Reconstructing dark energy potentials from parameterized deceleration parameters*

Wang Yu-Ting(王钰婷), Xu Li-Xin(徐立昕)†,
Lü Jian-Bo(吕剑波), and Gui Yuan-Xing(桂元星)

School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, *China*

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In this paper, the properties of dark energy are investigated according to the parameterized deceleration parameter $q(z)$, which is used to describe the extent of the accelerating expansion of the universe. The potential of dark energy $V(\phi)$ and the cosmological parameters, such as the dimensionless energy density Ω_ϕ, Ω_m, and the state parameter w_ϕ, are connected to it. Concretely, by giving two kinds of parameterized deceleration parameters $q(z) = a + bz/(1+z)$ and $q(z) = 1/2 + (az+b)/(1+z)^2$, the evolution of these parameters and the reconstructed potentials $V(\phi)$ are plotted and analysed. It is found that the potentials run away with the evolution of universe.

Keywords: dark energy(DE), deceleration parameter, reconstructed potential

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1. Introduction

Since 1998, type Ia supernova (SNe Ia) observations[1] show that the expansion of our universe is speeding up rather than slowing down. In recent years, many additional observational results, including current Cosmic Microwave Background (CMB) anisotropy measurements from the Wilkinson Microwave Anisotropy Probe,[2] and the data of the Large Scale Structure (LSS),[3] also strongly support this suggestion and have received considerable recognition. In order to understand the mechanism of the accelerating expansion of the universe, the authors have done plenty of work, which can be divided into two categories: on the one hand, we can carry on the revision to the geometry items on the left-hand side of the Einstein field equation ($R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 4\pi G T_{\mu\nu}$), and the famous work is $f(R)$ theory.[4] On the other hand, we can modify the material item on the right-hand side of this equation. Therefore, many reseachers have been making an effort to look for the matter that can lead to this evolutive history of the universe. Cosmic observations show that an exotic energy component called 'dark energy' (DE), with negative pressure, exists in the universe and its density accounts for two-thirds of the total energy density. In addition, there have been some studies which are accomplished to reconcile the two methods.[5] In the case of the latter, numerous models about DE are proposed in order to clarify the essence of DE. The simplest one is the cosmological constant Λ, with the equation of state (EOS) $w = P/\rho = -1$. However, it confronts us with two difficulties: the fine-tuning problem and the cosmic coincidence problem. An alternative offer is the dynamical DE to overcome the difficulties above. In this class of model, the energy form is provided by an evolving, scalar field ϕ, with a self-interaction potential $V(\phi)$, such as quintessence,[6] phantom,[7] quintom[8] and so forth. Furthermore, there are the generalized Chaplygin gas (GCG) model,[9] Holographic DE,[10] high dimension theories,[11] etc. Considering more and more DE models, obviously it is very meaningful to explore the properties of DE in a model independent manner.[12] Usually, we are able to parameterize the state parameter $w(z)$,[13] the energy density $\rho(z)$,[14] or the deceleration parameter $q(z)$[15] by using cosmological observations.

In view of the dynamical DE models, we can reconstruct the potential $V(\phi)$ from supernova observations.[16] The disadvantage of this kind of method is that we need to give a concrete form of a potential, which is usually selected to describe the desired properties of DE as well as possible, or an expanding form of a potential. Then, there have been some authors reconstructing the potential from the

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†Corresponding author. E-mail: lxxu@dlut.edu.cn

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parameterized parameters without assuming its form, such as the state parameter $w(z)$,[17] and the energy density $\rho(z)$,[18] both of which are used to describe the properties of DE. When we give parameterized parameters $w(z)$ and $\rho(z)$, to some extent we have limited some properties of DE. On the basis of the previous research, in turn we can think of reconstructing the potential by a parameter which can directly reflect the transition from the decelerating expansion to the accelerating expansion along with the evolving universe. Based on this consideration, in this paper we will reconstruct the potential of DE according to the parameterized parameter $q(z)$, being positive in the past and negative at present with the history of the universe, which accords with our idea.

Performing a comparison to the previous means, ours has its own advantages. Firstly, according to the expressions of $H(z) = H_0 \exp \int_0^z (1 + q(u)) \mathrm{d} \ln(1 + u)$ and $H^2(z) = H_0^2[\Omega_{\mathrm{m}0}(1 + z)^3 + (1 - \Omega_{\mathrm{m}0})(1 + z)^{3[1+w(z)]}]$, we can see that the constructed Hubble parameter $H(z)$ is independent of cosmological quantity $\Omega_{\mathrm{m}0}$ from the deceleration parameter $q(z)$. Also, it is well known that the Hubble parameter $H(z)$ is an observational quantity, then the cosmological quantities can be affected by $\Omega_{\mathrm{m}0}$ when we constrain them by combining the observational data.[19] But this problem disappears in the case of the deceleration parameter $q(z)$. Secondly, it is known that the state parameter $w(z)$ and the energy density $\rho(z)$ are made use of in describing the properties of DE. In other words, giving a parameterized state parameter $w(z)$ or a parameterized energy density $\rho(z)$ designates that we have assumed some properties of DE. For our method, we reconstruct the potential $V(\phi)$ by the parameterized deceleration parameter $q(z)$ without hypothesizing any evolutive mode of DE and probe DE completely grounding in the whole cosmological evolvement.

2. Construction of the potential and the evolution of cosmological parameters

Considering a spatially flat Friedmann–Robertson–Walker (FRW) model,[20] which consists of two components: the non-relativistic matter and the dynamical DE, we utilize a spatially homogeneous scalar field ϕ to describe DE.

With the metric $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)]$, the Einstein field equation can be written as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\mathrm{pl}}^2}(\rho_{\mathrm{m}} + \rho_\phi), \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\mathrm{pl}}^2}(\rho_{\mathrm{m}} + \rho_\phi + 3P_\phi), \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ_{m} is the matter density and $M_{\mathrm{pl}} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. The energy density ρ_ϕ and the pressure P_ϕ of the evolving scalar field ϕ are respectively given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{3}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{4}$$

where ϕ dot is the derivative of ϕ with respect to the time t and $V(\phi)$ is the potential of the evolving scalar field ϕ. From the equations above, we can easily obtain

$$V(\phi) = \frac{1}{2}(\rho_\phi - P_\phi), \tag{5}$$

$$\dot{\phi}^2 = \rho_\phi + P_\phi. \tag{6}$$

Obviously Eq. (1) can yield $\rho_\phi = 3M_{\mathrm{pl}}^2 H^2 - \rho_{\mathrm{m}}$, while Eqs. (1) and (2) can be combined to result in $P_\phi = (q - 1/2)2M_{\mathrm{pl}}^2 H^2$, where $q \equiv -\ddot{a}/(aH^2)$ is the deceleration parameter. Using $\rho_{\mathrm{m}} = \rho_{\mathrm{m}0}(1+z)^3$, where z is the redshift, and the relation between the scale factor and the redshift z, $a(t) = 1/(1 + z)$ with $a(t_0) = 1$, Eqs. (5) and (6) can be rewritten as

$$V(\phi(z)) = \rho_{\mathrm{m}0}\left[\frac{(2-q)H^2}{3\Omega_{\mathrm{m}0}H_0^2} - \frac{1}{2}(1+z)^3\right], \tag{7}$$

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}z}\right)^2 = M_{\mathrm{pl}}^2\left[\frac{2(1+q)}{(1+z)^2} - \frac{3\Omega_{\mathrm{m}0}H_0^2}{H^2}(1+z)\right], \tag{8}$$

where $\Omega_{\mathrm{m}0} \equiv \rho_{\mathrm{m}0}/3M_{\mathrm{pl}}^2 H_0^2$ is the dimensionless energy density, H_0 is the Hubble constant and the subscript 0 denotes the present value of a quantity at the redshift $z = 0$.

In order to realize the reconstruction $V(\phi)$ from the parameterized deceleration parameter $q(z)$, next we must deal with $H(z)$. The derivative of the Hubble parameter $H(z)$ with regard to the time t is $\dot{H} = -(1 + q)H^2$. Then there exists a contact between $H(z)$ and $q(z)$ by virtue of an integration:

$$\begin{aligned} H(z) &= H_0 \exp \int_0^z (1 + q(u)) \mathrm{d} \ln(1 + u) \\ &= H_0 f(z), \end{aligned} \tag{9}$$

where $f(z) \equiv \exp\int_0^z (1+q(u))\mathrm{d}\ln(1+u)$. Hence, we are capable of writing down the reconstructed equations for $V(z)$ and $(\mathrm{d}\phi/\mathrm{d}z)^2$ in term of $q(z)$:

$$V(\phi(z)) = \rho_{\mathrm{m}0}\left[\frac{(2-q)}{3\Omega_{\mathrm{m}0}}f^2(z) - \frac{1}{2}(1+z)^3\right], \quad (10)$$

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}z}\right)^2 = M_{\mathrm{pl}}^2\left[\frac{2(1+q)}{(1+z)^2} - 3\Omega_{\mathrm{m}0}(1+z)f^{-2}(z)\right]. \quad (11)$$

We define the dimensionless quantities $\tilde{V} \equiv V/\rho_{\mathrm{m}0}$ and $\tilde{\phi} \equiv \phi/M_{\mathrm{pl}}$. The reconstructed equations (10) and (11) can then be ulteriorly written as

$$\tilde{V}(\tilde{\phi}(z)) = \frac{(2-q)}{3\Omega_{\mathrm{m}0}}f^2(z) - \frac{1}{2}(1+z)^3, \quad (12)$$

$$\frac{\mathrm{d}\tilde{\phi}}{\mathrm{d}z} = -\sqrt{\frac{2(1+q)}{(1+z)^2} - 3\Omega_{\mathrm{m}0}(1+z)f^{-2}(z)}. \quad (13)$$

These are the main results in our paper, which reveal the relation between the potential of the scalar field $V(\phi)$ and the deceleration parameter $q(z)$. If we provide an effective parameterized deceleration parameter $q(z)$, we can obtain the reconstruction of the potential $V(\phi)$ by Eqs. (12) and (13). Here, we have completed the extraction and chosen the expression with the minus, namely $\dot{\phi} < 0$, which in fact has no effect on the result. If $\dot{\phi} > 0$, we only need to change the definition of ϕ to $-\phi$.

In addition, we present the state parameter w_ϕ, the dimensionless energy density Ω_ϕ, Ω_{m} on $q(z)$:

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{2q-1}{3[1-\Omega_{\mathrm{m}0}(1+z)^3 f^{-2}(z)]}, \quad (14)$$

$$\Omega_{\mathrm{m}} = \Omega_{\mathrm{m}0}(1+z)^3 f^{-2}(z), \quad (15)$$

$$\Omega_\phi = 1 - \Omega_{\mathrm{m}}. \quad (16)$$

The concrete expressions of these parameters can be readily attained after we provide an effective parameterized deceleration parameter $q(z)$. Concerning the parameters in the parameterized deceleration parameter $q(z)$, we are able to constrict them through the latest observations.[21,22]

In this paper, we consider two kinds of parameterized parameters as follows:

Parametrization 1 $q(z) = a + \dfrac{bz}{(1+z)^2}$

Just in order to write convenient formulae, we define $g(z) \equiv (1+z)^{-2(1+a+b)}\exp[2bz/(1+z)] \equiv f^{-2}(z)$. So we have for Parametrization 1:

$$\tilde{V}(z) = \frac{2-a-\dfrac{bz}{(1+z)^2}}{3\Omega_{\mathrm{m}0}}g^{-1}(z) - \frac{1}{2}(1+z)^3, \quad (17)$$

$$\frac{\mathrm{d}\tilde{\phi}}{\mathrm{d}z} = -\frac{\sqrt{2\left[1+a+\dfrac{bz}{(1+z)^2}\right] - 3\Omega_{\mathrm{m}0}(1+z)^3 g(z)}}{1+z}, \quad (18)$$

$$w_\phi = \frac{2\left[a+\dfrac{bz}{(1+z)^2}\right]-1}{3[1-\Omega_{\mathrm{m}0}(1+z)^3 g(z)]}, \quad (19)$$

$$\Omega_{\mathrm{m}} = \Omega_{\mathrm{m}0}(1+z)^3 g(z), \quad (20)$$

$$\Omega_\phi = 1 - \Omega_{\mathrm{m}}. \quad (21)$$

Parametrization 2 $q(z) = \dfrac{1}{2} + \dfrac{az+b}{(1+z)^2}$

It is the same as the case above and we define $h(z) \equiv \exp\left[-b - \frac{az^2-b}{(1+z)^2}\right]$ for convenience, then obtain

$$\tilde{V}(z) = \left[\frac{\dfrac{3}{2} - \dfrac{az+b}{(1+z)^2}}{3\Omega_{\mathrm{m}0}h(z)} - \frac{1}{2}\right](1+z)^3, \quad (22)$$

$$\frac{\mathrm{d}\tilde{\phi}}{\mathrm{d}z} = -\frac{\sqrt{2\left[\dfrac{3}{2}+\dfrac{az+b}{(1+z)^2}\right] - 3\Omega_{\mathrm{m}0}h(z)}}{1+z}, \quad (23)$$

$$w_\phi = \frac{2\left[\dfrac{1}{2}+\dfrac{az+b}{(1+z)^2}\right]-1}{3[1-\Omega_{\mathrm{m}0}h(z)]}, \quad (24)$$

$$\Omega_{\mathrm{m}} = \Omega_{\mathrm{m}0}h(z), \quad (25)$$

$$\Omega_\phi = 1 - \Omega_{\mathrm{m}}. \quad (26)$$

We plot the potentials $V(\phi)$ in Fig. 1 and the evolution of the cosmological parameters in Figs. 2 and 3. We obtain the values of the parameters by referring to Ref. [21] and taking $\Omega_{\mathrm{m}0} = 0.3$ as a priority.

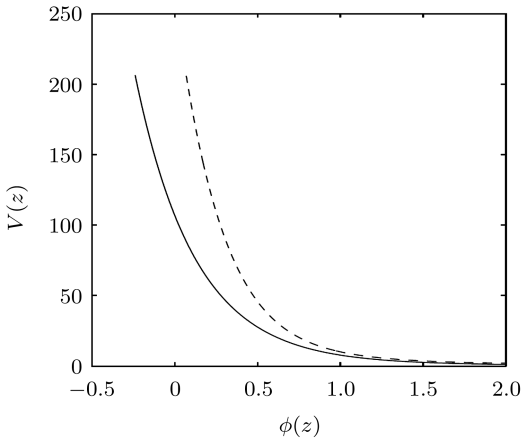


Fig. 1. The evolution of the reconstructed potentials with the scalar field ϕ.

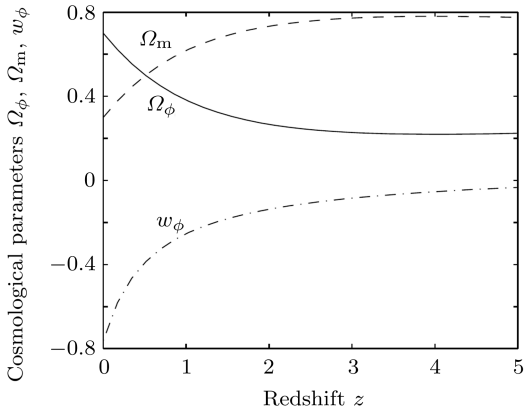


Fig. 2. The evolution of the cosmological parameters for Parametrization 1 with the redshift z, where $a = -0.3$ and $b = 0.8$ are used according to the observation constraint.

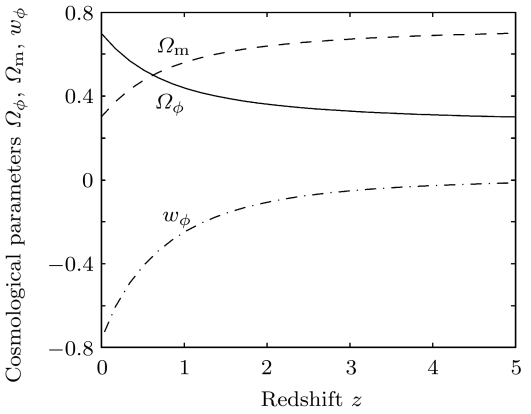


Fig. 3. The evolution of the cosmological parameters for Parametrization 2 with the redshift z, where $a = 0.1$ and $b = -0.8$ are used according to the observation constraint.

In theory, we can obtain the function $\phi(z)$ by integrating the differential equation (13) for a concrete $q(z)$. Then the expression of z with respect to ϕ can be obtained. After substituting $z(\phi)$ into equation (12), we are able to obtain the analytical formula of the reconstructed potential. However, as we have seen above, the differential equations are too intricate to be integrated analytically in our considered cases. Therefore, at last only can we numerically evaluate them with the help of the initial condition $\phi_0 = 2$. In fact, the value of ϕ_0 cannot change the shape or trend of the potential $V(\phi)$. It just leads to shift the graphics horizontally. From Fig. 1, it is demonstrated that the potentials decrease with the evolving scalar field ϕ and have similar behaviours for the two kinds of examples at the late period. By our numerical evaluation, we discover that the evolving scalar field ϕ linearly runs away with the increasing of redshift z. Figure 1 presents the change rates of the reconstructed potential $V(\phi)$, which are different between high redshift and low redshift, the former being faster than the latter.

Next, we are going to consider a simple form $q =$ constant, which is tenable within a small extension. Inserting it into Eqs. (12) and (13), we can hold on by considering discretely at low redshift and high one. For the low redshift, we can obtain an approximate analytic exponential expression:

$$V(\phi) = \frac{2 - q_c}{3\Omega_{\rm m0}} \exp\left[\frac{-(2q_c - 1)(\phi - \phi_0)}{\sqrt{2(1 + q_c) - 3\Omega_{\rm m0}}}\right]. \quad (27)$$

At high redshift, the result is different from that at low redshift. The formula is in square law:

$$V(\phi) = \frac{(2 - q_c)\left(q_c - \frac{1}{2}\right)^2}{4(1 + q_c)^4}(\phi - \phi_0)^2. \quad (28)$$

It is found that this expression is independent of the value of the dimensionless matter density $\Omega_{\rm m0}$.

3. Conclusions

In summary, the dark energy potentials can be reconstructed from parameterized deceleration parameters. For the two examples given above, we obtain the evolution of the reconstructed potential $V(\phi)$, the dimensionless energy density Ω_ϕ, Ω_m and the state parameter w_ϕ for a given parameterized deceleration parameter $q(z)$. The result shows that the reconstructed potentials $V(\phi)$ decrease with the evolving scalar field ϕ, namely that the reconstructed potentials $V(\phi)$ have the direction of going down with the expansion of the universe. Also it is found that the change rates of the reconstructed potential $V(\phi)$ are obviously diverse, which at $\phi < 1$ is quicker than that at $\phi > 1$. For the two cases, the variational trends of the reconstructed potentials $V(\phi)$ are very similar at the late period. In addition, analytically evaluating Eqs. (12) and (13) with $q =$ constant, we obtain two different approximate expressions and find the formula (28) does not contain the dimensionless matter density $\Omega_{\rm m0}$. As far as the theoretic view is concerned, we establish a relation between the deceleration parameter and the effective field theory in our means. With the progress of the observation techniques in the future, the parameters in $q(z)$ can be constrained more precisely, which can go a long way towards understanding the nature of dark energy for us.

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