Calculation of time-varying equivalent inductance and resistance of helical flux compression generators using the 2D filamentary method and dynamic matrix concept in the frequency domain

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In this paper we present an approach to calculate the time-varying equivalent inductance and resistance of helical flux compression generators accurately. This approach is based on the 2D filamentary method and uses the dynamic matrix concept. By using this method, it is possible to consider both the effects of the armature presence and frequency on the inductance and resistance of the HFCGs. The latter effect has not been discussed in previous open literature. The validation of the presented method has been verified by measurement results.

Keywords: helical flux compression generators, liner, seed current
PACC: 8630, 8760D, 5275F, 1340H

1. Introduction

Magnetic flux compression generators offer the largest pulsed power output per unit size or weight when compared with other more conventional systems.[1] These generators are capable of producing voltages of tens of kV and currents of tens to hundreds of MA.

In a flux compression generator (FCG), a chemical explosive is used to compress an initial magnetic flux by driving part of a conducting surface which contains the flux. Work done by the conductors moving against the magnetic field results in an increase in the electromagnetic energy.[2]

Various types of flux compression generators have been developed and tested during the past five decades. The most successful of them is the helical flux compression generator (HFCG) which is capable of producing a high energy output into large impedance loads, just as is needed for a practical pulsed power source.[1]

Two different approaches are applied to deal with the optimum design and to investigate the physical phenomenon and processes of HFCGs. One of them is the model of the equivalent circuit and the other is the magnetohydrodynamic (MHD) model. The model of the equivalent circuit is widely used for designing explosive flux compression generators because of its simple methodology, reliable calculation results, and saving time for optimum design.[3] In the latter approach the differential equation of the equivalent circuit which is time varying should be solved. The most important necessity for solving this equation is calculation of the time-varying inductance and resistance of the HFCG. In this paper we present an approach to calculate these parameters accurately. This approach is based on the 2D filamentary method in the frequency domain and uses the dynamic matrix concept. In spite of the fact that the HFCG is a pulsed device, it is usual that its parameters (inductance and resistance) are calculated or measured in the sinusoidal steady state at an averaged frequency.

2. Operating principle of FHCGs

As shown in Fig. 1, a helical generator has a long metal armature (liner) that is filled with high explosive and placed within a solenoid. The seed (priming) current is produced by a capacitor bank. As the capacitor discharges into the solenoid, the current generates a magnetic field in the space between the solenoid and armature. The explosive is detonated at one end, and the armature expands, resulting in

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a conical metal front moving with detonation velocity ($\sim$ 8 km/s). The detonation is so timed that this conical front shorts out the generator input at or near peak current or, equivalently, peak flux in the generator. The volume between the solenoid and armature decreases in both the radial and longitudinal directions. Flux conservation results in an enhanced magnetic field, which induces large current in the remaining loops of the solenoid and load.[4]

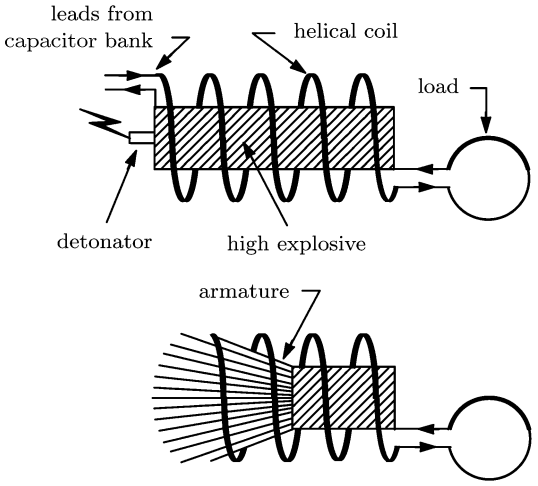


Fig. 1. The end-initiated explosively driven helical generator (above) prior to detonation and (below) during detonation.

3. Two-dimensional filamentary model

Equations describing a helical flux compression generator can be formulated from considerations of the superimposed B_θ and B_z magnetic fields produced, respectively, by the z and θ-directed armature currents.[5,6] As shown in Fig. 2 the HFCG can be decomposed into equivalent z and θ-current circuits.[7] The z-current circuit consists of the helical coil and the armature, with the load used to complete the circuit. The coil can be further divided into N rings, through which the same load current I_z flows azimuthally to generate the B_{zz} field. On the other hand, these rings, together with the armature, form a coaxial structure that generates the B_θ field. In order to model adequately the armature current in the θ-direction, which is induced by the time-dependent helical coil current I_z, the armature is divided into a number of separate, for example n, θ-circuits (rings), with a different current ($I_{\theta,i}$) flowing through each of them.[8]

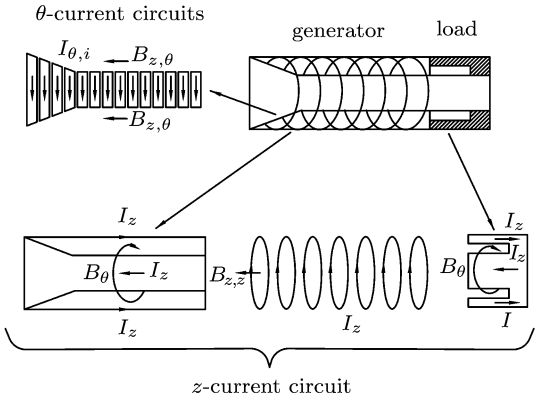


Fig. 2. Decomposition of the HFCG into a single z-circuit and multiple θ-circuits.

4. HFCG differential equation system

Based on the decomposed generator model, n+1 equivalent circuits are needed to describe the generator; they are shown in Fig. 3.

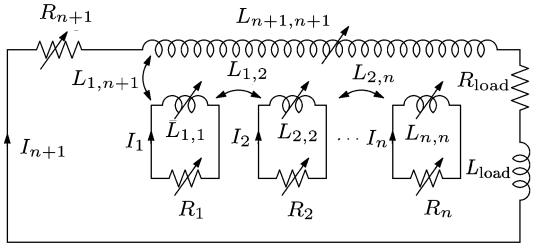


Fig. 3. The equivalent z and θ-circuits for the HFCG.

Using equivalent circuits, the differential equation system of the HFCG can be written as

$$[R(t)][I(t)] + \frac{\mathrm{d}}{\mathrm{d}t}\{[L(t)][I(t)]\} = 0, \qquad (1)$$

where $[L(t)]$ and $[R(t)]$ are the $(n+1)\times(n+1)$ time-varying inductance and resistance matrices of the z and θ-circuits respectively, and $[I(t)]$ is the current vector of the length $n+1$. The matrix of $[R(t)]$ is a diagonal matrix in which its n first elements are the resistance of armature rings and the last element of it is the helical coil resistance. Since the rings of the HFCG coil and armature are coaxial, we have used the following formula for determining the mutual inductance between ith and jth rings[9]

$$L_{ij} = 2\mu_0 \frac{\sqrt{r_i r_j}}{k}\left[\left(1-\frac{k^2}{2}\right)K(k) - E(k)\right], \qquad (2)$$

where r_i = radius of the ith ring, r_j = radius of the jth ring, $k^2 = 4r_i r_j/[(r_i + r_j)^2 + d_{ij}^2]$, d_{ij}=axial distance between the ith and jth rings.

$K(k)$ and $E(k)$ are the first and second types of elliptical integrals.

The self inductance of the ith ring of the coil or armature is given by[8]

$$L_i(r_i, h_i) = \mu_0 r_i \left(\ln \frac{8r_i}{h_i} - 0.5 \right). \tag{3}$$

Here h_i is the diameter of the ith ring conductor of the helical coil or axial dimension of the ith armature ring.

The total inductance in the z-circuit is

$$L_z = L_{zz} + L_{z\theta}. \tag{4}$$

The components L_{zz} and $L_{z\theta}$ are related to the B_{zz} and $B_{z\theta}$ fields, respectively. For the $B_{z\theta}$ field, the inductance $L_{z\theta}$ can be decomposed into two terms

$$L_{z\theta} = L_{\rm cyl} + L_{\rm con}, \tag{5}$$

where

$$L_{\rm cyl} = \frac{\mu_0 l_{\rm cyl}}{2\pi} \ln \frac{r_{\rm c}}{r_{\rm a}}, \tag{6}$$

and

$$L_{\rm con} = \frac{\mu_0}{2\pi} K \tag{7}$$

are the inductances of the corresponding cylindrical and conical parts of the coaxial structure, respectively, and

$$K = l_{\rm con} \ln \frac{r_{\rm c}}{r_{\rm a} + l_{\rm con} \tan\theta} - \frac{r_{\rm a}}{\tan\theta} \ln \frac{r_{\rm a} + l_{\rm con} \tan\theta}{r_{\rm a}}. \tag{8}$$

In Eq. (8) $r_{\rm c}$, $r_{\rm a}$, $l_{\rm cyl}$, $l_{\rm con}$ and θ are the coil and armature radii, the lengths of the cylindrical and conical parts of the armature and expanding angle, respectively.

For the B_{zz} field, the inductance L_{zz} is given by

$$L_{zz} = \sum_{i=1}^{N} \left(L_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} L_{ij} \right). \tag{9}$$

In the above equation, L_{ij} and L_i are taken from Eqs. (2) and (3) respectively. The n first diagonal elements of the $L(t)$ matrix are obtained from Eq. (3) and the nondiagonal elements of it (except the last row and column), are obtained from Eq. (2). The self inductance of the coil ($L_{n+1,n+1}$) is given by

$$L_{n+1,n+1} = L_z. \tag{10}$$

If the mutual inductance between the ith armature ring having radius $r_{{\rm a}i}$ and the jth coil ring having radius $r_{{\rm c}j}$ is taken to be $L_{ij}(r_{{\rm a}i},\ r_{{\rm c}j},\ d_{ij})$ then the mutual inductance between the ith armature ring and the helical coil is obtained

$$L_{n+1,i} = L_{i,n+1} = \sum_{j=1}^{N} L_{ij}(r_{{\rm a}i}, r_{{\rm c}j}, d_{ij}),$$
$$i = 1, 2, 3, \ldots, n. \tag{11}$$

The n first elements of the $R(t)$ matrix are given by

$$R_i(r_i, h_i) = \frac{2\pi\rho r_i}{\delta h_i}, \tag{12}$$

where ρ and δ are the electrical resistivity of the conductor and skin depth respectively. We have used the classical skin depth of current diffusion δ[1]

$$\delta = \sqrt{\frac{\rho}{\mu_0 \pi f}}, \tag{13}$$

in which μ_0 is the permeability of free space. We have considered the skin depth δ at a frequency f = 100 kHz which amounts to 0.2 mm for aluminum.

The resistance of the helical coil (last element of the $R(t)$ matrix) is obtained from

$$R_{n+1} = \sum_{i=1}^{N} R_i(r_i, d), \tag{14}$$

where R_i is the resistance of the ith ring of the helical coil and is given by

$$R_i(r_i, d) = \frac{2\rho r_i K_{\rm prox}}{\delta(d - \delta)}. \tag{15}$$

Here d is the diameter of the conductor and $K_{\rm prox}$ is the proximity effect coefficient given in Ref. [10].

5. Armature dynamics

To solve the system of equations (1), the time-varying inductance and resistance matrices $[L(t)]$ and $[R(t)]$ should be determined. As shown in Eqs. (2)–(15), the elements of these matrices are functions of the radii of the armature rings which are varied through HFCG operation. Most commonly accepted assumptions on the expansion behaviour of the armature are the constant expansion angle and the constant radial expansion velocity, which are theoretically based on research conducted on explosive-driven metals or based on some photos of deformed armature outer surfaces in explosive experiments. According to

the Gurney theory and the Taylor formula, the expansion angle of the armature is given by the following equation[11]

$$\theta_{\rm Gurney} = 2\sin^{-1}\left(\sqrt{2E}\Big/\left(2V_{\rm det}\sqrt{\left(\frac{M}{C}+\frac{1}{2}\right)}\right)\right), \tag{16}$$

where M, C, and $V_{\rm det}$ are the mass of the armature, the mass and velocity of the explosive respectively; and $\sqrt{2E}$ is called 'the Gurney velocity' of the explosive, while E is the specific energy of the explosive. The Gurney velocity can be approximated using the following equation[7]

$$\sqrt{2E} = \frac{V_{\rm det}}{3}. \tag{17}$$

Finite element calculations and experimental results indicate that the Gurney equation does not provide an accurate prediction of the armature expansion angle in the HFCG. Therefore, we have used a modified Gurney equation which is presented by Neuber and coworkers.[7] This modified equation is

$$\theta_{\rm Modified} = \exp(A + Ba)\,\theta_{\rm Gurney}, \tag{18}$$

where a is the ratio of wall thickness to radius of the armature, and the coefficients A and B are determined by the least square method to closely match the numerical results. For an aluminum armature charged with C-4 high explosive, the values of these coefficients are -0.0369 and -0.984 respectively.[7]

We have assumed that the expansion angle and the radial velocity of the armature are constant. Therefore the armature radius can be written as

$$r_{\rm a}(z,t) = r_{\rm a0} + (V_{\rm det}\,t - z)\tan\,\theta, \quad (r_{\rm a0} \le r_{\rm a} \le r_{\rm c}), \tag{19}$$

where $r_{\rm a0}$, z, θ and $r_{\rm c}$ are the initial armature radius, axial position, expansion angle and coil (stator) radius respectively. The armature radius as a function of z and t is shown in Fig. 4. The initial radius and length of this armature are 2 and 34 cm, respectively, and the radius of the stator coil is 4 cm.

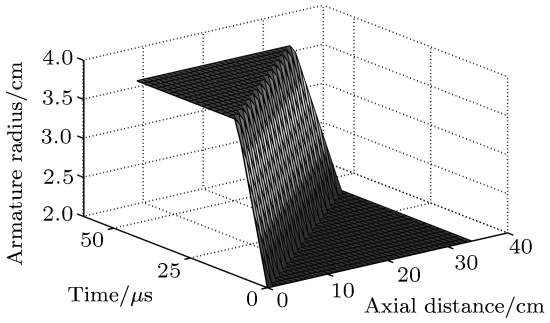


Fig. 4. Armature radius versus time and axial position

6. Derivation of time-varying equivalent inductance and resistance

For derivation of the time-varying equivalent inductance and resistance of a helical flux compression generator, we consider two phases for operation of it. The first phase begins with the fast expansion of the armature within the coil, up to the time that it makes contact with the helical coil. During this phase, due to time variation of armature radius (see Eq. (19)), the matrices $[L(t)]$ and $[R(t)]$ are time-variant, but the dimensions of them are constant. The second phase begins at the armature-helix contact time, up to the end of armature operation. In this phase the matrices $[L(t)]$ and $[R(t)]$ are also time-variant. In addition, due to explosion and removing armature rings from the circuit step by step, the dimensions of these matrices reduce with time. Because of this, we have named them 'the dynamic matrices'. In order to derive equivalent resistance and inductance, we consider the time-variant system (1) as an m time-invariant system at different instance and assume a sinusoidal voltage is applied to the z-circuit of Fig.3 at instance $t = t_i$ and the steady state is achieved.[12] Indeed, the sinusoidal signal is used as a test signal. Thus system (1) in the sinusoidal steady state would be

$$[R(t_i)][I_i] + {\rm j}\omega[L(t_i)][I_i] = [V_i], \tag{20}$$

where $[R(t_i)]$ and $[L(t_i)]$ are the $(n+1)\times(n+1)$ resistance and inductance matrices at instance $t = t_i$, $[I_i]$ is the current phasor vector of length $(n+1)$ and $[V_i]$ is a vector of length $(n+1)$ as

$$[V_i] = [0\ 0\ 0\ 0 \ldots\ V]. \tag{21}$$

The system (20) consists of n+1 equations. It is possible to consider n first equations of it as

$$[R_1(t_i)][I_{i1}] + {\rm j}\omega[L_1(t_i)][I_{i1}] + {\rm j}\omega I_{i\,n+1}[L_2(t_i)] = 0, \tag{22}$$

and the last equation as

$$R_{n+1}(t_i)I_{in+1} + {\rm j}\omega[L_2(t_i)]^{\rm T}[I_{i1}] + {\rm j}\omega L_{n+1,n+1}(t_i)I_{in+1} = V, \tag{23}$$

where $[R_1(t_i)]$ and $[L_1(t_i)]$ are the matrices resulting from elimination of the last row and column of

$[R(t_i)]$ and $[L(t_i)]$ respectively; $[I_{i1}]$ is the same as $[I_i]$ that has eliminated the last element of it; $R_{n+1}(t_i)$, $L_{n+1,n+1}(t_i)$, and I_{in+1} are the last components of $[R(t_i)]$, $[L(t_i)]$, and $[I_i]$ respectively; and $[L_2(t_i)]$ is a vector resulting from eliminating the last element of the last column of $[L(t_i)]$. From Eq. (22) we have:

$$[I_{i1}] = -\mathrm{j}\omega I_{in+1}[[R_1(t_i)] + \mathrm{j}\omega[L_1(t_i)]]^{-1}[L_2(t_i)]. \tag{24}$$

Substitution of Eq. (24) into Eq. (23) yields

$$\left\{ \begin{matrix} \omega^2[L_2(t_i)]^{\mathrm{T}}[[R_1(t_i)] + \mathrm{j}\omega[L_1(t_i)]]^{-1}[L_2(t_i)] \\ +R_{n+1}(t_i) + \mathrm{j}\omega L_{n+1,n+1}(t_i) \end{matrix} \right\} I_{in+1} = V. \tag{25}$$

Now we can write the equivalent impedance of the HFCG at the instance t_i as

$$Z_{\mathrm{eqi}} = \omega^2 [L_2(t_i)]^{\mathrm{T}}[[R_1(t_i)] + \mathrm{j}\omega[L_1(t_i)]]^{-1}[L_2(t_i)] + R_{n+1}(t_i) + \mathrm{j}\omega L_{n+1,n+1}(t_i). \tag{26}$$

Therefore, the equivalent inductance and resistance at the instance t_i are

$$R_{\mathrm{eqi}} = \mathrm{Re}(Z_{\mathrm{eqi}}), \tag{27}$$

$$L_{\mathrm{eqi}} = \mathrm{Im}(Z_{\mathrm{eqi}})/\omega. \tag{28}$$

In phase one of the operation, using Eqs. (26)–(28) the equivalent impedance, resistance and inductance at an arbitrary number of instances are calculated. During phase two, assuming n calculation steps, the dimensions of matrices $[R(t_i)]$ and $[L(t_i)]$ and vectors $[I_i]$ and $[V_i]$ at each step are reduced by one, with respect to the previous step. In addition, the turns of the helical coil are reduced with time. Therefore, under these considerations, the same equations (26)–(28) can be used to calculate equivalent impedance, resistance and inductance.

7. Calculations and comparison with measurement results

In order to calculate the time-varying equivalent inductance and resistance of the HFCG, we have written a MATLAB program that can compute the HFCG inductance and resistance matrices using the filamentary method from Eqs. (2)–(19) and then calculate equivalent impedance, resistance and inductance from Eqs. (26)–(28) at different instances and plot them versus time. Using the written program, the time-varying equivalent resistance and inductance of the HFCG have been calculated at a frequency of $f = 100$ kHz.

To verify our derived equations and our code, a small scale HFCG inductance simulator was made. As shown in Fig. 5, this simulator comprises a helical coil and an aluminum liner which can be moved through the helix. The liner consists of two parts: a cylindrical part and a conical part with Gurney angle. In order to achieve good electrical contact between the liner and helix wire, the inner insulation of the helix wire was machined off. To measure the inductance and resistance, the liner of the simulator was moved step by step through the helix and the inductance and resistance were measured by an RLC meter at a frequency of $f = 100$ kHz. As the speed of the shock wave in the HFCG is approximately constant, the time history of the HFCG parameters can be obtained easily. This technique has previously been used to measure the HFCG inductance collapsing.[13]

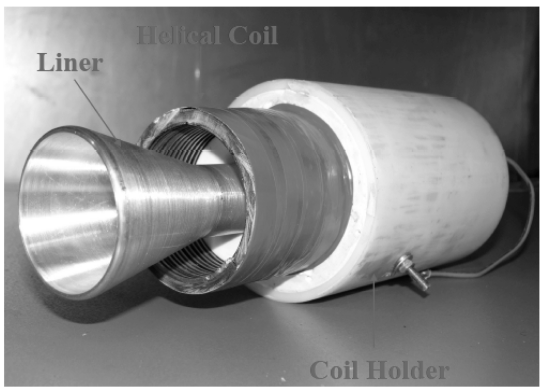


Fig. 5. Photo of the HFCG inductance simulator.

The simulator dimensions and parameters are shown in Table 1. The equivalent and coil (without armature) resistance and inductance of the HFCG are shown in Figs. 6 and 7. These figures indicate that due to the presence of the armature, the equivalent inductance decreases and the equivalent resistance increases. The generator losses can be increased due to the increasing equivalent resistance.

Table 1. Simulator dimensions and parameters.

coil length	118 mm
coil diameter	65 mm
armature diameter	32.5 mm
armature wall thickness	3 mm
No. of coil turns	37
pitch of coil	3.2 mm
conductor diameter	2.24 mm
conical part angle	14°

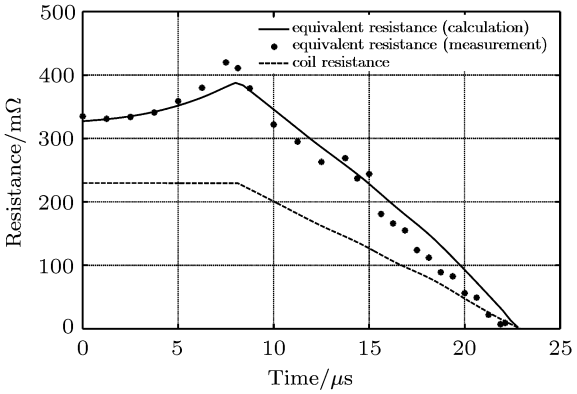


Fig. 6. The equivalent and coil (without armature) resistances versus time.

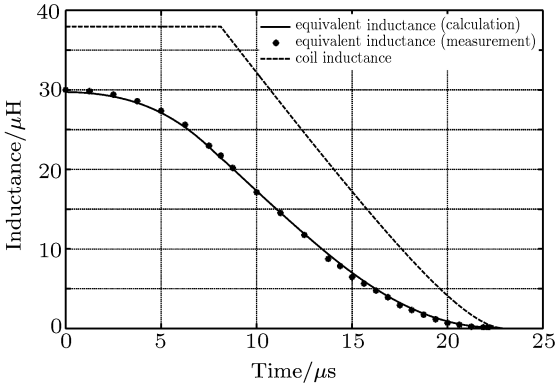


Fig. 7. The equivalent and coil (without armature) inductances versus time.

In addition, the calculated parameters were used to solve the differential equation of an HFCG and calculate the time history of its current. The HFCG current versus time is shown in Fig. 8. The experimental data in Fig. 8 have been taken from Ref. [13].

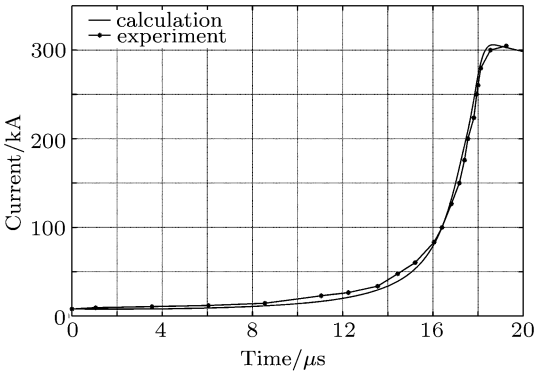


Fig. 8. The HFCG current versus time.

8. Conclusion

We have described an approach to determine the time-varying equivalent resistance and inductance of a helical flux compression generator which is capable of considering the effects of armature presence and frequency on these parameters. The main advantage of the presented approach is its simplicity, which is an important factor for design and optimization purposes.

It should be noted that the HFCG parameters which are calculated by our approach are frequency-dependent. Therefore it is important to consider a proper frequency in calculation of them. On the other hand, an HFCG is a pulsed device and its operating frequency cannot be determined explicitly. It is usual to consider its parameters (inductance and resistance) at an averaged frequency. The average frequency of the device is determined by applying the FFT technique to the approximated current waveform of the generator (for instance an exponential current waveform).

As shown in Figs. 6–8, the calculation results demonstrate a good agreement with measurement and experimental results.

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