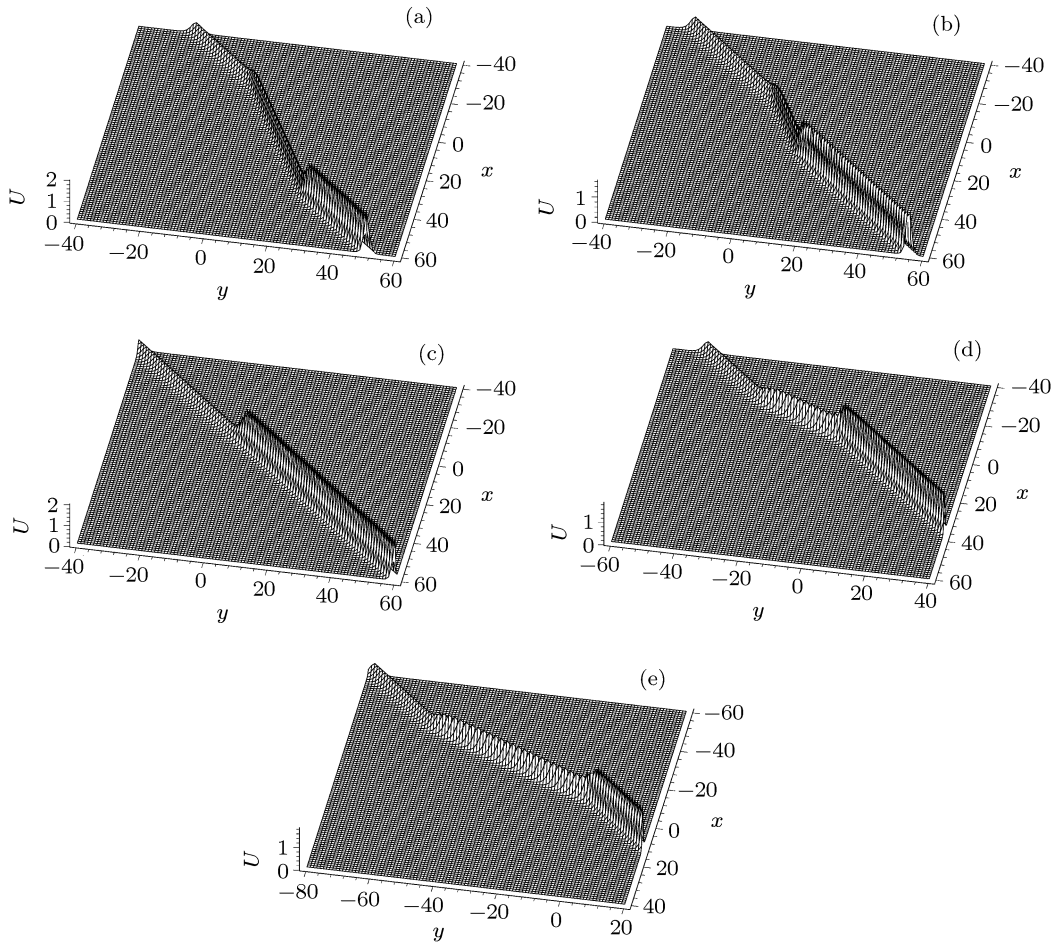


Fig. 1. Evolutional plots of three solitoffs expressed by Eq. (26) with Eqs. (29) and (30) at times $t = -6$ (a), -3 (b), 0 (c), 3 (d), and 6 (e).

It can be seen that before the interaction (Figs. 1(a) and 1(b)), two solitoffs situated separately on the lines $x-y-3t=0$ and $2x-2y+3t=0$ are connected by a solitoff lying on the line $x-2y-3t=0$ at the points $(3t,0)$ and $(-6t,-\frac{9}{2}t)$ respectively. During the interaction (Fig. 1(c)), the middle solitoff disappears. After the interaction (Figs. 1(d) and 1(e)), a fourth solitoff appears, located on the line $2x-y+3t=0$ and joins in the remaining two solitoffs at the points $(-6t,-9t)$ and $(-\frac{3}{2}t,0)$ respectively.

Alternatively, this interaction phenomenon can be viewed as an interaction among three solitoffs (visible) and one ghost solitoff (invisible).

3.2. Fusion of two solitoffs and two antisolitoffs

If the parameters are fixed to be

$$k_1=-k_2=K_1=1,\quad \omega_1=3,$$
$$\omega_2=4,\quad K_2=2,\quad C=0, \tag{31}$$

then solution (26) with Eqs. (29) and (31) describes the fusion interaction among four solitoffs (Fig. 2).

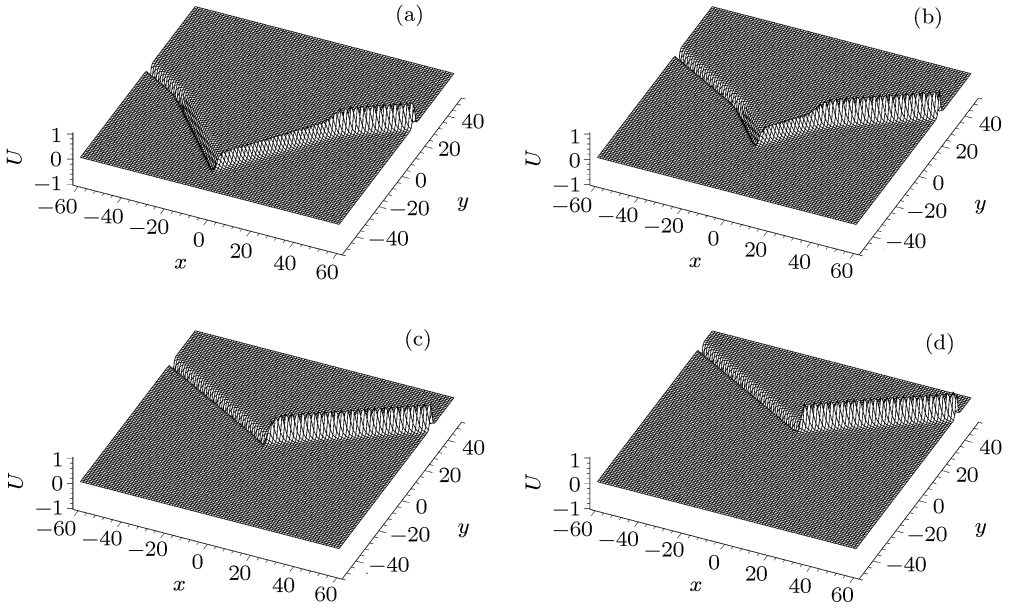


Fig. 2. Plots of fusion interaction among four solitoffs given by Eq. (26) with Eqs. (29) and (31) at times $t=-10$ (a), -5 (b), 0 (c), and 5 (d).

It can be observed from Fig. 2 that before the interaction, there are two solitoffs (with positive amplitudes) and two antisolitoffs (with negative amplitudes) while after the interaction, only one solitoff and one antisolitoff survive. Specifically, before the interaction (Fig. 2(a)), two solitoffs are located on the lines $x-2y+3t=0$ and $x-y+3t=0$ separately and connected with each other at $(-3t,0)$; two antisolitoffs are situated on the lines $x+y-4t=0$ and $x+2y-4t=0$ separately and coalesce together at $(4t,0)$ into a "V"-like structure with the apex at $(\frac{1}{2}t,\frac{7}{2}t)$. With time passing by, the solitoff on the line $x-2y+3t=0$ and the antisolitoff on the line $x+y-4t=0$ become shorter (Fig. 2(b)), and disappear during the interaction (Fig. 2(c)). After the interaction (Fig. 2(d)), the remaining solitoff and antisolitoff coalesce together at $(\frac{1}{2}t,\frac{7}{4}t)$ into a stable solitoff–antisolitoff pair.

3.3. Fission of a solitoff–antisolitoff pair

To obtain the fission interaction between solitoffs, we set the parameters to be

$$k_1=K_1=-k_2=2,\quad \omega_1=1,\quad \omega_2=-3,\quad C=0. \tag{32}$$

This fission phenomenon is illustrated in Fig. 3. It is seen that before the interaction, there are one solitoff situated on the line $2x-2y+t=0$ and one antisolitoff on the line $2x+2y+3t=0$ and they are joined at $(-t,-\frac{1}{2}t)$ to form a solitoff–antisolitoff pair. During the interaction, one solitoff lying on the line $6x-y+3t=0$ and one antisolitoff on the line $6x+y+9t=0$ are generated, which not only join each other at the apex $(-t,-3t)$ but also connect with the existing solitoff and antisolitoff at the points $(-\frac{1}{2}t,0)$ and $(-\frac{3}{2}t,0)$ respectively. After the interaction, two solitoffs and two antisolitoffs coalesce into a whole entity.

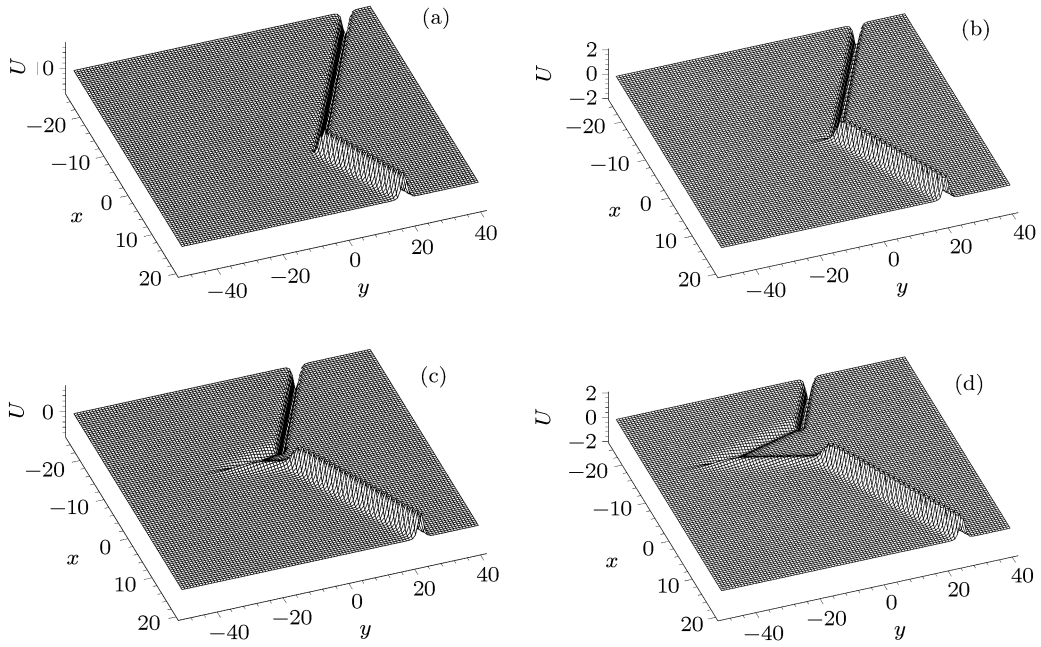


Fig. 3. Fission interaction between two solitoffs given by Eq. (26) with Eqs. (29) and (32) at times $t=-5$ (a), 0 (b), 5 (c), and 10 (d).

3.4. Reconnection of two solitoff–antisolitoff pairs

Equation (26) with Eq. (29) may also show a reconnection interaction of solitoffs. Figure 4 shows the evolutional plots of the reconnection interaction of two solitoff–antisolitoff pairs with the parameters

$$\omega_1=\omega_2=-k_1=K_1=1,\quad k_2=-K_2=\frac{1}{3},\quad C=0. \tag{33}$$

It is observed that there are two solitoff and antisolitoff pairs before the interaction (Figs. 4(a) and 4(b)). One of the pairs is composed of one solitoff situated on the line $x+y+3t=0$ and one antisolitoff on the line $x-3y+3t=0$ in the area $x>0$ with its apex at $(-3t,0)$ and a velocity of -3 along the x direction. The other pair consists of one solitoff located on the line $x+y-t=0$ and one antisolitoff on $3x-y-3t=0$ in the area $x<0$ with its apex being at $(t,0)$ and the velocity being 1 along the x direction. The two pairs stably move toward each other along the x direction.

During the interaction (Figs. 4(c) and 4(d)), these two solitoff–antisolitoff pairs are opened and then reconnected to form two new solitoff–antisolitoff pairs. Specifically, the solitoff standing on the line $x-3y+3t=0$ is reconnected with the antisolitoff on the line $x+y-t=0$ to form one pair with the apex at $(0,t)$ and a velocity of 1 along the y direction in the area $y>0$, and the other solitoff and antisolitoff, perching on the lines $3x-y-3t=0$ and $x+y+3t=0$, respectively, are reconnected to form another new pair with the apex at $(0,-3t)$ and a velocity of -3 along the y direction in the area $y<0$.

After the interaction (Figs. 4(d) and 4(e)), these two reconnected solitoff–antisolitoff pairs stably move

against each other along the y direction.

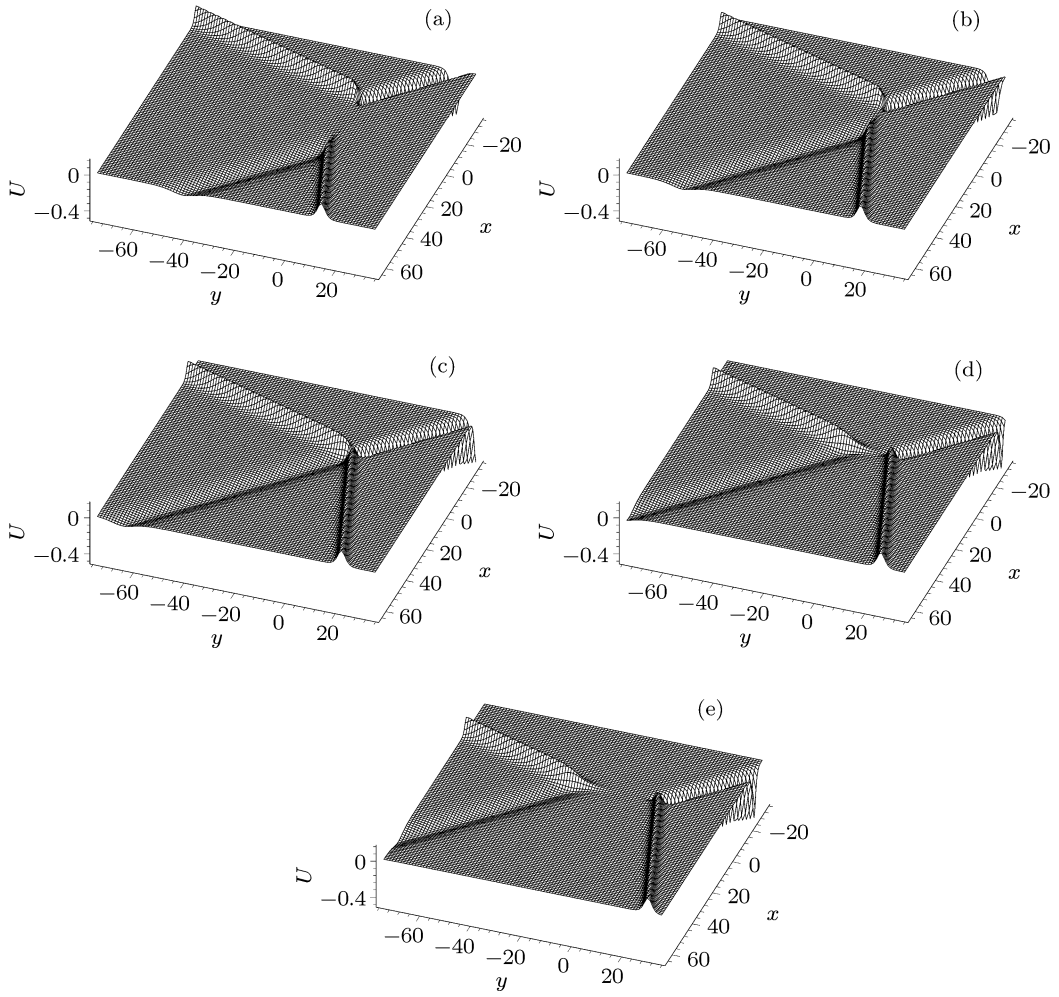


Fig. 4. Evolutional plots of reconnection interaction between two solitoffs and two anitisolitoffs given by Eqs. (26) with Eqs. (29) and (33) at times $t = -8$ (a), -4 (b), 0 (c), 4 (d), and 8 (e).

3.5. Annihilation of two solitoffs and two antisolitoffs

We find that under the parameters

$$\omega_1 = \omega_2 = -k_1 = K_1 = 1, \quad k_2 = -K_2 = \frac{1}{3}, \quad C = 0, \tag{34}$$

solution (27) with Eq. (29) can be a kind of special interaction of two solitoffs and two antisolitoffs, here termed "annihilation interaction" (Fig. 5).

It is seen that before the interaction (Fig. 5(a)), two solitoffs situated separately on the lines $x - y - t = 0$ and $x - y + 3t = 0$ and two antisolitoffs located separately on the lines $x + 3y + 3t = 0$ and $3x + y - 3t = 0$ constitute a trapeziform structure. They move closer and closer (Fig. 5(b)) and are then combined into an entity during the interaction (Fig. 5(c)). Afterwards, the amplitude of the entity reduces at a very fast rate while the shape remains unchanged (Figs. 5(d) and 5(e)). Actually, from time $t = 0$ to $t = 15$, the height of the combined entity decreases from about 0.06 to about $2.4 \times \mathrm{e}^{-8}$. The magnification of this entity shows a consolidated two-dipole-type-dromion structure (Figs. 5(f) and 5(g)). Since two solitoffs and two antisolitoffs almost disappear immediately after the interaction, we refer to this interaction phenomenon as the annihilation

of two solitoff and antisolitoff pairs.

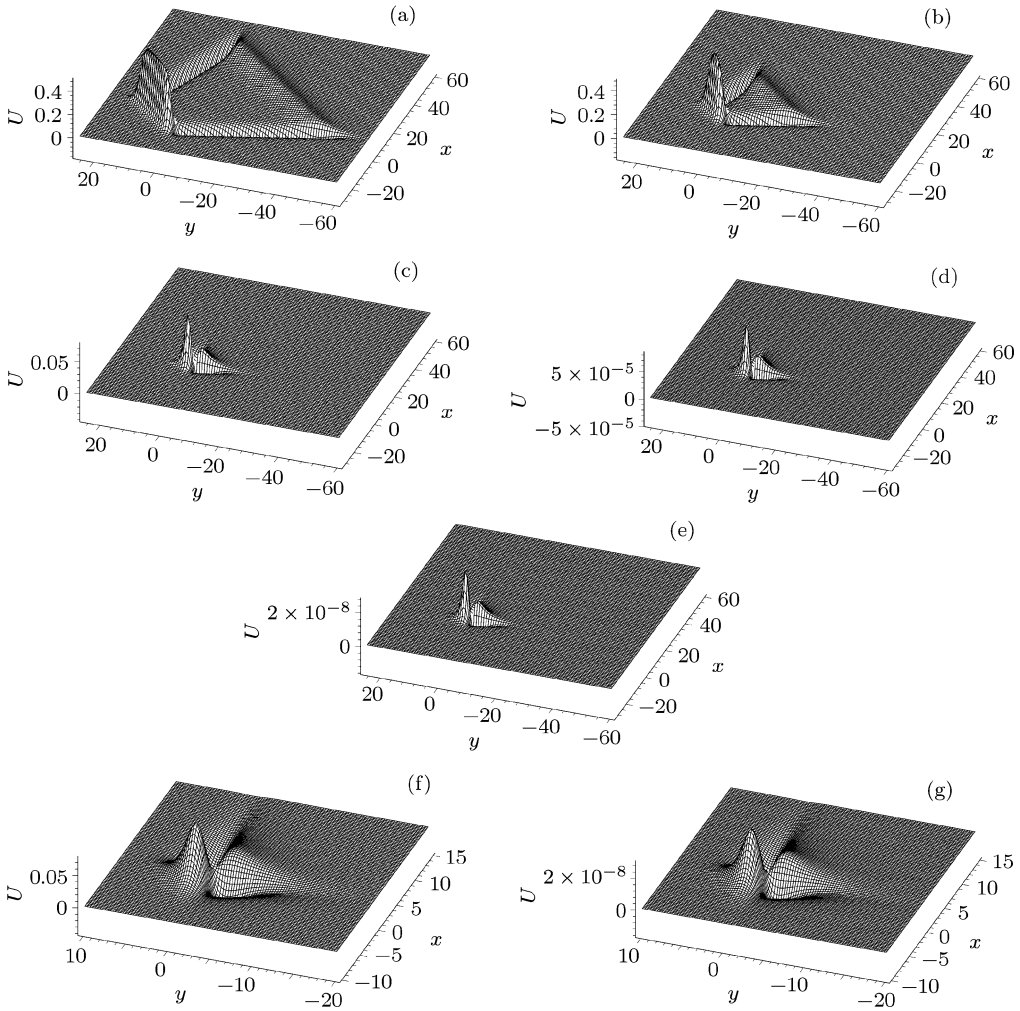


Fig. 5. Evolutional plots of the annihilation of two solitoffs and two antisolitoffs expressed by Eq. (27) with Eqs. (29) and (34) at times $t = -15$ (a), -7 (b), 0 (c), 7 (d), and 15 (e), and panels (f) and (g) are closeups of panels (c) and (e), respectively.

4. Summary and discussion

A simple and effective modified tanh function expansion method is used to obtain the exact solution of the (2+1)-dimensional ANNV system. The introduction of two arbitrary functions permit us to construct abundant nonlinear coherent structures. In this paper, we assume the arbitrary functions to be some exponential functions with some constant parameters. As a result, we can generate solitoffs and study their fission, fusion, reconnection and annihilation interaction behaviours of solitoffs. The reconnection and annihilation interactions of solitoffs are observed for the first time. It is noted that a similar reconnection phenomenon has been found in a plasma system.[13]

Solution (27) with the selection (29) can be a reconnection interaction of solitoffs as long as $k_1k_2 < 0$ and $K_1K_2 < 0$. In this case, the values of ω_1 and ω_2 determine the moving directions of two solitoff–antisolitoff pairs, either coming toward each other along the x direction, reconnected and then going away from each other along the y direction, or *vice versa*. Nonetheless, it is difficult to fix the parameter conditions for the fission and the fusion interactions.

The nonlinear structures and the interaction behaviours are both very abundant. For example, figure 6 depicted according to the solution expression

$$U = \frac{\left(\mathrm{e}^{\frac{1}{2}x+t} - \mathrm{e}^{-\frac{1}{2}x+t} + 4\,\mathrm{e}^{2x-t} - 4\,\mathrm{e}^{-2x-t}\right)\left(3\,\mathrm{e}^{y} - 3\,\mathrm{e}^{-y} + \mathrm{e}^{\frac{1}{3}y} - \mathrm{e}^{-\frac{1}{3}y}\right)}{3\left(\mathrm{e}^{\frac{1}{2}x+t} + \mathrm{e}^{-\frac{1}{2}x+t} + \mathrm{e}^{2x-t} + \mathrm{e}^{-2x-t} + \mathrm{e}^{y} + \mathrm{e}^{-y} + \mathrm{e}^{\frac{1}{3}y} + \mathrm{e}^{-\frac{1}{3}y}\right)^2}, \tag{35}$$

reveals a fission interaction of two solitoff–antisolitoff pairs, that is, two solitoff–antisolitoff pairs fission into four solitoffs and four antisolitoffs during the interaction.

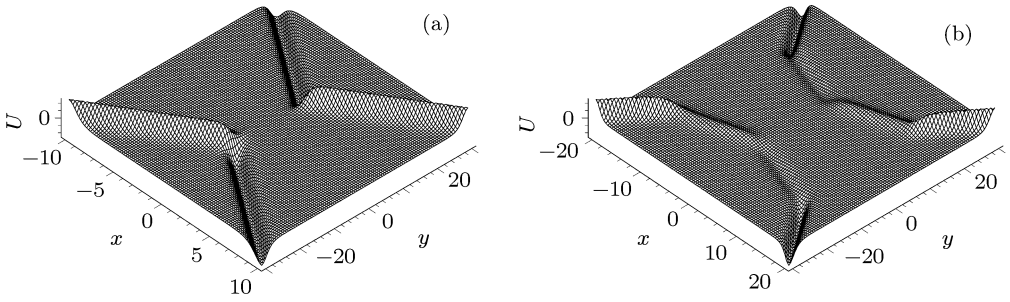


Fig. 6. Fission interaction of two solitoff–antisolitoff pairs expressed by Eq. (35) at times $t = -10$ (a) and 10 (b).

Similar interactions, especially the reconnection and the annihilation interactions, are expected to be valid for other nonlinear structures and exist in other nonlinear systems. Since the ANNV equation was discovered only in the mathematical framework, the future study will focus on some models with strong physical background. In addition, it is worth while finding the possible applications of these various interaction phenomena.

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References

[1] Chow K W, Grimshaw R H J and Ding E 2005 *Wave Motion* **43** 158
[2] Radhakrishnan R, Kundu A and Lakshmanan M 1999 *Phys. Rev.* E **60** 3314
[3] Calogero F and Degasperis A 2005 *Physica* D **200** 242
[4] Lou S Y 2002 *J. Phys.* A: *Math. Gen.* **35** 10619
[5] Lou S Y 2003 *J. Phys.* A: *Math. Gen.* **36** 3877
[6] Lou S Y, Hu H C and Tang X Y 2005 *Phys. Rev.* E **71** 036604
[7] Tang X Y, Lou S Y and Zhang Y 2002 *Phys. Rev.* E **66** 046601
[8] Yang J R and Mao J J 2008 *Chin. Phys.* B **17** 4337
[9] Li H M 2008 *Chin. Phys.* B **17** 759
[10] Deng X J, Yang Z Z and Han L B 2009 *Chin. Phys.* B **18** 3169
[11] Song J and Yang L G 2009 *Chin. Phys.* B **18** 2873
[12] Zhao L N, Tong Z S and Lin J 2009 *Chin. Phys.* B **18** 2352
[13] Nishinari K, Abe K and Satsuma J 1993 *J. Phys. Soc. Jpn.* **62** 2021
[14] Tang X Y, Li J M and Lou S Y 2007 *Phys. Scr.* **75** 201
[15] Malfliet W 1992 *Am. J. Phys.* **60** 650
[16] Li Z B and Wang M L 1993 *J. Phys.* A: *Math. Gen.* **26** 6927
[17] Parkes E J and Duffy B R 1996 *Comput. Phys. Commun.* **98** 288
[18] Duffy B R and Parkes E J 1996 *Phys. Lett.* A **214** 271
[19] Fan E G 2000 *Phys. Lett.* A **277** 212
[20] Fan E G, Zhang J and Benny H 2001 *Phys. Lett.* A **291** 376
[21] Chen Y, Wang Q and Lang Y H 2005 *Z. Naturforsch* a **60** 127
[22] Darwish A and Fan E G 2004 *Chaos, Solitons and Fractals* **20** 609
[23] Zheng C L, Fang J P and Chen L Q 2005 *Chaos, Solitons and Fractals* **23** 1741
[24] Dai C H and Zhang J F 2006 *J. Math. Phys.* **47** 043501
[25] Camassa R and Holm D D 1993 *Phys. Rev. Lett.* **71** 1661
[26] Constantin A and McKean H P 1999 *Commun. Pure. Appl. Math.* **52** 949
[27] Constantin A and Escher J 1998 *Commun. Pure. Appl. Math.* **51** 475
[28] Lenells J 2004 *Int. Math. Res. Not.* **10** 485
[29] Constantin A and Molinet L 2000 *Commun. Math. Phys.* **211** 45
[30] Lou S Y 1995 *J. Phys.* A: *Math. Gen.* **28** 7227
[31] Boiti M, Leon J J P, Manna M and Penpinelli F 1986 *Inverse Problem* **2** 271
[32] Boiti M, Leon J J P, Manna M and Penpinelli F 1987 *Inverse Problem* **3** 25
[33] Clarkson P A and Mansfield E L 1994 *Nonlinearity* **7** 975
[34] Radha R and Lakshmanan M 1994 *J. Math. Phys.* **35** 4746
[35] Lou S Y and Ruan H Y 2001 *J. Phys.* A: *Math. Gen.* **34** 305
[36] Lou S Y and Hu X B 1997 *J. Math. Phys.* **38** 6401
[37] Hu H C, Tang X Y, Lou S Y and Liu Q P 2004 *Chaos, Solitons and Fractals* **22** 327
[38] Ruan H Y and Chen Y X 2004 *J. Phys.* A: *Math. Gen.* **37** 2709
[39] Tang X Y and Lou S Y 2002 *Chaos, Solitons and Fractals* **14** 1451
[40] Tang X Y and Lou S Y 2003 *J. Math. Phys.* **44** 4000