Leader-following formation control of multi-agent networks based on distributed observers*

Luo Xiao-Yuan(罗小元)†, Han Na-Ni(韩娜妮), and Guan Xin-Ping(关新平)

Institute of Electrical Engineering, Yanshan University, Qinhuangdao 066004, *China*

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To investigate the leader-following formation control, in this paper we present the design problem of control protocols and distributed observers under which the agents can achieve and maintain the desired formation from any initial states, while the velocity converges to that of the virtual leader whose velocity cannot be measured by agents in real time. The two cases of switching topologies without communication delay and fixed topology with time-varying communication delay are both considered for multi-agent networks. By using the Lyapunov stability theory, the issue of stability is analysed for multi-agent systems with switching topologies. Then, by considering the time-varying communication delay, the sufficient condition is proposed for the multi-agent systems with fixed topology. Finally, two numerical examples are given to illustrate the effectiveness of the proposed leader-following formation control protocols.

Keywords: formation control, distributed observer, multi-agent system, graph theory

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1. Introduction

In recent years, great attention has been paid to the multi-agent network from different views, such as consensus, collective behaviours of flocks and swarms, formation control for multi-robot systems, algebraic connectivity of complex network, and so on, among which the leader-following formation control is an especially interesting issue.

There are many researches about formation control.[1−12] With focusing on consensus and cooperation issues, Rodrigues *et al.*[1] discussed the distributed formation control of a multi-agent system based on graph theory. Its aim is to achieve and maintain a formation from any initial condition. Dimarogonas and Kyriakopoulos[2] proposed a feedback control strategy which makes a multi-agent system converge to a desired formation configuration in the cases of agents with single integrator and nonholonomic unicycle-type kinematics. Anderson *et al.*[3] investigated the formation of three points moving in the plan, where each agent is required to maintain a desired distance from its neighbours, and each agent is allowed to use the local knowledge of its neighbours. Olfati-Saber and Murray[4] presented a graph theoretical framework for the formal definition of formation, the issues that appeared in graph realizations and unicity, and their connections to stability of formations. Consolini *et al.*[5] investigated the leader-following formation control strategy that is applied to the nonholonomic mobile robots. Chen *et al.*[6] considered the formation control of multi-agent systems described as the first-order and second-order model. In Ref. [7], Yang *et al.* considered the formation control and obstacle avoidance for a group of nonholonomic mobile robots. Liu *et al.*[8] studied the problem of modeling and controlling leader-following formation of mobile robots based on a second-order kinematics model. Wen and Sorensen[9] investigated the limitations of current leader-following formation control strategies and a consensus-based approach with fully available group trajectory information. Also, a unified, distributed formation control architecture that accommodates an arbitrary number of group leaders and arbitrary information flow among vehicles was proposed. Gu and Hu[10] considered a formation control problem of mobile robots, and they were also concerned with the problems of the implementation of distributed formation control and analysis of the effect of network communication on the closed-loop systems. Simultaneous tracking and formation control were investigated for a team of autonomous agents that evolve dynamically in a space containing a measurable vector field.[11]

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†Corresponding author. E-mail: xyluo@ysu.edu.cn

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However, the above researches mainly focused on the formation control and little attention has been paid to the leader-following formation control for a multi-agent network when the leader's velocity cannot be measured in real time. In Ref. [1], the authors pointed out that the formation's velocity will converge to the average network velocity. In Ref. [13], the leader-following consensus problem with a switching interconnection topology is considered. Distributed observers are designed on the assumption that the velocity of the leader cannot be measured in real time.[14] Instead, in this paper, we design distributed observers for formation control rather than consensus problems and the communication delay is also considered in the cases of switching topologies and fixed topology.

In this paper, based on the graph theory and control theory, the distributed observers for a multi-agent system described as second-order equations are designed under the assumption that the virtual leader's velocity cannot be measured in real time. By using the Lyapunov stability theory, the stability of a multi-agent system is analysed. The cases of switching topology without communication delay and fixed topology with time-varying communication delay are both investigated.

This paper is organized as follows. Some background of graph theory is presented in Section 2. Section 3 describes the problem formulation. The results are presented in Section 4; and simulation results are given in Section 5 to demonstrate the obtained results. The conclusion drawn from the present study is presented in Section 6.

2. Graph theory

Some background of algebraic graph theory and matrix theory will be introduced in this section.

An agent in the formation can be regarded as a node, and the interconnection topology can conveniently be described as a graph. The following are some basic concepts about graph theory (the details can be found in Refs. [15] and [16]).

Let $G(V,\ \varepsilon,\ A)$ be a weighted graph of order n, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of n nodes with indexes belonging to a finite index set $I = \{1, 2, \ldots, n\}$, $\varepsilon \subseteq V \times V$ is the set of edges, in which each edge $e \in \varepsilon$ is a pair of vertices (i, j) such as $i \neq j$. If $(i, j) \in \varepsilon$, then we say that i and j are adjacent, or j is a neighbour of i. We define that the adjacency matrix $A = [a_{ij}]$ of a graph G is the integer matrix with rows and columns indexed by the vertices of G, such that $(i, j) \in E$ is equal to $a_{ij} = 1$, else $a_{ij} = 0$.

If a graph G with $\forall (i, j) \in E \Rightarrow (j, i) \in E$, then we say that graph G is an undirected graph. Every formation can be described by an underlying undirected graph. The vertices of the graph represent the agents in the formation and the edges represent the communications between agents. The edges of undirected graph denote that the communications between agents are mutual. In this paper, we consider the network topologies under the undirected graph.

The Laplacian of the graph is denoted by $L = D - A$, where $D = [d_{ij}]$ is a diagonal matrix with $d_{ii} = \sum_{j=1}^{n} a_{ij}$. The neighbours' set of node i is labeled by $N_i = \{v_j \in V : (v_i, v_j) \in \varepsilon\}$.

3. Problem formulation

Consider a multi-agent system with n agents and a virtual leader, a simple and undirected graph $G = (V, \varepsilon, A)$ can be used to describe the network of these n followers and the virtual leader, in which one node represents an agent. The dynamics of the n followers is described by the following second-order equations

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad i = 1, 2, \ldots, n, \qquad (1)$$

where $x_i(t)$ and $v_i(t)$ are position and velocity variables of the node i (or the agent i), respectively, and $u_i(t)$ is the control input.

The virtual leader can be described as

$$\dot{x}_0(t) = v_0, \qquad (2)$$

where x_0 represents the position and v_0 is the constant but unknown velocity of the virtual leader.

In this paper, we aim at designing control protocols and distributed observers to make the followers converge to virtual leader's velocity and form the desired formation when the velocity of the virtual leader cannot be measured in real time.

4. Main results

The distributed observers, which are used to estimate the virtual leader's velocity followed by all of the n followers, will be designed for the formation control on the assumption that the virtual leader's velocity cannot be measured in switching topologies without communication delay and fixed topology with time-varying communication delay. The Lyapunov stability theory is adopted to demonstrate the convergence

of the designed protocols and distributed observers in the following theorems.

4.1. Observer and control protocol design for switching topologies without communication delay

In practice, the multi-agent networks may be changing just because of the variation of the relationship between agents. Suppose that there is an infinite sequence of bounded, nonoverlapping, continuous time-intervals $[t_j, t_{j+1}), j = 0, 1, \ldots$, which starts at $t_0 = 0$. And we define a piecewise-constant switching signal $\sigma_N : [0, \infty) \to W = \{1, 2, \ldots, N\}$ that represents successive switching times. $N_i(t)$ denotes the set of labels of those agents that are neighbours of agent i at time t.

Since the leader's velocity cannot be measured in real time, the followers need to estimate it throughout the formation process. In this part, we will design n distributed observers for all followers.

Denote $\hat{v}_i$ as the estimate velocity of v_0 by agent $i, i = 1, 2, \ldots, n$. For formation control, the following observers and protocols are designed

$$\dot{\hat{v}}_i = -k_3 \sum_{j \in N_i(t)} (x_i - x_j - r_{ji}), \tag{3}$$

$$u_i = -k_1 \sum_{j \in N_i(t)} (x_i - x_j - r_{ji}) - k_2(v_i - \hat{v}_i). \tag{4}$$

Because of the existence of the distributed observer, every follower can obtain the estimate velocity of the virtual leader. The first terms of the protocol (4) can make the agents form the desired formation based on the estimate velocity; $r_{ij} = -r_{ji}$ is the relative distance between agent i and agent j. The unknown constants k_1, k_2, and k_3 to be determined can make the protocols and observers more adaptive by changing them.

By using Eqs. (3) and (4), the dynamics of system (1) can be summarized as follows:

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= -k_1(L_\sigma x(t) - \mathrm{diag}(A_\sigma R)) - k_2(v - \hat{v}), \\ \dot{\hat{v}} &= -k_3(L_\sigma x(t) - \mathrm{diag}(A_\sigma R)), \end{aligned} \tag{5}$$

where, $x = (x_1,\ x_2,\ \ldots,\ x_n)^{\mathrm{T}} \in R^n$, $v = (v_1,\ v_2,\ \ldots,\ v_n)^{\mathrm{T}} \in R^n$, $\hat{v} = (\hat{v}_1,\ \hat{v}_2 \ldots,\ \hat{v}_n)^{\mathrm{T}} \in R^n$ are followers' position, velocity and estimate velocity of the leader, respectively. The switching signal, $\sigma : [0, \infty) \to W$, is a piecewise constant. A_σ is the adjacent matrix, and its element $a_{ij} = 1$ if $e_{ij} \in \varepsilon$, else, $a_{ij} = 0$. The Laplacians matrix L_σ is defined as $L_\sigma = D_\sigma - A_\sigma$, where D_σ is a diagonal matrix with $d_{ii} = \sum_{j=1}^{n} a_{ij}$. $R = [r_{ij}]_{n \times n}$ is the matrix that is used to present the prescribed distance information, and $\mathrm{diag}(A_\sigma R)$ represents the distance vector whose elements are the diagonal ones of the matrix $A_\sigma R$. Except for R, all the matrixes vary when the topology changes.

Considering the followers (1) and leader (2), we have the following result.

Theorem 1 For the dynamics of system (5), if there exists a symmetric positive matrix P satisfying

$$\begin{pmatrix} -P & 0 \\ 0 & E_\sigma^{\mathrm{T}} P + P E_\sigma \end{pmatrix} < 0, \tag{6}$$

where

$$E_\sigma = \begin{pmatrix} 0_n & L_\sigma & 0_n \\ -k_1 I_n & -k_2 I_n & k_2 I_n \\ -k_3 I_n & 0_n & 0_n \end{pmatrix},$$

then

$$\lim_{t \to \infty} |x_i - x_j - r_{ji}| = 0, \quad \lim_{t \to \infty} |v_i - v_0| = 0;$$

namely, the agents can achieve the desired formation and converge to the virtual leader's velocity.

Proof Let

$$\begin{aligned} \bar{x} &= L_\sigma x(t) - \mathrm{diag}(A_\sigma R), \\ \bar{v} &= v - v_0 \otimes 1_n, \\ \bar{\hat{v}} &= \hat{v} - v_0 \otimes 1_n, \end{aligned} \tag{7}$$

where the symbol $\otimes$ represents the Kronecker product, and $1_n = (1, 1, \ldots, 1)^{\mathrm{T}} \in R^n$.

Denote $\varepsilon(t) = (\ \bar{x}^{\mathrm{T}}(t)\ \ \bar{v}^{\mathrm{T}}(t)\ \ \bar{\hat{v}}^{\mathrm{T}}(t)\ )^{\mathrm{T}}$, then we can obtain the error dynamic system

$$\dot{\varepsilon}(t) = \begin{pmatrix} 0_n & L_\sigma & 0_n \\ -k_1 I_n & -k_2 I_n & k_2 I_n \\ -k_3 I_n & 0_n & 0_n \end{pmatrix} \varepsilon(t) = E_\sigma \varepsilon(t). \tag{8}$$

Define a Lyapunov function

$$V = \varepsilon^{\mathrm{T}}(t) P \varepsilon(t).$$

The time derivation of V along the solution of system (8) is

$$\dot{V} = \varepsilon^{\mathrm{T}}(E_\sigma^{\mathrm{T}} P + P E_\sigma)\varepsilon.$$

Therefore, if there exist a symmetric positive matrix P and positive constants k_1, k_2, and k_3 that make $E_\sigma^{\mathrm{T}} P + P E_\sigma$ be negative, then the error system (8) is asymptotically stable. The proof is completed.

Remark 1 It should be pointed out that all the possible E_σ share a common Lyapunov function (CLF) $V = \varepsilon^{\mathrm{T}}(t)P\varepsilon(t)$ and the matrix P satisfies formula (6) in each time interval. The CLF plays an important role in the research of switched systems. The research of switched system is of great significance, because the network topology is hard to maintain one topology in practice.

4.2. Observer and control protocol design for fixed topology with time-varying communication delay

In practical multi-agent network, there are many cases that cause the communication delay, just like the communication delay between sensors. The time delay influences the performance, even at the stability of the multi-agent system. Therefore, it is quite necessary and practical to consider the communication delay in multi-agent networks. However, there are few researches about the formation for multi-agent systems with such a delay. In this part, the time-varying communication delay will be dealt with for multi-agent systems with fixed topology. We will give a sufficient condition which can guarantee error system's stability.

We design the following control protocols and observers for agents in fixed topology.

$$\dot{\hat{v}}_i = -k_3 \sum_{j\in N_i} (x_i(t-\tau(t)) - x_j(t-\tau(t)) - r_{ji}), \quad (9)$$

$$u_i = -k_1 \sum_{j\in N_i} (x_i(t-\tau(t)) - x_j(t-\tau(t)) - r_{ji}) - k_2(v_i - \hat{v}_i), \quad (10)$$

where $\tau(t)$ is the time-varying communication delay and satisfies $0 < \tau(t) < \hat{\tau}$.

The structures of observer (9) and control protocol (10) have the same form as Eqs. (3) and (4). The difference lies in that the time-varying communication delay is considered in fixed topology.

Applying Eqs. (8) and (9) to system (1), we can obtain

$$\begin{aligned} &\dot{x} = v,\\ &\dot{v} = -k_1(Lx(t-\tau(t)) - \mathrm{diag}(AR)) - k_2(v - \hat{v}),\\ &\dot{\hat{v}} = -k_3(Lx(t-\tau(t)) - \mathrm{diag}(AR)). \end{aligned} \quad (11)$$

Let

$$\begin{aligned} \bar{x} &= Lx - \mathrm{diag}(AR),\\ \bar{v} &= v - v_0 \otimes 1_n,\\ \bar{\hat{v}} &= \hat{v} - v_0 \otimes 1_n. \end{aligned}$$

Denote $\delta(t) = (\bar{x}^{\mathrm{T}}(t) \quad \bar{v}^{\mathrm{T}}(t) \quad \bar{\hat{v}}^{\mathrm{T}}(t))^{\mathrm{T}}$, then we have the dynamics of the error system

$$\dot{\delta}(t) = E\delta(t) + F\delta(t-\tau(t)), \quad (12)$$

where,

$$E = \begin{pmatrix} 0_n & L & 0_n \\ 0_n & -k_2 I_n & k_2 I_n \\ 0_n & 0_n & 0_n \end{pmatrix}, \quad F = \begin{pmatrix} 0_n & 0_n & 0_n \\ -k_1 I_n & 0_n & 0_n \\ -k_3 I_n & 0_n & 0_n \end{pmatrix}.$$

Before Theorem 2 is given, we introduce a lemma which will be useful in the following part.

Consider the following system

$$\begin{aligned} &\dot{x} = f(x_t), \quad t > 0,\\ &x(\theta) = \varphi(\theta), \quad \theta \in [-\hat{\tau}, 0], \end{aligned} \quad (13)$$

where $x_t(\theta) = x(t+\theta), \forall\theta \in [-\hat{\tau}, 0]$, and $f(0) = 0$. Let $C([-\hat{\tau}, 0], R^n)$ be a Banach space of continuous functions defined in an interval $[-\hat{\tau}, 0]$, taking values in R^n with the topology of uniform convergence, and with a norm $\|\varphi\|_c = \max_{\theta\in[-\hat{\tau},0]} \|\varphi(\theta)\|$. The following result is for the stability of system (13) (the details can be found in Ref. [11]).

Lemma 1 (Lyapunov–Razumikhin theorem[17]): Let ϕ_1, ϕ_2, and ϕ_3 be continuous, nonnegative, nondecreasing functions with $\phi_1(s) > 0$, $\phi_2(s) > 0$, $\phi_3(s) > 0$ for $s > 0$ and $\phi_1(0) = \phi_2(0) = 0$. For system (13), suppose that the function $f : C([-\hat{\tau}, 0], R^n) \to R$ takes bounded sets of $C([-\hat{\tau}, 0], R^n)$ in bounded sets of R^n. If there is a continuous function $V(t, x)$ such that

$$\phi_1(\|x\|) \le V(t, x) \le \phi_2(\|x\|), \quad t \in R, x \in R^n.$$

In addition, there exists a continuous nondecreasing function $\phi(s)$ with $\phi(s) > s$, $s > 0$ such that $\dot{V}(t, x) \le -\phi_3(\|x\|)$ if $V(t+\theta, x(t+\theta)) < \phi(V(t, x(t)))$, $\theta \in [-\hat{\tau}, 0]$, then the solution $x = 0$ is uniformly asymptotically stable.

Now, the Theorem 2 is presented below:

Theorem 2 For system (12), if there exists a symmetric positive matrix $\tilde{P}$ satisfying

$$\begin{pmatrix} M^{\mathrm{T}}\tilde{P} + \tilde{P}M + \hat{\tau}q\tilde{P} & \tilde{P}FE \\ * & \dfrac{-\tilde{P}}{\hat{\tau}} \end{pmatrix} < 0, \quad (14)$$

where $M = E + F$, $\hat{\tau}$ is the upper bound of communication delay, and $q > 1$ is a known constant. Then, the error system is uniformly asymptotically stable with the observers (9) and control protocols (10), and the

agents can form the desired formation and converge to the virtual leader's velocity.

Proof Define the following Lyapunov–Razumikhin function

$$V(t) = \delta^{\mathrm{T}}(t)\tilde{P}\delta(t).$$

By Leibniz–Newton formula

$$\begin{aligned}\delta(t-\tau) &= \delta(t) - \int_{-\tau}^{0} \dot{\delta}(t+s)\mathrm{d}s \\ &= \delta(t) - E\int_{-\tau}^{0} \delta(t+s)\mathrm{d}s \\ &\quad - F\int_{-2\tau}^{-\tau} \delta(t+s)\mathrm{d}s,\end{aligned}$$

we obtain

$$\begin{aligned}\dot{\delta}(t) &= E\delta(t) + F\left(\delta(t) - E\int_{-\tau}^{0} \delta(t+s)\mathrm{d}s \right. \\ &\quad \left. - F\int_{-2\tau}^{-\tau} \delta(t+s)\,\mathrm{ds}\right).\end{aligned}$$

Thus, from $F^2 = 0$, the above equation can be rewritten as

$$\begin{aligned}\dot{\delta}(t) &= (E+F)\delta(t) - FE\int_{-\tau}^{0} \delta(t+s)\mathrm{d}s \\ &= M\delta(t) - FE\int_{-\tau}^{0} \delta(t+s)\mathrm{d}s.\end{aligned}$$

And for any x, $y \in R^n$ and any symmetric positive-definite matrix $\bar{R} \in R^{n\times n}$, we have

$$\pm 2x^{\mathrm{T}}y \le x^{\mathrm{T}}\bar{R}^{-1}x + y^{\mathrm{T}}\bar{R}y.$$

Then, the time derivation of $V(t)$ along the solution of the system (12) is

$$\begin{aligned}\dot{V}(t) &= 2\delta^{\mathrm{T}}(t)\tilde{P}M\delta(t) - 2\delta^{\mathrm{T}}(t)\tilde{P}FE\int_{-\tau}^{0} \delta(t+s)\mathrm{d}s \\ &\le 2\delta^{\mathrm{T}}(t)\tilde{P}M\delta(t) + \tau\delta^{\mathrm{T}}(t)\tilde{P}FE\tilde{P}^{-1}E^{\mathrm{T}}F^{\mathrm{T}}\tilde{P}\delta(t) \\ &\quad + \int_{-\tau}^{0} \delta^{\mathrm{T}}(t+s)\tilde{P}\delta(t+s)\mathrm{d}s.\end{aligned}$$

Take $\phi(s) = qs$, $q > 1$. In the case of $V(\delta(t+s)) < qV(\delta(t))$, $-\hat{\tau} \le s \le 0$, we have

$$\begin{aligned}\dot{V} &\le \delta^{\mathrm{T}}(t)(M^{\mathrm{T}}\tilde{P} + \tilde{P}M)\delta(t) \\ &\quad + \tau\delta^{\mathrm{T}}(t)(\tilde{P}FE\tilde{P}^{-1}E^{\mathrm{T}}F^{\mathrm{T}}\tilde{P} + q\tilde{P})\delta(t) \\ &\le \delta^{\mathrm{T}}(t)(M^{\mathrm{T}}\tilde{P} + \tilde{P}M \\ &\quad + \hat{\tau}\tilde{P}FE\tilde{P}^{-1}E^{\mathrm{T}}F^{\mathrm{T}}\tilde{P} + \hat{\tau}q\tilde{P})\delta(t) \\ &= \delta(t)^{\mathrm{T}}H\delta(t).\end{aligned}$$

If $H < 0$, according to the Lyapunov theory, the error system (12) is asymptotically stable. Using Schur's complement theorem,[18] we have

$$H < 0 \Leftrightarrow \begin{pmatrix} M^{\mathrm{T}}\tilde{P} + \tilde{P}M + \hat{\tau}q\tilde{P} & \tilde{P}FE \\ * & \dfrac{-\tilde{P}}{\hat{\tau}} \end{pmatrix} < 0.$$

Therefore, the conclusion can be drawn by using Lemma 1. The proof is completed.

Remark 2 The Lyapunov–Razumikhin theorem is adopted to illustrate the sufficient condition for a multi-agent system. By solving the linear matrix inequality (LMI) (14), the maximal allowable upper bound of the time delay can be obtained.

Remark 3 It is necessary to note that if the topology is not desired, then there will not exist appropriate parameters satisfying the sufficient conditions presented in Theorem 1 and Theorem 2.

5. Simulation results

In this section, we will give the simulation studies to illustrate the effectiveness of the approach obtained in the previous section. In the following two subsections, we present the simulation results to illustrate that the followers can form the desired formation and converge to the virtual leader's velocity in two cases.

5.1. Simulation for switching topology without time delay

For simplicity, we consider the multi-agent network with three followers and a virtual leader, and the switching topologies are shown in the following graphs in Fig. 1.

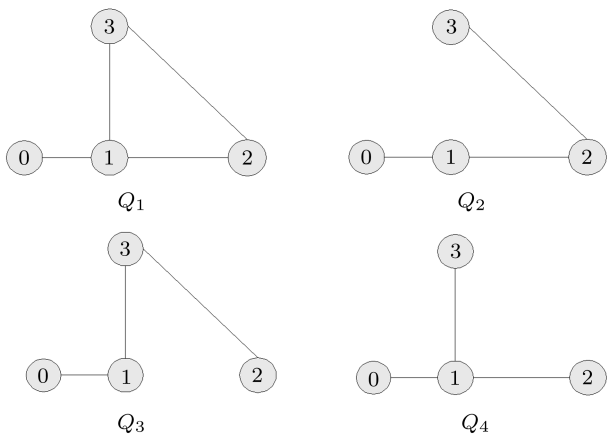


Fig. 1. The four possible topologies.

Q_1, Q_2, Q_3 and Q_4 are four possible topologies of three followers and one virtual leader which are connected, and agent 1 is always connected to the virtual leader. The agent labeled "0" is the virtual leader and its velocity is assumed to be constant but unknown. The other three agents are followers, and the distances between each other are represented by the above graphs. In the one-dimensional simulation, we assume that the distance between agent 1 and agent 2, or that between agent 2 and agent 3 is 1. For clarity, we assume that the distance between agent 1 and agent 2, or that between agent 1 and agent 3 is 2 in the two-dimensional simulation. Moreover, the three followers form a right-angled triangle.

Figure 2 shows a finite state machine with four states, which depicts the states of a multi-agent network with switching topologies; and it starts at Q_1 and switches to the next topology every 5 seconds.

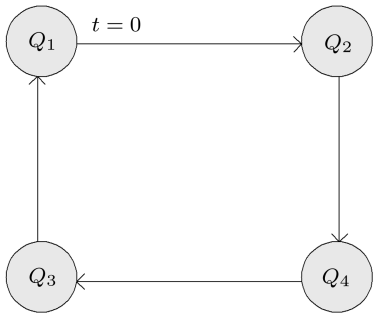


Fig. 2. A finite machine with four states denoting the states of a network with switching topology.

Figure 3 shows the trajectories of $\bar{x}$, $\bar{v}$ and $\bar{\dot{v}}$ of the error system (8). The nine states asymptotically reach the origin separately and show that the error system is stable. Moreover, figure 3 also indicates that the agents can form the desired formation and the followers' velocity can converge to the virtual leader's velocity.

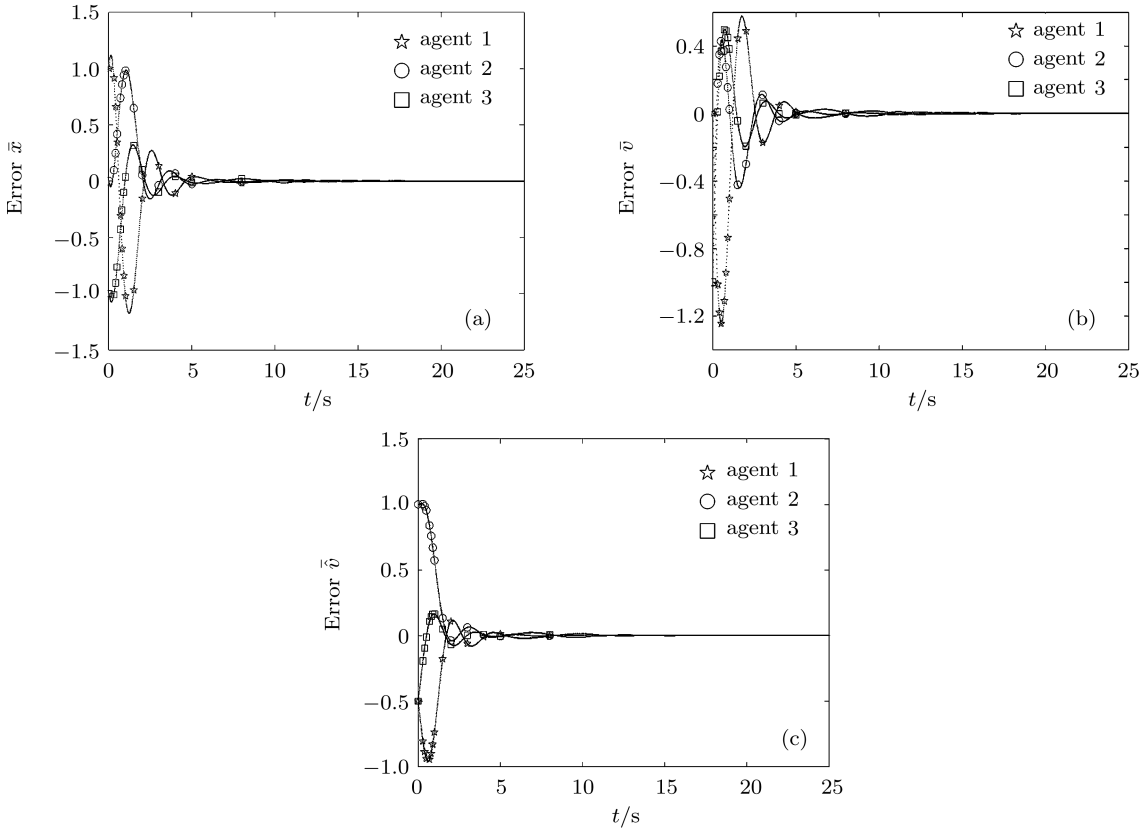


Fig. 3. The state error trajectories of the network with switching topology.

Figure 4 shows the one-dimensional trajectories of states x_1, x_2, and x_3, the three followers form the desired formation (a line) and they keep the nominated distance between each other. Figure 5 shows the state trajectories in the two-dimensional simulation. The three followers finally achieve a right-angled triangle. These two figures show the effectiveness of the designed control protocol and distributed observers.

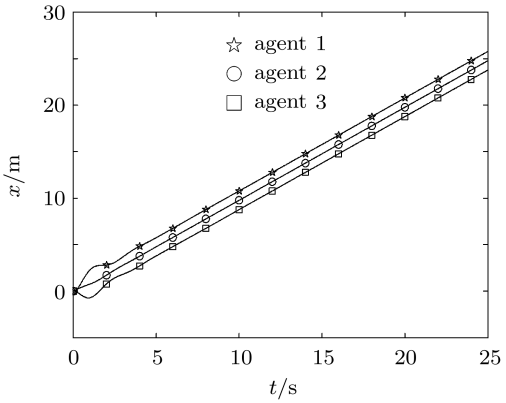


Fig. 4. The one-dimensional state trajectories.

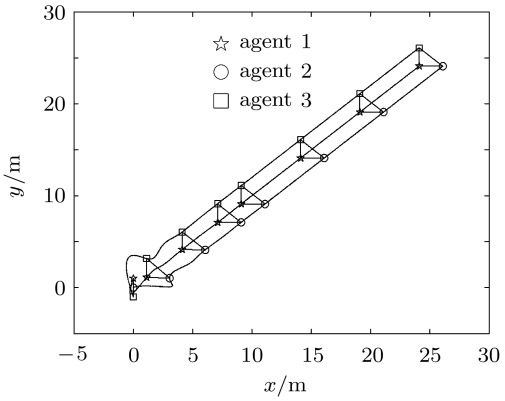


Fig. 5. The two-dimensional state trajectories in switching topology.

States v_1, v_2, and v_3 shown in Fig. 6 finally reach the same value and figure 7 shows that the states $\hat{v}_1$, $\hat{v}_2$, $\hat{v}_3$ converge to a constant velocity. These two figures are obtained from the one-dimensional simulation and the results are similar to two-dimensional simulation. It means that the agents and observers can converge to the same velocity.

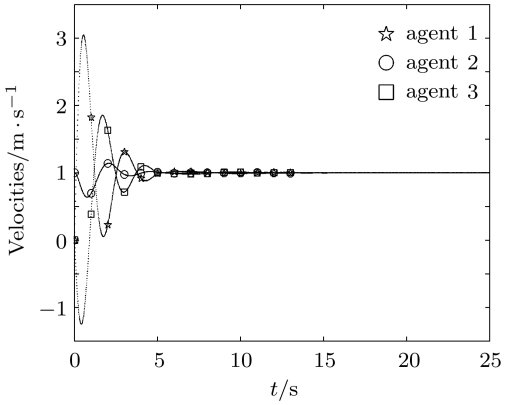


Fig. 6. The one-dimensional state trajectories v_1, v_2, and v_3.

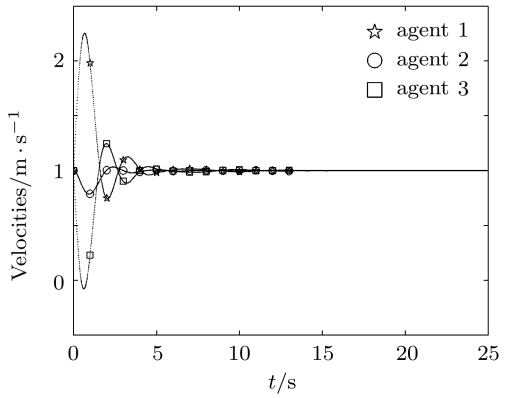


Fig. 7. The one-dimensional state trajectories $\hat{v}_1$, $\hat{v}_2$, and $\hat{v}_3$.

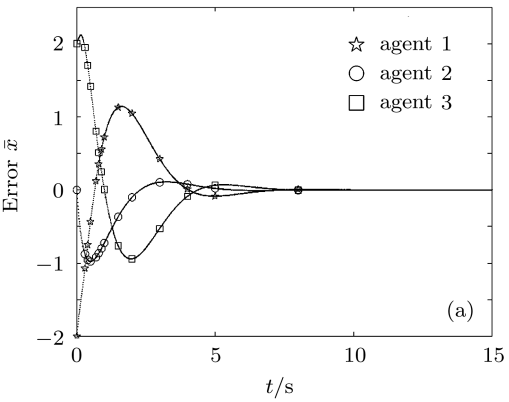
5.2. Simulation for fixed topology with varying time delay

In this subsection, we are only concerned with the one-dimensional simulation. The fixed topology of the four agents is shown by Q_1 in Fig. 1.

We solve the LMI (14), and the upper bound of varying communication of τ can be obtained, which is $\hat{\tau} = 0.1205$, while $k_1 = 3$, $k_2 = 7$, $k_3 = 0.3$. The following simulation results are obtained on the assumption that the varying communication delay $\tau = 0.12|\sin(t)|$.

Figure 8 shows the trajectories of $\bar{x}$, $\bar{v}$ and $\bar{\hat{v}}$ of the error system (12). It is seen that the error system (12) is asymptotically stable. And it also indicates that the followers can form the desired formation and converge to the virtual leader's velocity, which is demonstrated by the simulation results in Figs. 9 and 10.

Figure 9 shows the one-dimensional trajectories of x_1, x_2, and x_3 with varying communication delay. It can be seen that the agents finally form the desired formation and keep nominated distance, which indicates that the designed control protocols and distributed observers are effective.



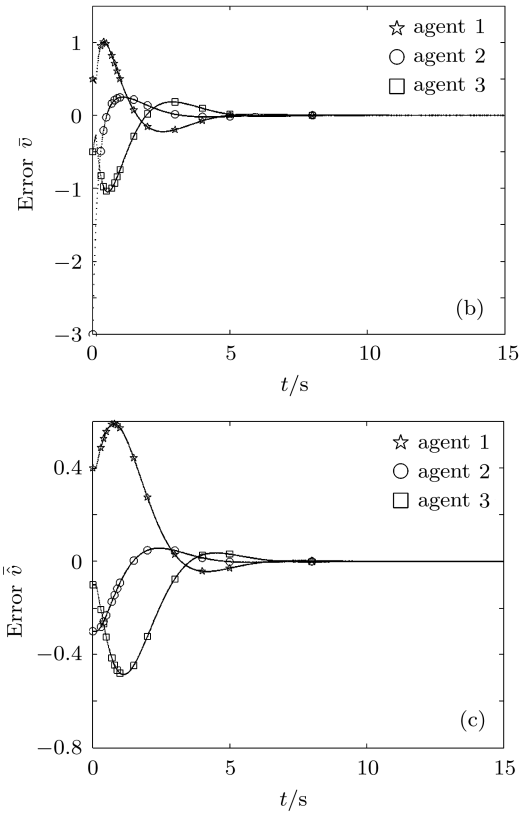


Fig. 8. The state error trajectories of the network with varying time delay.

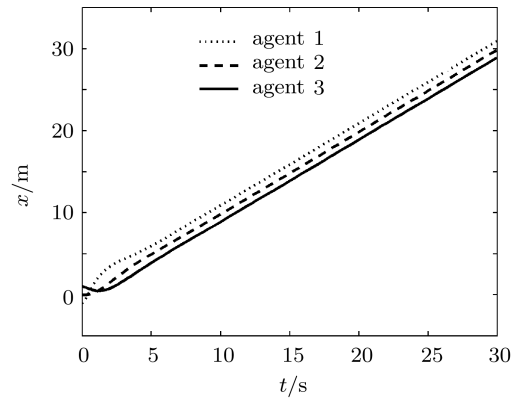


Fig. 9. The state trajectories x_1, x_2, and x_3 with varying time delay.

Figure 10 shows the trajectories of v_1, v_2, and v_3 with varying communication delay and figure 11 shows the trajectories of $\hat{v}_1$, $\hat{v}_2$, $\hat{v}_3$. The followers' velocities in Fig. 10 asymptotically reach the same value as those in Fig. 11, and they illustrate the effectiveness of the observers. The velocities of the agents reach consensus quickly.

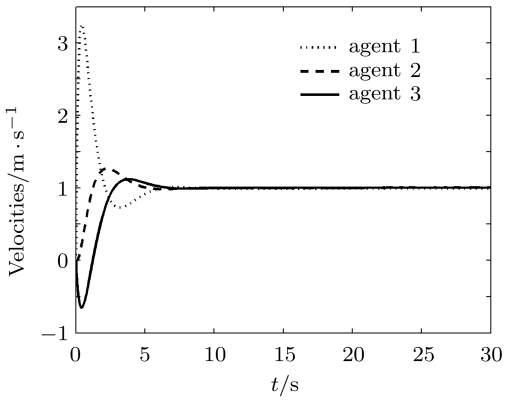


Fig. 10. The state trajectories v_1, v_2, and v_3 with varying time delay.

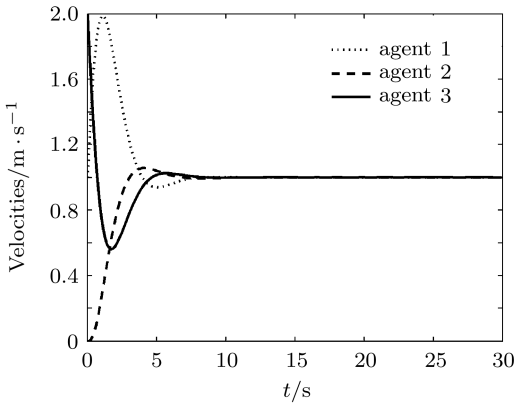


Fig. 11. The state trajectories $\hat{v}_1$, $\hat{v}_2$, and $\hat{v}_3$ with varying time delay.

6. Conclusion

This paper investigated the formation control of multi-agent networks when the virtual leader's velocity is unknown. The cases of switching topology without time delay and fixed topology with varying time delay have been studied. We have illustrated that the agents with the designed controllers and observers can achieve and maintain the desired formation, as well as converge to the virtual leader's velocity. And the numerical simulations indicate their effectiveness. Our future work will involve the formation control when there exits disturbances.

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