Maintenance of cooperation induced by punishment in public goods games*

Wang Zhen(王 震)[a], Xu Zhao-Jin(许照锦)[b],
Huang Jian-Hua(黄建华)[a], and Zhang Lian-Zhong(张连众)[a]†

[a] *School of Physics, Nankai University, Tianjin* 300071, *China*
[b] *School of Science, Tianjin University of Technology, Tianjin* 300384, *China*

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In this paper, we study the public goods games with punishment by adopting the well-known approximate best response dynamics. It shows that the evolution of cooperation is affected by two aspects when other parameters are fixed. One is the punishment mechanism which can avoid the dilemma of lacking investment, and the other is the degree of rationality. Theoretical analysis and numerical results indicate that the existence of punishment mechanism and distribution of rationality are the keys to the enhancement of cooperation level. We also testify that they can heavily influence the payoffs of system as well. The findings in this paper may provide a deeper understanding of some social dilemmas.

Keywords: public goods games, punishment, degree of rationality, evolution

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1. Introduction

The evolution of cooperation is a fundamental problem in both animal and human societies.[1−7] Due to its abundance, it has drawn considerable attention from sociologists, economists, biologists, physicists, and others. In particular, public goods games and its variations have become powerful tools to investigate the evolution of cooperation.[7−12] However, the emergency of cooperation is always an intense challenge: if cooperation brings a benefit to other individuals at the cost of its own benefit, it will disappear, which contradicts Darwinian evolution. All such characteristics have been addressed by "the tragedy of commons".[13,14]

In the classical form of public goods games (PGGs), the game consists of N individuals, who must simultaneously decide whether to cooperate or defect, and whose payoffs depend on their options. We generally assume that, for example, an individual can endow a fixed money c into a common pool. All the individuals have the opportunity to invest their money knowing that the total amount in the pool will be tripled, and then shared equally among N individuals, irrespective of whether they contributed. It is obvious that if every individual makes investment, the games will end up with the maximal payoff. However, each individual faces temptation to defect, because it is possible to obtain a higher benefit by exploiting others' investments rather than his own investment. Therefore, the rational individuals will decide to freely exploit other individuals' contributions. If such behaviour spreads widely, finally the dilemma that no one will invest money and their capital will not increase any more emerges. In traditional game theory, this unfortunate result of defection persisting and cooperation dying out is known as the Nash equilibrium.[12,15]

During the past decades, the original form of PGG has been extended for different types of games including voluntary PGG, spatially structured PGG, and so on.[8−10,16−26] Besides, mechanisms on promoting cooperation and corresponding updating rules have been proposed to explain how the cooperation can be sustained in a system of selfish individuals. The most prominent are replicator dynamics with voluntary PGG and the random sequential updates with spatially networks games.[4,8,9,11,12,21,24,25] In volun-

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†Corresponding author. E-mail: zhanglz@nankai.edu.cn
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tary PGG, replicator dynamics, which plays a vital part in facilitating cooperation, leads to a rock-scissor-paper cycle, because every individual possesses the capability of imitating or learning more successful strategies for higher profit.[8,10] However, if one of all the strategies vanishes, it cannot be recovered forever and the overall number of strategies will decrease. For spatially structured PGG, each individual can adopt a strategy from one of his nearest neighbours with a probability dependent on their payoff difference, which has been presented by random sequential updates.[9,16,17,25,26] When the varying of the model parameters, like the noise, and the structure of connectivity are considered, the equilibrium frequency of cooperation is also changing. Moreover, it has been turned out that if the trend of choosing defection exceeds a threshold value in various kinds of spatial evolutionary games, cooperation will disappear. That is, spatial structure is not necessarily beneficial to cooperation.

In this paper, a punishment mechanism is introduced into the classical PGG to study how cooperation can be maintained. Such a mechanism is natural in social systems to overcome the dilemma.[27,28] In addition, the degree of rationality K is proposed to explain the equilibrium of cooperation and defection. We explicate that punishment can revert the extinction of cooperation, irrespective of the initial conditions. Moreover, since irrationality is a part of everyday life, we argue that the explicit variations of the degree of rationality K present a significant way to promote cooperation in the games.

2. Model

Our model is defined as follows. The PGG with punishment is played in a finite population. From time to time, a group of N individuals are chosen randomly to form such a population. Simultaneously, they must decide either to cooperate or to defect. Moreover, we assume that cooperators (individuals choosing cooperation) are offered the opportunity to punish the defectors (individuals choosing defection) by imposing a small fine μ. Although defectors still exploit the profit of cooperators, they will incur a cost of ν multiplied by the number of defectors ($\nu > \mu$). Thus, the payoffs of N players depend on the choice of their strategies. If $n_{\rm c}$ and $n_{\rm d}$ denote the numbers of cooperators and defectors in the games respectively (i.e., $n_{\rm c}+n_{\rm d} = N$), the payoffs for cooperators $p_{\rm c}$ and defectors $p_{\rm d}$ is given by

$$p_{\rm c} = cr\frac{n_{\rm c}}{N} - c - \mu, \quad (1 \le n_{\rm c} \le N), \tag{1}$$

$$\begin{aligned} p_{\rm d} &= cr\frac{n_{\rm c}}{N} - \nu(N - n_{\rm c}) \\ &= cr\frac{n_{\rm c}}{N} - \nu n_{\rm d}, \quad (1 \le n_{\rm c} \le N), \end{aligned} \tag{2}$$

where r $(1 < r < N)$ represents the interest rate of the common pool. Obviously, if no cooperators show up (i.e., $n_{\rm c} = 0$), it will not yield any payoffs for both strategies. While all the individuals choose to cooperate (i.e., $n_{\rm c} = N$), the payoff for defectors spontaneously equals zero.

In order to calculate the average payoff values for cooperators and defectors, we let $x_{\rm c}$ and $x_{\rm d}$ ($x_{\rm c}+x_{\rm d} = 1$) respectively represent the frequencies of cooperators (c) and defectors (d). We obtain

$$P_{\rm c} = crx_{\rm c} - c + c{x_{\rm d}}^N - \mu + \mu{x_{\rm d}}^N, \tag{3}$$

$$P_{\rm d} = crx_{\rm c} - cr{x_{\rm c}}^N + N\nu{x_{\rm d}}^N - N\nu x_{\rm d}. \tag{4}$$

In this model, the evolution of the probability x_i ($i = c, d$) is governed by the approximate best response dynamics[15]

$$\dot{x_i} = \frac{c_i(\boldsymbol{X})}{\sum_j c_j(\boldsymbol{X})} - x_i \tag{5}$$

with

$$c_i(\boldsymbol{X}) = \left[P_i - \max_j P_j + K\right]_+, \quad (i, j = c, d), \tag{6}$$

where $\mu_+ = \max(\mu, 0)$ denotes the positive part. Only strategies which lose at most K compared to the maximal payoff are used in this approximate best response. K characterizes the degree of rationality. In the case of $K \to 0$, individuals own perfectly rationality, where individuals always adopt the best strategy, and the final equilibrium state is Nash equilibrium (NE). When $K > 0$, it denotes bounded rationality. The equilibrium is not completely in accordance with NE, because there is the possibility to select the worse strategy in such a case. $K \to \infty$ implies that individuals are completely irrational, and their choices of strategies are stochastic processes. Thus, K has obvious effect in our model. Whatever the initial state is, individuals will follow cooperation or defection with equilibrium probabilities dependent on the values of K.

Clearly, $\dot{x_i} = 0$ means the system reaches the equilibrium state. Substituting this result into Eq. (5), we obtain

$$x_i = \frac{c_i(\boldsymbol{X})}{\sum_j c_j(\boldsymbol{X})}. \tag{7}$$

In the case of equilibrium state, both strategies have the fixed frequencies, and do not depend on the composition of the sampled group.

3. Numerical results

For the quantitative analysis numerical simulations are performed to illustrate the varying frequencies of cooperators and defectors when punishment mechanism is introduced into PGG. Figure 1 shows that the mechanism can maintain the evolution of cooperation to avoid the dilemma. In the case of $K \to 0$, i.e. all the individuals are perfectly rational, the equilibrium state is NE. In PGG with punishment , NE is that enough high frequency can be obtained by cooperators, whereby cooperators are extinct in classical PGG. [Figs. 1(a), 1(b) and 1(c)]. It indicates that punishment mechanism plays a vital role in the evolution of cooperation. When $K > 0$, i.e. in the case of bounded rationality, the equilibrium states of PGG with and without punishment possess a common characteristic: cooperators' frequencies are increased compared with NE, which also means the dilemma is significantly inhibited in classical PGG [Figs. 1(d), 1(e) and 1(f)]. Thus, the increase of the degree of irrationality will directly facilitate cooperation in PGG. In addition, we notice that if the value of K is fixed, PGG with initial strategies of various frequencies will reach the same equilibrium state. When varying the value of K, the final results will change correspondingly. Naturally, the equilibrium state is related to the value of K.

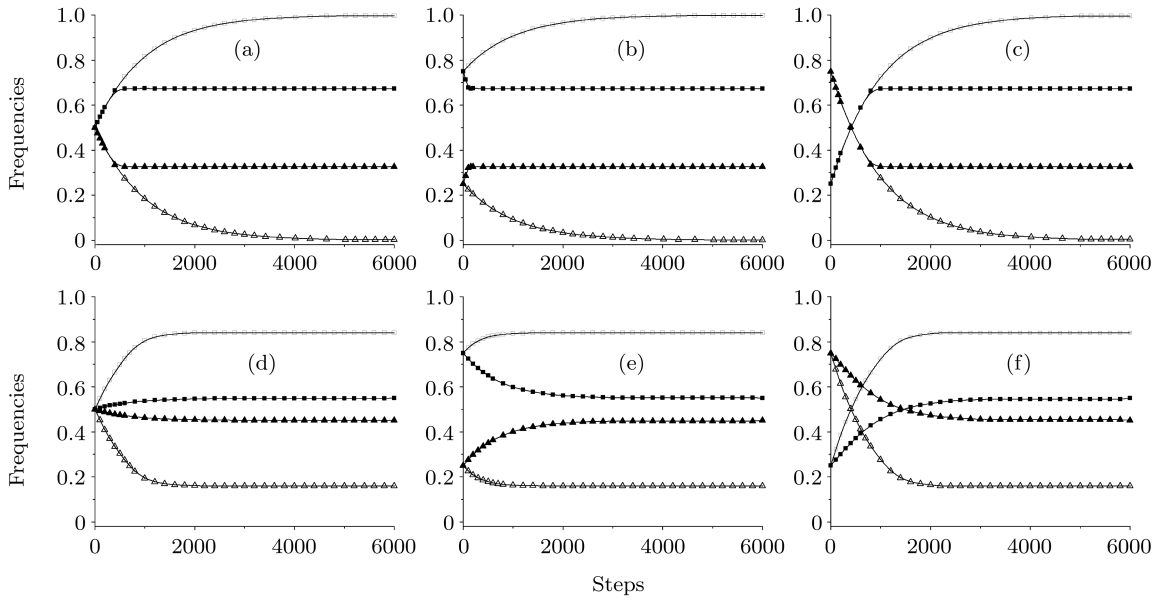


Fig. 1. Imitation for different values of the degree of rationality K in public good games. The values of K are 10^{-4} and 1.0 for the top and bottom rows respectively. Symbols indicate results for 6000 continuous steps. The closed (open) triangles and squares denote cooperators and defectors in public goods games with (without) punishment respectively. (Parameters: $N = 6$, $r = 3.6$, $c = 1.25$, $\nu = 0.35$ and $\mu = 0.1$.)

Further information about how the equilibrium state is affected by the parameter K is provided by Fig. 2. For $K \to 0$, the yielded result is NE. In PGG with punishment, cooperation is effectively promoted. NEs of the system are converted from global state of defection that is reasonable in classical PGG to the emergence of more than 30% cooperators, which severely restrains the dilemma. With the value of K gradually increasing, the frequency of cooperators is also increasing irrespective of PGG with or without punishment. However, the trend of increasing is more intense in PGG without punishment than that with it. It indicates that the evolution of cooperation is very sensitive to the change of K in classical PGG, while cooperation facilitation is the joint effect of punishment mechanism and diversely distributed rationality in our model.

According to Fig. 2, it is safe to predict that the

frequency of cooperators becomes higher and defectors' frequency becomes lower as K getting greater, for cooperation is highly promoted under the condition of bounded rationality. When $K \to \infty$, i.e., all the individuals are completely irrational, the existence frequency of defectors will be identical with that of cooperators, because the selection of strategies will become a stochastic procedure in such a situation, so that each strategy possesses the same probability to be chosen.

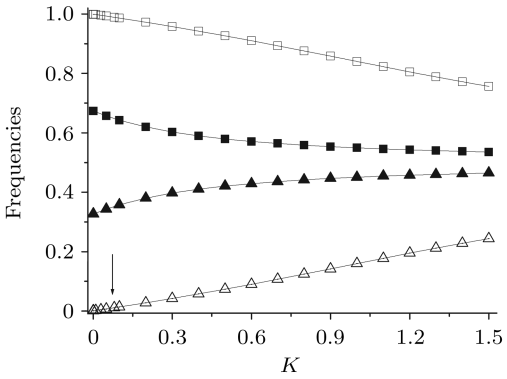


Fig. 2. Average frequencies of strategies in the equilibrium as a function of the degree of rationality K. The closed (open) triangles and squares correspond to cooperators and defectors in PGG with (without) punishment. The black arrow indicates the point of cooperators bursting. (Parameters: $N = 6$, $r = 3.6$, $c = 1.25$, $\nu = 0.35$ and $\mu = 0.1$.)

Now we focus our attention on the average payoffs for both strategies. In fact, according to Eq. (6), it is easy to point out that strategies' payoffs always possess the same orders as their frequencies. In the case of $K \to 0$, for the classical PGG lacking investment, naturally individuals are not able to gain any profit. While cooperators and defectors obtain suitable payoffs in PGG with punishment. Thus, the result is evident that punishment mechanism raises payoffs for strategies and sustains cooperation. Shown as Figs. 1 and 2, cooperators' frequency increases and defectors' drops when the value of K becomes larger. However, cooperators' frequency is still lower than that of defectors. Based on the above discussion, we confirm that both strategies are able to acquire payoffs which are in agreement with the orders of frequencies as K increases, and defectors can obtain more profit than cooperators for higher value of K (see Fig. 3). Simultaneously, it is also obvious that corresponding strategies' payoffs increase to some extent compared with that of small K, even if cooperators' payoff will decrease partly in classical PGG.

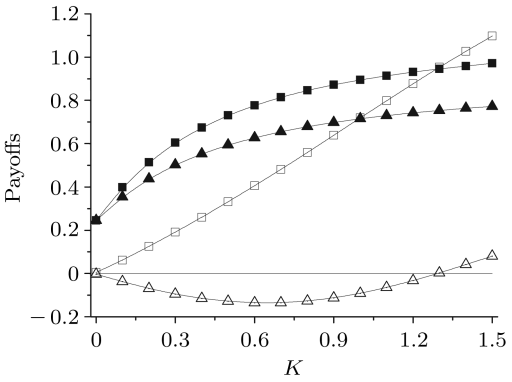


Fig. 3. Average payoffs of strategies as a function of the degree of rationality K. The closed (open) triangles and squares represent cooperators and defectors in PGG with (without) punishment. (Parameters: $N = 6$, $r = 3.6$, $c = 1.25$, $\nu = 0.35$ and $\mu = 0.1$.)

Finally, it is explicit that with increasing K, the intervals of frequencies between cooperators and defectors are diminishing, but the intervals of their payoffs are enlarging, especially in classical PGG. Because the fines against defectors become more and more insignificant, which means that punishment mechanism and the degree of rationality affect their payoffs jointly.

4. Conclusion

In this work, within the framework of PGGs we have analytically and quantitatively studied how punishment mechanism affects the games. This kind of mechanism has also been recognized to be more natural in the real society system. Our numerical investigations have clearly shown that PGGs with punishment possess the ability of refraining the dilemma and stimulating cooperation. Meanwhile, another significantly introduced conception, the degree of rationality, plays an important role in the improvement of cooperation as well. Our results imply that a better distribution of cooperation in PGGs is induced by punishment mechanism or by increasing the degree of rationality K. The average payoffs of both strategies have also verified that the survival of cooperation is the result of the parameter K and punishment mechanism.

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