Precise relative rotation sensing using slow light*

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(Received 12 May 2008; revised manuscript received 16 June 2009)

A novel relative rotation sensor based on slow light is proposed and analysed. A theoretical analysis shows that the high sensitivity of the proposed rotation sensor is achieved through an electromagnetically-induced-transparency medium. Unlike the tradition detection method, the idea of rotation sensing is to detect group delay between counter-propagating wave packets. It can be used to realize an ultra-precise rotation sensor.

Keywords: Sagnac effect, relative rotation, group delay

PACC: 4262E, 4250

Rotation sensing has been intensively investigated due to its important applications in the fields of navigation, spaceflight, military engineering, geophysics and relative rotation study. In the recent years, with the maturation of the slow-light technique,[1−4] much attention has been paid to the use of the concept of slow light to boost the rotational sensitivity of rotation sensors. Leonhardt and Piwnicki[5] first proposed the idea of a slow light interferometer. Later, Zimmer and Fleischauer[6] presented an alternative hybrid light and matter-wave interferometer, based on slow light momentum transfer from light to atoms. Subsequently, Steinberg[7] and Matsko and Savchenkov[8] studied the effects of photonic crystal defect cavities and whispering gallery mode cavities respectively, in order to utilize the dispersive characteristics of the 'slow light' structure to enhance sensor sensitivity. Scheuer and Yariv[9] presented a coupled-resonator slow-light waveguide structure to realize highly compact integrated rotation sensors and gyroscopes. Recently, Peng *et al.*[10,11] proposed a coupled-resonator-induced transparency structure to construct a miniature highly sensitive gyroscope. Shahriar *et al.*[12,13] envisioned the slow light induced dispersive drag used to enhance sensitivity in relative rotation sensing.

As is well known, the phase difference induced by the Sagnac effect is detected for obtaining the rotating rate of a rotation sensor. In the present paper, based on the concept of slow light, we present a relative rotation sensor device, by which the group delay between counterpropagating wave packets can be directly detected, to obtain the rotating rate, and the explicit expression of the group delay between the counterpropagating wave packets is deduced.

Figure 1 shows a schematic diagram of the relative rotation sensor.

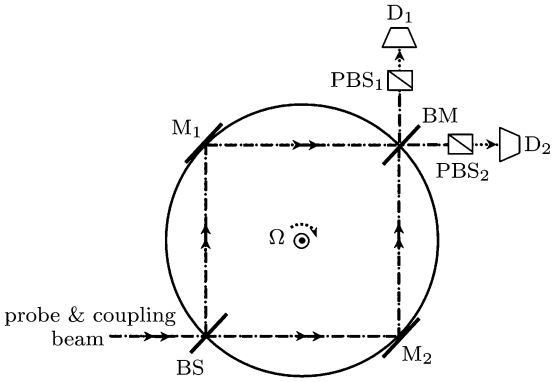


Fig. 1. Schematic diagram of relative rotation sensor.

In the device, the probe beam (dot line) and coupling beam (dash line) are cross-linearly polarized. They pass around the device in a clockwise sense (from beam splitter (BS) to mirror 1 (M_1), and then cast into bilateral mirror (BM)) and in a counterclockwise sense (from BS to mirror 2 (M_2), and then hit BM), respectively. In order to accurately measure the group delay of probe beams, the coupling beams are filtered by the two polarizing beam splitters (PBS). The reserved probe beams are incident on detector 1 (D_1) and detector 2 (D_2) respectively, and then enter

*Project supported by the National Natural Science Foundation of China (Grant Nos. 60272075, 60478014 and 60870006), and the National High Technology Research and Development Program of China (Grant No. 2007AA12Z112).

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into a two-channel digital oscilloscope for comparison. Thus the group delay between the counterpropagating waves can be obtained. The device, which is constrained to propagate in a square path with a length of side L, is filled with electromagnetically-induced-transparency (EIT) medium. $v = L\Omega/\sqrt{2}$ is the tangent velocity of rotation at BS, M and BM. $v_n = L\Omega/2$ is the projection of v along the direction of wave propagation. For the clockwise (+) and counterclockwise (−) directions, the group velocities $v^{\pm}$ in the laboratory frame, the time $t^{\pm}$ for wave packets to travel from BS to BM and the effective distances $L^{\pm}$ from BS to BM are related as follows:

$$L^{\pm} = 2L \pm L\Omega t^{\pm}/2, \quad t^{\pm} = L^{\pm}/v^{\pm}. \tag{1}$$

Thus, the group delay between the counterpropagating wave packets can be obtained to be

$$\Delta t = t^{+} - t^{-} = \frac{2L\left(v^{-} - v^{+} + L\Omega\right)}{\left(v^{+} - L\Omega/2\right)\left(v^{-} + L\Omega/2\right)}. \tag{2}$$

If the device is filled with the nondispersive medium, the group velocities $v^{\pm}$ correspond to phase velocities $v_p^{\pm}$ as follows:

$$v_p^{\pm} = \frac{c/n \pm \Omega L/2}{1 \pm \Omega L/2cn}. \tag{3}$$

Substituting expression (3) into expression (2), we can obtain the group delay in a nondispersive medium:

$$\Delta t_0 = \frac{2L^2\Omega}{c^2}, \tag{4}$$

which is identical to the time delay of a traditional optical Sagnac interferometer.

If the device is filled with a highly dispersive medium, such as the EIT medium, and only the medium was rotated, relative rotation takes place between the medium and the rest of the device. Obviously, the rotating medium creates a local Doppler shift in the frequency and wave vector of the probe light, while the coupling light is not affected by the Doppler effect. In this case, the group velocity in the laboratory frame is[14]

$$v^{\pm} = \pm\sqrt{v_{\mathrm{g}}\left(\omega^{\pm}\right)^2 + 2c\delta v_{\mathrm{g}}\left(\omega^{\pm}\right) + \Omega^2L^2/4}, \tag{5}$$

where $v_{\mathrm{g}}\left(\omega^{+}\right)$ is the clockwise group velocity in the rotating frame, $v_{\mathrm{g}}\left(\omega^{-}\right)$ is the counterclockwise group velocity in the rotating frame, $\delta = (\omega - \omega_0)/\omega_0$ is the normalized detuning, ω_0 is the resonance frequency of the slow-light beam for a medium at rest, and ω is the frequency of the probe light in the laboratory frame. Expression (5) is valid only within a narrow window of two-photon resonance,[15] i.e.

$$|\omega' - \omega_0| < \varepsilon v_{\mathrm{g}}'\omega_0/c, \tag{6}$$

where ε is of the order of 10^{-3},[15] and ω' is the probe light frequency in the rotating frame. We consider that a pulse with the carrier frequency ω_0 propagates in the laboratory frame, thus the normalized detuning δ is zero. Therefore, the probe light frequency in the rotating frame can be written as

$$\omega^{\pm} = \omega_0\left(1 \mp \Omega L/2c\right). \tag{7}$$

Correspondingly, the detuning can be written as

$$\delta'^{\pm} = \omega_0 - \omega^{\pm} = \pm\Omega L/2c. \tag{8}$$

By substituting $\delta = 0$ into expression (5), the following results can be obtained:

$$v^{\pm} = \pm\sqrt{v_{\mathrm{g}}\left(\omega^{\pm}\right)^2 + \Omega^2L^2/4}. \tag{9}$$

Note, the group velocity expressed by expression (9) is valid if only the detuning expressed by expression (8) satisfies expression (6). Therefore, substituting expression (8) into expression (6), the range of the rotating rate Ω can be deduced as

$$-2\varepsilon v_{\mathrm{g}}/L < \Omega < 2\varepsilon v_{\mathrm{g}}/L. \tag{10}$$

As is well known, in the narrow window of two-photon resonance expressed by expression (6), it can be easily known that n_{g} is symmetrical about $\delta' = 0$, and the detuning values in the clockwise direction and the counterclockwise direction are $\mp\Omega L/2c$ according to expression (8). Therefore, it can be obtained that $n_{\mathrm{g}}\left(\delta'^{+}\right) = n_{\mathrm{g}}\left(\delta'^{-}\right)$, i.e. $n_{\mathrm{g}}\left(\omega^{+}\right) = n_{\mathrm{g}}\left(\omega^{-}\right)$. Subsequently, we can obtain

$$v_{\mathrm{g}}\left(\omega^{+}\right) = v_{\mathrm{g}}\left(\omega^{-}\right) = V_{\mathrm{g}}. \tag{11}$$

Substituting expression (11) into expression (9), the result can be written as

$$v^{\pm} = \pm\sqrt{V_{\mathrm{g}}^2 + \Omega^2L^2/4}, \tag{12}$$

where sign '+' indicates that the propagation direction of light is the same as that of the rotating medium, and sign '−' indicates that the propagation direction of light is opposite to that of the rotating medium. Substituting expression (12) into expression (2), we can obtain the group delay to be

$$\Delta t = n_{\mathrm{g}}^2\Delta t_0, \tag{13}$$

where $\Delta t_0 = 2L^2\Omega/c^2$ is the time delay of the conventional Sagnac interferometer. Evidently, the group delay Δt of the rotation sensor is higher than the time delay of the conventional Sagnac interferometer. Generally, the time delay of the conventional Sagnac interferometer is too small to be detected, so one detects the phase difference $\Delta\varphi = \omega\Delta t_0$ for obtaining the rotating rate (see, for example, Ref. [13]), where ω is the frequency of the light source. In fact, the existence of ω in the phase difference can introduce the error generated by the imbalance of central wavelength. For the proposed rotation sensor in this paper, the group delay is detected to obtain the rotating rate, and the detection method is obviously different from that for traditional interferometric and resonant optical gyroscopes. Evidently, expression (13) indicates that we can slow down the speed of light in the EIT medium to enhance the detection sensitivity. It is noticeable that the experimental observations of slow light reported in atomic and solid media by using EIT have shown the group index $n_{\rm g}$ to be as large as 10^2–10^8.[1] Therefore, one can use this ultra-precise rotation sensor for sensing the relative rotation between the EIT medium and the rest of the rotation sensor.

Expression (13) is derived on the premise that the rotating rate Ω belongs to the range of expression (10). Only in this range can two-photon resonance happen, and then we can obtain the effective group velocity in the laboratory frame. As an example, we consider the rotation sensor with $L = 0.2$ m and $\varepsilon = 10^{-3}$. Figure 2(a) shows the curve of the group delay as a function of the group index $n_{\rm g}$ at the rotating rate $\Omega = 0.01°/{\rm s}$. As expected, the relation between group delay and group index is quadratic (rather than linear). Figure 2(b) shows the curve of the group delay as a function of rotating rate Ω at group index $n_{\rm g}$=10^5. The range of rotating rate Ω can be obtained from expression (10): -30 Hz$< \Omega < 30$ Hz, i.e. $-1718.87°/{\rm s} < \Omega < 1718.87°/{\rm s}$. This detected range can be controlled by adjusting both group index $n_{\rm g}$ and length L according to expression (10). As shown in the two figures, it can be easily known that the group delay is of the order of milliseconds, so the group delay can be easily detected by the slow light technique. All in all, the introduction of the slow light technique can greatly enhance the sensitivity of the rotation sensor.

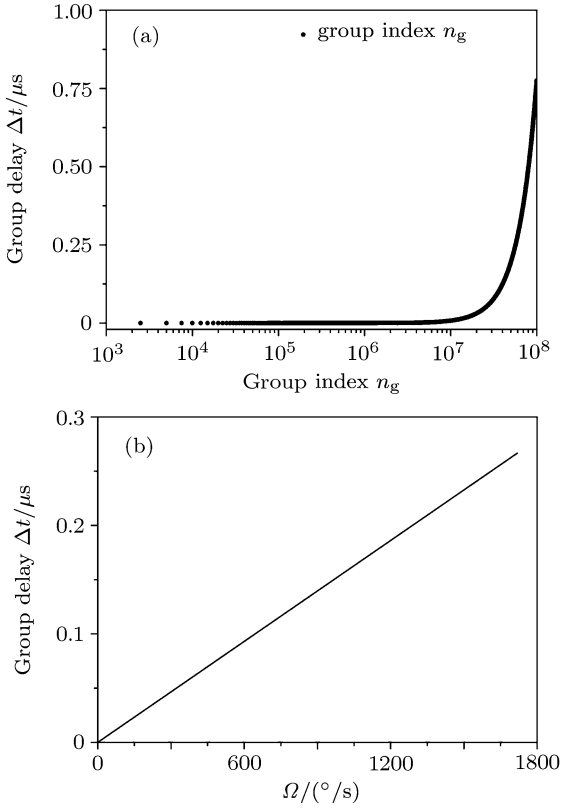


Fig. 2. (a) Group delay versus the group index $n_{\rm g}$ for $\Omega = 0.01°/{\rm s}$ and (b) group delay versus the rotating rate Ω for $n_{\rm g} = 10^5$. The parameters are $L = 0.2$ m, $\varepsilon = 10^{-3}$.

In summary, we propose and analyse a relative rotation sensor whose sensitivity in measuring relative rotation is greatly enhanced via the EIT medium based on the slow light technique. We show that one can enhance the sensitivity to provide a direct readout of the group delay between counterpropagating wave packets.

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