The effects of self-fields on the electron trajectory and gain in a two-stream electromagnetically pumped free-electron laser with axial guiding field

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A theory of a two-stream free-electron laser in a combined electromagnetic wiggler (EMW) is developed, in which we use an axial-guide magnetic field and take into account the effects of the self-fields. The electron trajectories and the small signal gain are derived. The stability of the trajectories, the characteristics of the linear-gain, and the normalised maximum gain are studied numerically. The results show that there are nine stable groups of orbits in the presence of self-fields instead of seven groups reported in the absence of the self-field. It is also shown that the normalised gains of four groups of the orbits are decreasing and those for the rest of them are increasing with growing $\bar{\Omega}_0$. Furthermore, it is found that the two-stream laser with self-field enhances the maximum gain in comparison with the single stream case.

Keywords: free electron laser, two-stream

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1. Introduction

The free-electron laser (FEL) makes use of the unstable interaction of a relativistic electron beam with transverse wiggler magnetic field to generate coherent electromagnetic radiation. The electromagnetic wiggler can produce a shorter wavelength as compared to the magnetostatic wiggler. As a result, the electromagnetic wiggler has advantages over the magneto static wiggler for generation of very short wavelength radiation whenever a limitation on the beam energy is encountered.[1−3] Two-stream free-electron laser amplifiers have been studied extensively during the last few decades.[4−7] Dispersion relation and electron trajectories with EM wiggler and axial field have been studied by the author.[8,9] The effect of two-stream instability on the gain of FELs with axial guide field has been investigated in Ref. [10]. Although there is a considerable literature on the theory of two-stream free-electron lasers, most treatments heretofore have neglected the influence of the equilibrium self-fields of the electron beams. Regarding the low energy of the beams, an axial magnetic field is usually used to guide the electron beams. In such a configuration, equilibrium self-electric and self-magnetic fields, generated by the charge and current densities of the beams, can have considerable effects on the equilibrium orbits. The purpose of this paper is to investigate the effects of the self-fields on electron trajectory and gain in a two-stream free-electron laser with electromagnetic wave wiggler and axial guiding field. The paper is organised as follows. In Section 2, the analysis of single-particle trajectories of two streams in the presence of axial guiding field, EMW and self-fields are given. Section 3 is devoted to the derivation of the small-signal gain. The numerical results and conclusions are given in Section 4.

2. Single particle orbits

The physical configuration consists of a two-stream relativistic electron beam propagating through a circularly polarised electromagnetic wave wiggler in the presence of the axial-guide magnetic field, $B = B_0\hat{z}$. The electromagnetic wave wiggler can be characterised by

$$\begin{aligned}\boldsymbol{B}_{\rm w} &= B_{\rm w}[\hat{x}\cos(k_{\rm w}z+\omega_{\rm w}t)\\ &\quad+\hat{y}\ \sin(k_{\rm w}z+\omega_{\rm w}t)],\end{aligned}\tag{1}$$

$$\begin{aligned}\boldsymbol{E}_{\rm w} &= -E_{\rm w}[\hat{x}\ \sin(k_{\rm w}z+\omega_{\rm w}t)\\ &\quad-\hat{y}\cos(k_{\rm w}z+\omega_{\rm w}t)],\end{aligned}\tag{2}$$

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with

$$E_{\rm w} = \frac{\omega_{\rm w} B_{\rm w}}{c k_{\rm w}}.$$

The beam's equilibrium self-electric and self-magnetic fields are

$$\boldsymbol{E}_s(\boldsymbol{x}) = -2\pi \sum_k n_k e(x\ \hat{x} + y\ \hat{y}), \tag{3a}$$

$$\boldsymbol{B}_s(\boldsymbol{x}) = -2\pi \sum_k n_k e\ \beta_{zk}(-y\ \hat{x} + x\ \hat{y}), \tag{3b}$$

and small amplitude radiation fields,

$$\delta \boldsymbol{E} = -\omega\, \delta A\, [\hat{x} \sin(kz - \omega t) + \hat{y} \cos(kz - \omega t)], \tag{4a}$$

$$\delta \boldsymbol{B} = k \delta A\, [\hat{x} \cos(kz - \omega t) - \hat{y} \sin(kz - \omega t)]. \tag{4b}$$

The relativistic equations of motion for an electron with rest mass m and charge $-e$ moving with velocity $\boldsymbol{v}$ in these combined fields are

$$\frac{{\rm d}\boldsymbol{v}}{{\rm d}t} = -\frac{e}{m\gamma}[\boldsymbol{E} - \frac{1}{c^2} - \boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{E}) + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}], \tag{5a}$$

and

$$\frac{{\rm d}\gamma}{{\rm d}t} = -\frac{e}{mc^2}\boldsymbol{v}\cdot\boldsymbol{E}, \tag{5b}$$

with the aid of Eq. (5) and field Eqs. (1)–(3) the normalised constant transverse velocity of each beam is given by

$$\beta_{{\rm w}i} = \frac{\hat{\Omega}_{\rm w}(\beta_{z\,0i} + \beta_{\rm p})^2}{[-\hat{\omega}_{\rm p1}^2 - \hat{\omega}_{\rm p2}^2 + \beta_{z\,0i}(\hat{\omega}_{\rm p1}^2\beta_{z\,01} + \hat{\omega}_{\rm p2}^2\beta_{z\,02}) + \hat{\Omega}_0(\beta_{z\,0i} + \beta_{\rm p}) - \gamma_i(\beta_{z\,0i} + \beta_{\rm p})^2]}, \quad i = 1, 2, \tag{6}$$

where $\hat{\Omega}_{\rm w} = eB_{\rm w}/m_{\rm e}c^2k_{\rm w}$, $\hat{\Omega}_0 = eB_0/m_{\rm e}c^2k_{\rm w}$, $\hat{\omega}_{{\rm b}i}^2 = 2\pi e^2 n_{{\rm b}i}/mc^2k_{\rm w}^2$ and $\beta_{\rm p} = \omega_{\rm w}/ck_{\rm w}$ denote the normalised wiggler frequency, normalised cyclotron frequency, normalised beam frequency and normalised phase velocity of the EMW, respectively.

Since energy is a constant for motion of the steady-state trajectory, we require that $\beta_{{\rm w}\,i}$ and $\beta_{z\,i}$ are related via

$$\beta_{{\rm w}0\,i}^2 + \beta_{z\,0i}^2 + \gamma_{0\,i}^{-2} - 1 = 0, \quad i = 1,\ 2; \tag{6a}$$

using Maxwell equations together with Eqs. (6), the dispersion relation becomes

$$\beta_{\rm p}^2 - 1 + \frac{2\,\hat{\omega}_{\rm p1}^2(\beta_{z\,01} + \beta_{\rm p})^2}{[-\hat{\omega}_{\rm p1}^2 - \hat{\omega}_{\rm p2}^2 + \beta_{z\,01}(\hat{\omega}_{\rm p1}^2\beta_{z\,01} + \hat{\omega}_{\rm p2}^2\beta_{z\,02}) + \hat{\Omega}_0(\beta_{z\,01} + \beta_{\rm p}) - \gamma_1(\beta_{z\,01} + \beta_{\rm p})^2]}$$
$$+ \frac{2\,\hat{\omega}_{\rm p2}^2(\beta_{z\,2} + \beta_{\rm p})^2}{[-\hat{\omega}_{\rm p2}^2 - \hat{\omega}_{\rm p1}^2 + \beta_{z\,02}(\hat{\omega}_{\rm p2}^2\beta_{z\,02} + \hat{\omega}_{\rm p1}^2\beta_{z\,01}) + \hat{\Omega}_0(\beta_{z\,02} + \beta_{\rm p}) - \gamma_2(\beta_{z\,02} + \beta_{\rm p})^2]} = 0. \tag{7}$$

The stability of orbits can be determined by small perturbations about the steady-state orbits. To investigate the orbital stability, we assume $\beta_{1\,i} = \beta_{1\,i} + \delta\beta_{1\,i}$, $\beta_{2\,i} = \delta\beta_{2\,i}$, $\beta_{z\,i} = \beta_{z\,0i} + \delta\beta_{z\,i}$, $X_{1\,i} = \delta X_{1\,i}$, $X_{2\,i} = X_{{\rm w}0\,i} + \delta X_{{\rm w}\,i}$, $\beta_{\rm p} = \beta_{\rm p0} + \delta\beta_{\rm p}$ and $\gamma_{1\,i} = \gamma_{0\,i} + \delta\gamma_{1\,i}$, $i = 1, 2$.

Here $\boldsymbol{X}$ represents the transverse displacement. The orbit equations for each of the streams are

$$\frac{\rm d}{{\rm d}t}\delta\beta_{1\,i} = -B1_i\ \delta X_{1\,i} + A1_i\ \delta\beta_{2\,i}, \tag{8}$$

$$\frac{\rm d}{{\rm d}t}\delta\beta_{2\,i} = -D2_i\ \delta X_{2\,i} + A2_i\ \delta\beta_{1\,i} + C2_i\ \delta\gamma_i, \tag{9}$$

$$\frac{\rm d}{{\rm d}t}\delta\beta_{z\,i} = -B3_i\ \delta X_{1\,i} + A3_i\ \delta\beta_{2\,i}, \tag{10}$$

$$\frac{{\rm d}\delta\gamma_i}{{\rm d}t} = A4_i\,\delta\beta_{2\,i} + B4_i\,\delta X_{1\,i}, \tag{11}$$

$$\frac{{\rm d}\delta X_{1\,i}}{{\rm d}t} = A6_i\,\delta\beta_{1\,i} + B6_i\delta\beta_{z\,i} + C6_i\,\delta X_{2\,i}, \tag{12}$$

$$\frac{{\rm d}\delta X_{2\,i}}{{\rm d}t} = A5_i\delta\beta_{2\,i} + B5_i\,\delta X_{1\,i}, \tag{13}$$

where

$$A1_{1\,i} = (\beta_{\rm p} + \beta_{z0\,i}) - \frac{\hat{\Omega}_0}{\gamma_i} - \beta_{{\rm w}0\,i}\,X_{{\rm w}0\,i}\,\alpha_i + \frac{\beta_{{\rm w}0\,i}\,\beta_{\rm p}\,\hat{\Omega}_{\rm w}}{\gamma_i}, \quad i = 1,\ 2, \tag{14a}$$

$$\alpha_1 = \frac{1}{\gamma_1}(\hat{\omega}_{\rm p1}^2 + \hat{\omega}_{\rm p2}^2), \quad \alpha_2 = \frac{1}{\gamma_2}(\hat{\omega}_{\rm p2}^2 + \hat{\omega}_{\rm p1}^2), \quad i = 1,\ 2, \tag{14b}$$

$$B1_i = (-\beta_{{\rm w}0\,i}^2 + 1)\alpha_i - \beta_{z0\,i}{\rm P}_i, \quad i = 1,\ 2, \tag{14c}$$

$$P_1 = \frac{1}{\gamma_1}(\hat{\omega}_{\rm p1}^2\,\beta_{z0\,1} + \hat{\omega}_{\rm p2}^2\,\beta_{z0\,2}), \quad i = 1,\ 2, \tag{14d}$$

$$P_2 = \frac{1}{\gamma_2}(\hat{\omega}_{\rm p2}^2\,\beta_{z0\,2} + \hat{\omega}_{\rm p1}^2\,\beta_{z0\,1}), \quad i = 1,\ 2, \tag{14e}$$

$$A2_i = \frac{\hat{\Omega}_0}{\gamma_i} - (\beta_{\rm p} + \beta_{z0\,i}), \quad i = 1,\ 2, \tag{15a}$$

$$B2_i = -\beta_{{\rm w}0\,i} - \frac{\hat{\Omega}_{\rm w}}{\gamma_i} - P_i X_{{\rm w}0\,i}, \quad i = 1,\ 2, \tag{15b}$$

$$C2_i = -(\beta_{\rm p} + \beta_{z0\,i})\frac{\beta_{\rm w0\,i}}{\gamma_i}, \quad i = 1,\, 2, \tag{15c}$$

$$D2_i = \alpha_i - \beta_{z0\,i}{\rm P}_i, \quad i = 1,\, 2, \tag{15d}$$

$$A3_1 = \frac{1}{\gamma_1}[\hat{\Omega}_{\rm w}(1 + \beta_{z0\,1}\beta_{\rm p}) + \hat{\omega}^2_{\rm p2}X_{\rm w0\,1}(\beta_{z0\,2} - \beta_{z0\,1})], \tag{16a}$$

$$A3_2 = \frac{1}{\gamma_2}[\hat{\Omega}_{\rm w}(1 + \beta_{z0\,2}\beta_{\rm p}) + \hat{\omega}^2_{\rm p1}X_{\rm w0\,2}(\beta_{z0\,1} - \beta_{z0\,2})], \tag{16b}$$

$$B3_1 = \frac{1}{\gamma_1}\beta_{\rm w0\,1}\hat{\omega}^2_{\rm p2}(\beta_{z0\,2} - \beta_{z0\,1}), \tag{16c}$$

$$B3_2 = \frac{1}{\gamma_2}\beta_{\rm w0\,2}\hat{\omega}^2_{\rm p1}(\beta_{z0\,1} - \beta_{z0\,2}), \tag{16d}$$

$$A4_{_i} = -\beta_{\rm p}\hat{\Omega}_{\rm w} + \gamma_i\alpha_i X_{\rm w0\,i}, \quad i = 1,\, 2, \tag{17a}$$

$$B4_i = \gamma_i\beta_{\rm w0\,i}\alpha_i, \quad i = 1,\, 2, \tag{17b}$$

$$A5_i = 1, \quad A6_i = 1, \quad i = 1,\, 2, \tag{18a}$$

$$B5_i = -(\beta_{z0\,i} + \beta_{\rm p}), B6_i = X_{\rm w0\,i}, \quad i = 1,\, 2, \tag{18b}$$

$$C6_i = (\beta_{z0\,i} + \beta_{\rm p}) \quad i = 1,\, 2. \tag{18c}$$

Differentiating Eqs. (9) and (12) with respect to time and employing Eqs. (8)–(18), we obtain two homogeneous higher order differential equations

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{1\,i}\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{2\,i}\right)\delta v_{2\,i} = 0, \tag{19a}$$

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{1\,i}\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{2\,i}\right)\delta X_{1\,i} = 0, \tag{19b}$$

where

$$\Omega^2_{1\,i} = \frac{(\omega^2_{1\,i} + \omega^2_{2\,i}) - [(\omega^2_{1\,i} - \omega^2_{2\,i})^2 + 4N_iM_i]^{1/2}}{2}, \tag{20a}$$

$$\Omega^2_{2\,i} = \frac{(\omega^2_{1\,i} + \omega^2_{2\,i}) + [(\omega^2_{1\,i} - \omega^2_{2\,i})^2 + 4N_iM_i]^{1/2}}{2}, \tag{20b}$$

with

$$\omega^2_{1\,i} = -(A2_iA1_i + B2_iA3_i + D2_iA5_i + A4_iC2_i), \tag{21a}$$

$$\omega^2_{2\,i} = -(A6_iB1_i + C6_iB5_i + B6_iB3_i), \tag{21b}$$

$$N_i = A6_iA1_i + B6_iA3_i + A5_iC6_i, \tag{21c}$$

$$M_i = A2_iB1_i + C2_iB4_i + D2_iB5_i + B2_iB3_i. \tag{21d}$$

Let us assume that all the variables oscillate with the same frequency ω like

$$\delta\beta_{1\,i} = A\exp({\rm i}\,\omega t). \tag{22}$$

Using this assumption Eq. (19) becomes

$$\bar{\omega}^2 - (\Omega^2_{1\,i} + \Omega^2_{2\,i})\bar{\omega} - \Omega^2_{1\,i}\Omega^2_{2\,i} = 0,$$

where

$$\bar{\omega} = \omega^2. \tag{23}$$

This equation has two roots for $\bar{\omega}(\bar{\omega}_1 = \Omega^2_{1\,i}$ and $\bar{\omega}_2 = \Omega^2_{2\,i})$. The stability of the steady-state two-stream electron trajectory requires that these roots, $\bar{\omega}_1$ and $\bar{\omega}_2$, must be real and positive.

3. Small signal gain

In this section we wish to study small signal gain; to end this job, it is sufficient to substitute Eqs. (4) in to Eqs. (8)–(19), and then by doing some straightforward algebra, we obtain

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{1\,i}\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{2\,i}\right)\delta v_{2\,i} = [K_i\omega^2_{2\,i} + M_iL_i]\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{24a}$$

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{1\,i}\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega^2_{2\,i}\right)\delta X_{1\,i} = [L_i\,\omega^2_{1\,i} + N_i\,K_i]\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{24b}$$

$$\frac{{\rm d}\delta\beta_{3\,i}}{{\rm d}t} = -B3_i\ \delta X_{1\,i} + A3_i\ \delta\beta_{2\,i} + (k - \omega\beta_{z0\,i})\beta_{\rm w0\,i}\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{24c}$$

where $\psi_i = [(k + k_{\rm w})\beta_{z0\,i} - (\omega - \omega_{\rm w})]t$,

$$K_i = A2_i(\omega - k\beta_{z0\,i} - \omega\,\beta^2_{\rm w0\,i}) + B2_i\beta_{\rm w0\,i}(k - \omega\beta_{z0\,i}) + C2_i\omega\gamma_i\beta_{\rm w0\,i}, \tag{25}$$

$$L_i = A6_i(\omega - k\beta_{z0\,i} - \omega\beta^2_{\rm w0\,i}) + B6_i\beta_{\rm w0\,i}(k - \omega\beta_{\rm w0\,i}). \tag{26}$$

With the aid of Eqs. (24a) and (24b), Eq. (24c) takes the following form

$$\frac{{\rm d}\delta\beta_{3i}}{{\rm d}t} = \frac{\phi_i}{\Omega^2_{1\,i}\,\Omega^2_{2\,i}}\,\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{27}$$

where

$$\phi_i = A3_i(K_i\omega^2_{2\,i} + L_iM_i) + B3_i(L\omega^2_{i1} + N_iK_i) + (k - \omega\beta_{\rm w0\,i})\beta_{\rm w0\,i}(\omega^2_{i1}\,\omega^2_{2\,i} - M_iN_i). \tag{28}$$

The combination of Maxwell equations and Eq. (27) yields

$$2k\frac{\mathrm{d}}{\mathrm{d}z}\delta a = -2\sum_{i=1}^{2}\hat{\omega}_{\mathrm{b}i}^2\,\beta_{\mathrm{w}0\,i}\langle\sin\psi_i\rangle. \tag{29}$$

Here

$$\delta a = \frac{e\delta A}{m_{\mathrm{e}}c},$$

$$\langle\sin\psi_i\rangle = -\frac{(\omega-\omega_{\mathrm{w}})\,\phi_i}{2\,\Omega_{1\,i}^2\,\Omega_{2\,i}^2\beta_{z0\,i}^3\,\gamma_i\,\Delta k_i^2} \times[\sin(\Delta k_i z)-\Delta k_i z\cos(\Delta k_i z)], \tag{30}$$

and

$$\Delta k_i = k+1-\frac{\omega-\beta_{\mathrm{p}}}{\beta_{z0\,i}}.$$

Using the definition of gain over some length L:[11]

$$G_L \equiv \frac{\delta a(z=L)-\delta a(z=0)}{\delta a(z=0)}, \tag{31}$$

and employing Eqs. (29) and (30), we obtain

$$G_L = -\frac{L^3\hat{\omega}_{\mathrm{b}1}^2\beta_{\mathrm{w}0\,1}(1-(\omega_{\mathrm{w}}/\omega))\phi_1}{8\Omega_{11}^2\Omega_{21}^2\beta_{z0\,1}^2\gamma_1}\frac{\mathrm{d}}{\mathrm{d}\theta_1}\left(\frac{\sin\theta_1}{\theta_1}\right)^2 - \frac{L^3\hat{\omega}_{\mathrm{b}2}^2\beta_{\mathrm{w}0\,2}(1-(\omega_{\mathrm{w}}/\omega))\phi_2}{8\,\Omega_{12}^2\Omega_{22}^2\beta_{z0\,2}^2\gamma_2}\frac{\mathrm{d}}{\mathrm{d}\theta_2}\left(\frac{\sin\theta_2}{\theta_2}\right)^2, \tag{32}$$

where

$$\theta_{1,2} = \frac{\Delta k_{1,2}\,L}{2}.$$

Neglecting the effects of self-fields and vanishing $\omega_{\mathrm{p}2}$, this result is reduced to the reported gain formula.[11]

4. Numerical results and conclusions

A numerical study of the steady-state relativistic two-stream electron trajectories in an EMW in the presence of self-fields and axial magnetic field has been made. The typical dependence of the axial velocity on B_0 was obtained by simultaneously solving Eqs. (6) and (7). The electron energy was taken to be $\gamma_1 = 4$ and $\gamma_2 = 4.1$; normalised wiggler magnetic and normalised plasma frequencies of the two beams are chosen to be $\hat{\Omega}_{\mathrm{w}} = 0.2$, $\hat{\omega}_{\mathrm{p}1} = 0.212 = \hat{\omega}_{\mathrm{p}2}$, respectively. Figures 1(a) and 1(b) show the graph of the normalised axial velocity of two-beam, β_{z1} and β_{z2}, as a function of the normalised cyclotron frequency, $\bar{\Omega} = \hat{\Omega}_0/\gamma_1$. As seen in these figures, the electron trajectories in the presence of self-fields have nine distinguishable groups of orbits rather than seven groups reported in the absence of self-fields.[9] The normalised electromagnetic wave wiggler frequency, $\beta_{\mathrm{p}} = \omega_{\mathrm{w}}/ck_{\mathrm{w}}$, is depicted in Fig. 2 as a function of the normalised cyclotron frequency, $\bar{\Omega}_0$.

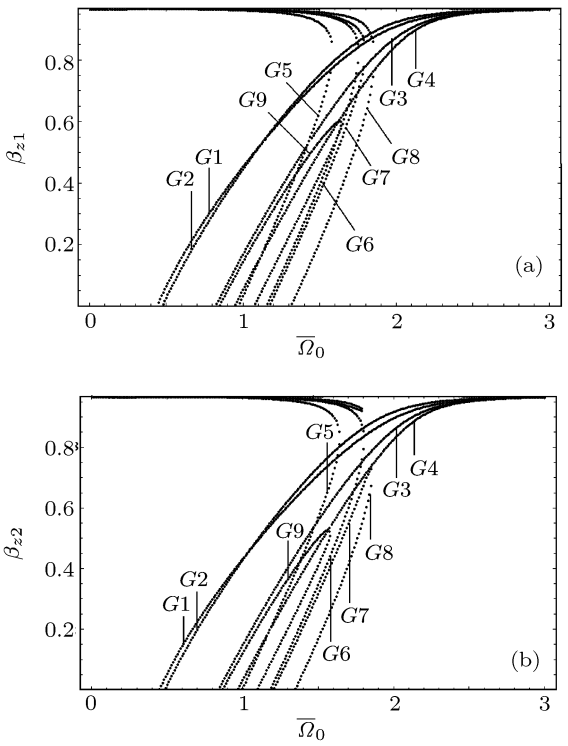


Fig. 1. (a) Graph of the normalised axial velocity (β_{z1}) of the steady-state orbits as a function of the normalised cyclotron frequency ($\bar{\Omega}_0$) in the case of two streams. The chosen parameters are: $\hat{\Omega}_{\mathrm{w}} = 0.2$, $\hat{\omega}_{\mathrm{p}1} = 0.212$ and $\hat{\omega}_{\mathrm{p}2} = 0.212$. (b) Graph of the normalised axial velocity (β_{z2}) of the steady-state orbits as a function of the normalised cyclotron frequency ($\bar{\Omega}_0$) in the case of two streams. The chosen parameters are: $\hat{\Omega}_{\mathrm{w}} = 0.2$, $\hat{\omega}_{\mathrm{p}1} = 0.212$ and $\hat{\omega}_{\mathrm{p}2} = 0.212$.

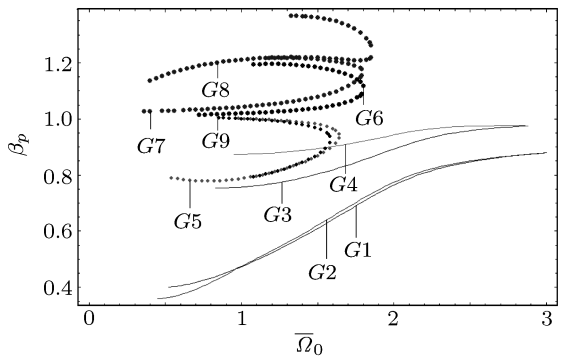


Fig. 2. (Colour online) Plot of the normalised wiggler phase velocity ($\hat{\beta}_{\mathrm{p}}$) as a function of the normalised cyclotron frequency ($\bar{\Omega}_0$) in the case of a two-stream laser for given parameters: $\hat{\Omega}_{\mathrm{w}} = 0.2$ and $\hat{\omega}_{\mathrm{p}1} = \hat{\omega}_{\mathrm{p}2} = 0.212$.

The roots of Eq. (23) ($\bar{\omega}_1$ and $\bar{\omega}_2$) as a function of $\bar{\Omega}_0$ are illustrated in Figs. 3 and 4. As seen in these figures $\bar{\omega}_1$ and $\bar{\omega}_2$ are real and positive. Therefore, the nine groups are stable, in the given range for $\bar{\Omega}_0$.

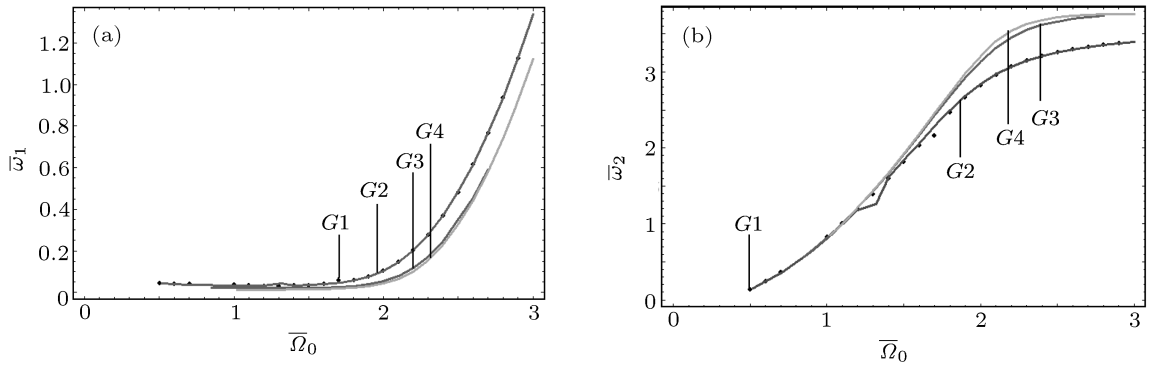


Fig. 3. (a) Graph of $\bar{\omega}_1$ versus $\bar{\Omega}_0$ corresponding to the G (I–IV) orbits. The choice of system parameters is $\hat{\Omega}_{\rm w} = 0.2$, $\hat{\omega}_{\rm p1} = \hat{\omega}_{\rm p2} = 0.212$. (b) Graph of $\bar{\omega}_2$ versus $\bar{\Omega}_0$ corresponding to the G (I–IV) orbits. The choice of system parameters is $\hat{\Omega}_{\rm w} = 0.2$, $\hat{\omega}_{\rm p1} = \hat{\omega}_{\rm p2} = 0.212$.

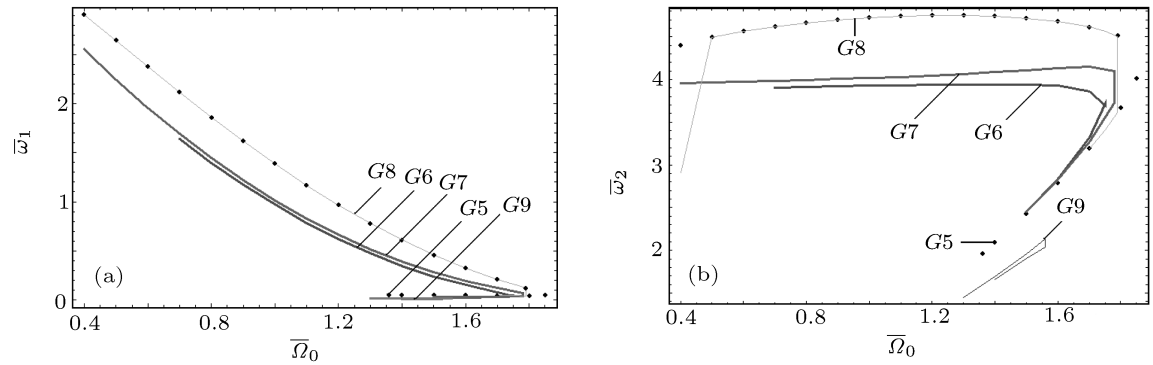


Fig. 4. (a) Graph of $\bar{\omega}_1$ versus $\bar{\Omega}_0$ corresponding to the G (V–IX) orbits. The choice of system parameters is $\hat{\Omega}_{\rm w} = 0.2$, $\hat{\omega}_{\rm p1} = \hat{\omega}_{\rm p2} = 0.212$. (b) Graph of $\bar{\omega}_2$ versus $\bar{\Omega}_0$ corresponding to the G (V–IX). The choice of system parameters is $\hat{\Omega}_{\rm w} = 0.2$, $\hat{\omega}_{\rm p1} = \hat{\omega}_{\rm p2} = 0.212$.

The maximum gain [normalised to the value of $(G_L)_{\rm Max}$ for $B_0 = 0$] as a function of normalised cyclotron frequency is depicted in Figs. 5(a) and 5(b). Figure 5(a) shows that the gain decreases with increasing $\bar{\Omega}_0$, to fall to its minimum and have a small rise until it reaches its asymptotes. The order of the normalised gains, in the region $1.8 \leq \bar{\Omega}_0 \leq 2.4$, is $\bar{G}_{\rm gIV} > \bar{G}_{\rm gIII} > \bar{G}_{\rm gII} > \bar{G}_{\rm gI}$($\bar{G}_{\rm gi}$ denotes the normalised maximum gain for group i). In contrast to Fig.5a, the normalised gain of groups presented in Fig. 5(b) is enhanced by increasing $\bar{\Omega}_0$ until it reaches its asymptotes. The order of these normalised gains is $\bar{G}_{\rm gV} > \bar{G}_{\rm gVI} > \bar{G}_{\rm gVII} > \bar{G}_{\rm VIII}$ in the range of $0.4 \leq \bar{\Omega}_0 \leq 1.5$. A comparison between the normalised maximum gains for two-stream and single stream is given in Figs. 6(a) and 6(b). The results show that the two-stream enhances the maximum gain.

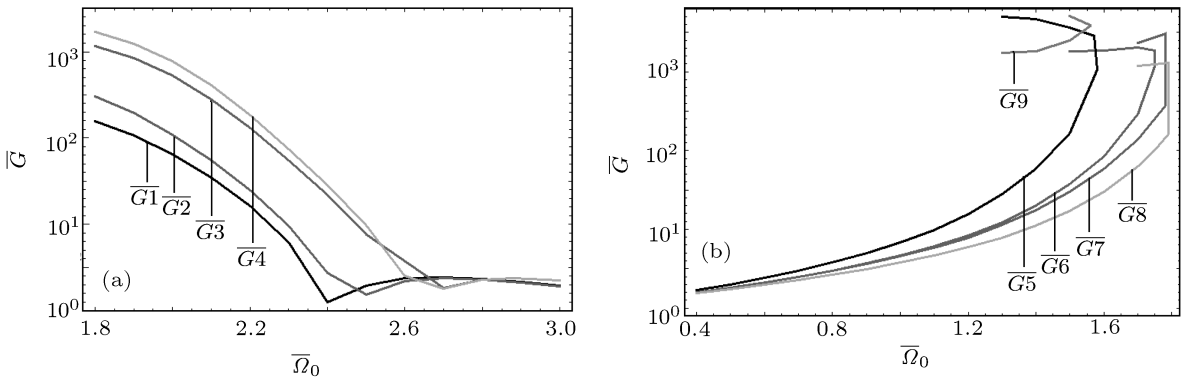


Fig. 5. (a) Plot of normalised maximum gain ($\bar{G}$) with respect to cyclotron frequency ($\bar{\Omega}_0$) for G (I–IV). The chosen parameters are $\hat{\Omega}_{\rm w} = 0.2$, $\hat{\omega}_{\rm p1} = 0.212$ and $\hat{\omega}_{\rm p2} = 0.212$. (b) Plot of normalised maximum gain ($\bar{G}$) with respect to cyclotron frequency ($\bar{\Omega}_0$) for G (V–IX). The chosen parameters are $\hat{\Omega}_{\rm w} = 0.2$, $\hat{\omega}_{\rm p1} = 0.212$ and $\hat{\omega}_{\rm p2} = 0.212$.

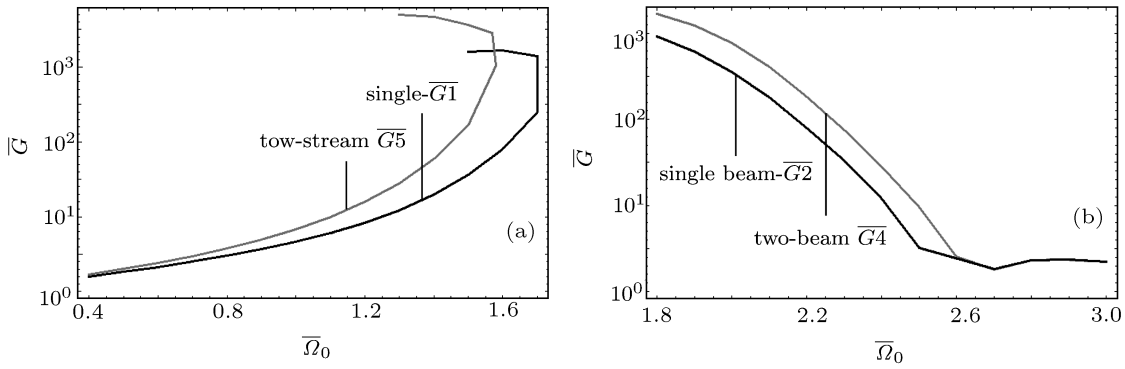


Fig. 6. (a) Graph of the normalised maximum gain $\bar{G}$ for two-stream ($G5$) and single stream ($G1$) as a function of the normalised cyclotron frequency ($\bar{\Omega}_0$). (b) Graph of the normalised maximum gain $\bar{G}$ for two-stream ($G4$) and single stream ($G2$) as a function of the normalised cyclotron frequency ($\bar{\Omega}_0$).

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