Left-handedness without absorption in the four-level Y-type atomic medium*

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In this paper we have investigated three external fields interacting with the four-level Y-type atomic system described by the density-matrix approach. The results show that left-handedness with zero absorption is achieved. The zero absorption property displays the possibility of manipulation by varying the phase and the intensity of the coupling field. Also, the zero absorption property may be used to amplify the evanescent waves that have been lost in imaging by traditional lenses. We propose an approach to obtain a negative refractive medium with zero absorption and the possibility of enhanceingthe imaging resolution in realizing 'superlenses'.

Keywords: zero absorption, left-handedness, negative refractive index, negative permittivity and permeability

PACC: 4250, 4262A, 4270, 7820D

1. Introduction

Materials with simultaneous negative electric permittivity and magnetic permeability have attracted much attention.[1−4] These materials have extraordinary properties, such as negative refraction, inverse Doppler shift and backward-directed Cherenkov radiation cone, etc.[5] The electric field, the magnetic field and the wave vector of an electromagnetic wave propagating in such a material obey the left-hand rule and thus it is called a left-handed material (LHM) or negative refraction index material, which was first introduced by Veselago many years ago.[5] In 2000, Pendry[6] proposed that a class of 'superlenses' could be made by LHMs. Such lenses may overcome the traditional limitation in resolution by the wavelength of the light used and make 'perfect' images. Since then, research on such a superlens and LHM has been booming. Driven by the goals of realizing a 'perfect lens', many technologies are used in demonstrating negative refraction such as composite materials,[1,2,7−17] transmission line simulation,[18] photonic crystal structures[19−24] and photonic resonant materials.[25−28] However, the authors in Refs. [29] and [30] pointed out that LHM lenses can indeed amplify evanescent waves that have been lost in imaging by traditional lenses; the presence of absorption, even a small amount, plays a crucial rule in recovery of the lost evanescent waves, thereby controlling the quality of imaging, and makes LHM lenses less perfect. So, realization of LHM without absorption is still the key challenge, which is particularly important in the optical regime.[31−34]

With the realization of LHM without absorption in mind, here we theoretically put forward a four-level Y-type dense atomic vapour scheme based on the quantum coherence effect to realize left-handedness with zero absorption. In our scheme, the contribution of the Lorentz–Lorenz local field of the system should be considered. Under some appropriate conditions, the system shows simultaneous negative permittivity and permeability as well as zero absorption.

The paper is organized as follows: In Section 2, we present our model and its expressions for the electric permittivity, magnetic permeability and refractive index. In Section 3, we present numerical results and their discussion. This is followed by concluding remarks in Section 4.

2. Theoretical model

Consider a four-level Y-type atomic ensemble interacting with three optical fields, i.e. the coupling beam, probe light and signal field. The atomic config-

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uration is schematically shown in Fig. 1. The strong resonant coherent field with frequency ω_3 and Rabi frequency Ω_3' couples states $|1\rangle$ and $|4\rangle$ (with transition frequency ω_{14}). The weak probe field with frequency ω_1 interacts with the transition $|1\rangle$ to $|2\rangle$ (transition frequency ω_{12}), the electric (E) and magnetic (B) components of the probe field couple state $|2\rangle$ to state $|1\rangle$ by an electric dipole transition, and to state $|4\rangle$ by a magnetic dipole transition, while the weak signal field with frequency ω_2 and Rabi frequency Ω_2 drives the transition $|2\rangle$ to $|3\rangle$ (transition frequency ω_{23}). The parities in this system are $|2\rangle$ even, $|4\rangle$ even, $|1\rangle$ odd or vice versa; 2 γ_1 and $2\gamma_4$ are spontaneous decay rates from the levels $|1\rangle$ and $|4\rangle$ to the level $|2\rangle$, while $2\gamma_2$ corresponds to the decay rate from $|2\rangle$ to $|3\rangle$.

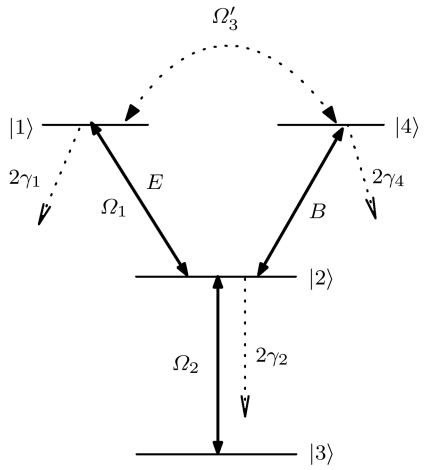


Fig. 1. The energy level scheme of the four-level Y-type atom driven by three microwave fields.

The Hamiltonian under the dipole and rotation-wave approximation is given by (throughout the paper we assume $\hbar = 1$),

$$\hat{H} = \Delta_2|2\rangle\langle 2| + (\Delta_1 + \Delta_2)|1\rangle\langle 1| + (\Delta_4 + \Delta_2)|4\rangle\langle 4| - (\Omega_1|1\rangle\langle 2| + \Omega_2|2\rangle\langle 3| + \Omega_3'|1\rangle\langle 4| + \text{H.c.}), \quad (1)$$

where the detunings of the probe and signal light field are $\Delta_1 = \omega_{12} - \omega_1$ and $\Delta_2 = \omega_{23} - \omega_2$, respectively. The dynamical evolution of the system including spontaneous emission is governed by the equation of motion of the density matrix,

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}[H,\rho] - \sum_{i,j=1,2,3,4} \gamma_{i,(i\neq 3)}\{|j\rangle\langle j|,\hat{\rho}\} + 2\gamma_1|2\rangle\langle 1|\rho|1\rangle\langle 2| + 2\gamma_2|3\rangle\langle 2|\rho|2\rangle\langle 3|, \quad (2)$$

where $[\hat{A},\ \hat{B}]$ and $\{\hat{A},\ \hat{B}\ \}$ denote the commutator and anti-commutator of the operators $\hat{A}$ and $\hat{B}$, respectively. Here we treat the Rabi frequencies Ω_1 and Ω_2 as real parameters, Ω_3' as complex parameters: $\Omega_3' = \Omega_3\,\mathrm{e}^{\mathrm{i}\Phi_3}$, where Φ_3 is the phase of the coupling field. The matrix elements ρ_{ij} $(i,j = 1,2,3,4)$ obey the differential equations as follows:

$$\dot{\rho_{11}} = -2\gamma_1\rho_{11} - (\mathrm{i}\Omega_1\rho_{12} + \mathrm{i}\Omega_3\rho_{14}\,\mathrm{e}^{-\mathrm{i}\Phi_3} + \text{H.c.}), \quad (3a)$$

$$\dot{\rho_{33}} = 2\gamma_2\rho_{22} + (\mathrm{i}\Omega_2\rho_{23} + \text{H.c.}), \quad (3b)$$

$$\dot{\rho_{44}} = -2\gamma_4\rho_{44} - (\mathrm{i}\Omega_3\rho_{41}\,\mathrm{e}^{-\mathrm{i}\Phi_3} + \text{H.c.}), \quad (3c)$$

$$\dot{\rho_{12}} = -(\gamma_1 + \gamma_2 + \mathrm{i}\Delta_1)\rho_{12} - \mathrm{i}\Omega_1\rho_{11} - \mathrm{i}\Omega_2\rho_{13} + \mathrm{i}\Omega_1\rho_{22} + \mathrm{i}\Omega_3\rho_{42}\,\mathrm{e}^{\mathrm{i}\Phi_3}, \quad (3d)$$

$$\dot{\rho_{13}} = -(\gamma_1 + \mathrm{i}\Delta_1 + \mathrm{i}\Delta_2)\rho_{13} - \mathrm{i}\Omega_2\rho_{12} + \mathrm{i}\Omega_1\rho_{23} + \mathrm{i}\Omega_3\rho_{43}\,\mathrm{e}^{\mathrm{i}\Phi_3}, \quad (3e)$$

$$\dot{\rho_{14}} = -(\gamma_1 + \gamma_4 - \mathrm{i}\Delta_1)\rho_{14} + \mathrm{i}\Omega_1\rho_{24} - \mathrm{i}\Omega_3(\rho_{11} - \rho_{44})\,\mathrm{e}^{\mathrm{i}\Phi_3}, \quad (3f)$$

$$\dot{\rho_{23}} = -(\gamma_2 + \mathrm{i}\Delta_2)\rho_{23} + \mathrm{i}\Omega_1\rho_{13} - \mathrm{i}\Omega_2\rho_{22} + \mathrm{i}\Omega_2\rho_{33}, \quad (3g)$$

$$\dot{\rho_{24}} = -(\gamma_2 + \gamma_4)\rho_{24} + \mathrm{i}\Omega_1\rho_{14} + \mathrm{i}\Omega_2\rho_{34} - \mathrm{i}\Omega_3\rho_{21}\,\mathrm{e}^{\mathrm{i}\Phi_3}, \quad (3h)$$

$$\dot{\rho_{34}} = -(\gamma_4 - \mathrm{i}\Delta_2)\rho_{34} + \mathrm{i}\Omega_2\rho_{24} - \mathrm{i}\Omega_3\rho_{31}\,\mathrm{e}^{\mathrm{i}\Phi_3}, \quad (3i)$$

where the above density matrix elements obey the conditions: $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$ and $\rho_{ij} = \rho_{ji}^*$.

In the following, we will discuss the electric and magnetic responses of the medium to the probe field. It should be noted that here the atoms are assumed to be nearly stationary (e.g., at a low temperature) and hence any Doppler shift is neglected. When discussing how the detailed properties of the atomic transitions between the levels are related to the electric and magnetic susceptibilities, one must make a distinction between macroscopic fields and the microscopic local fields acting upon the atoms in the vapour. In a dilute vapour, there is little difference between the macroscopic fields and the local fields that act on any atoms (molecules or group of molecules).[35] But in dense media with closely packed atoms (molecules), the polarization of neighbouring atoms (molecules) gives rise to an internal field at any given atom in addition to the average macroscopic field, so that the total fields at the atom are different from the macroscopic fields.[35] In order to achieve negative permittivity and permeability, here the chosen vapour with atomic concentration $N = 0.25 \times 10^{24}$ m^{-3} should be dense, so that one should consider the local field effect, which results from the dipole–dipole interaction between neighbouring atoms. Next, we first obtain the atomic electric and magnetic polarizations, and then

consider the local field correction to the electric and magnetic susceptibilities (and hence to the permittivity and permeability) of the coherent vapour medium. With the formula of the atomic electric polarizations $\gamma_{\rm e} = 2d_{21}\rho_{12}/\epsilon_0 E_{\rm p}$, where $E_{\rm p} = \hbar\Omega_{\rm p}/d_{21}$, one can have

$$\gamma_{\rm e} = \frac{2d_{21}^2\rho_{12}}{\epsilon_0\hbar\Omega_{\rm p}}. \tag{4}$$

In a similar fashion, by using the formulae of the atomic magnetic polarizations $\gamma_{\rm m} = 2\mu_0\mu_{42}\rho_{24}/B_{\rm p}$,[35] and the relation between the microscopic local electric and magnetic fields $E_{\rm p}/B_{\rm p} = c$ we can obtain an explicit expression for the atomic magnetic polarizability. Here μ_0 is the permeability of vacuum, c is the speed of light in vacuum. Then, we have obtained the microscopic physical quantities $\gamma_{\rm e}$ and $\gamma_{\rm m}$. In order to achieve a significant magnetic response, the levels $|1\rangle$ and $|4\rangle$ should be degenerate approximately to the level $|2\rangle$. However, what we are interested in is the macroscopic physical quantities such as the electric and magnetic susceptibilities, which are the electric permittivity and magnetic permeability. The electric and magnetic Clausius–Mossotti relations can reveal the connection between the macroscopic and microscopic quantities. According to the Clausius–Mossotti relation,[35] one can obtain the electric susceptibility of the atomic vapour medium

$$\chi_{\rm e} = N\gamma_{\rm e}\cdot\left(1 - \frac{N\gamma_{\rm e}}{3}\right)^{-1}. \tag{5}$$

The relative electric permittivity of the atomic medium reads $\varepsilon_{\rm r} = 1 + \chi_{\rm e}$. Meanwhile, the magnetic Clausius–Mossotti formula[36]

$$\gamma_{\rm m} = \frac{1}{N}\left(\frac{\mu_{\rm r} - 1}{\frac{2}{3} + \frac{\mu_{\rm r}}{3}}\right) \tag{6}$$

shows the connection between the macroscopic magnetic permeability $\mu_{\rm r}$ and the microscopic magnetic polarizations $\gamma_{\rm m}$. It follows that the relative magnetic permeability of the atomic vapour medium is

$$\mu_{\rm r} = \frac{1 + \frac{2}{3}N\gamma_{\rm m}}{1 - \frac{1}{3}N\gamma_{\rm m}}. \tag{7}$$

Substituting the expressions of $\varepsilon_{\rm r}$ and $\mu_{\rm r}$ into $n = -\sqrt{\varepsilon_{\rm r}\mu_{\rm r}}$,[5] we can obtain the refractive index of the LHM. In the above, we have obtained the expressions for the electric permittivity, magnetic permeability and refractive index of the four-level atomic medium. In the section that follows, we will find solutions to the density-matrix equations (3) under the steady-state condition.

3. Results and discussion

We consider the atomic vapour medium with $N = 0.25 \times 10^{24}$ m^{-3}, and examine the case of dense media where the Lorentz–Lorenz local field corrections play a significant role. As pointed out in the following numerical example, the magnitude of the applied coherent optical field's Rabi frequency is (10^6/s), which is larger than the radiative decay constants. Thus, the effects of dephasing, and collisional broadening can be neglected, and the coherence effect can be maintained and even increased in the atomic vapour under consideration. With the steady solution of the matrix equations (3) we can obtain the coherent terms ρ_{12} and ρ_{24}, and with the expressions for the atomic electric and magnetic polarizability (4)–(7), we will present a numerical example to show that strong electric and magnetic responses can truly arise in the four-level coherent vapour medium under certain appropriate conditions. The strong electric and magnetic responses can even simultaneously lead to negative permittivity and permeability at certain wide frequency bands of the probe light. The real parts of the relative permittivity and permeability as well as refractive index are plotted in Figs. 2 and 3. We examine the left-handedness and the absorption behaviours of the four-level Y-type atomic system.

In Fig. 2, we plot real parts of the permittivity (dashed curve) and permeability (dotted curve) (in Fig. 2(a)), the refractive index versus the probe detuning Δ_1/γ (in Fig. 2(b)) for the parameter values $\Omega_3 = 2.8\gamma$ and $\Phi_3 = \pi/3$. The other parameters are scaled by γ: $\gamma_1 = \gamma_4 = 0.001\gamma$, $\gamma_2 = 0.005\gamma$, $\Omega_1 = 0.8\gamma$, $\Omega_2 = 0.12\gamma$, $\Delta_2 = 0.001\gamma$, $\gamma = 10^6$. It can be seen from Fig. 2(a) that the relative dielectric permittivity $\varepsilon_{\rm r}$ has a negative real part in the probe frequency detuning range [–100γ; 100γ]. The real part of $\mu_{\rm r}$ is also negative in this frequency detuning range. The negative permittivity and permeability are simultaneously exhibited in the four-level coherent atomic vapour. In Fig. 2(b), the grey and solid curves correspond to the real and imaginary parts of the refractive index. The grey curve shows that the real part of the refractive index has a negative value in the same probe frequency detuning range as in Fig. 2(a). This means

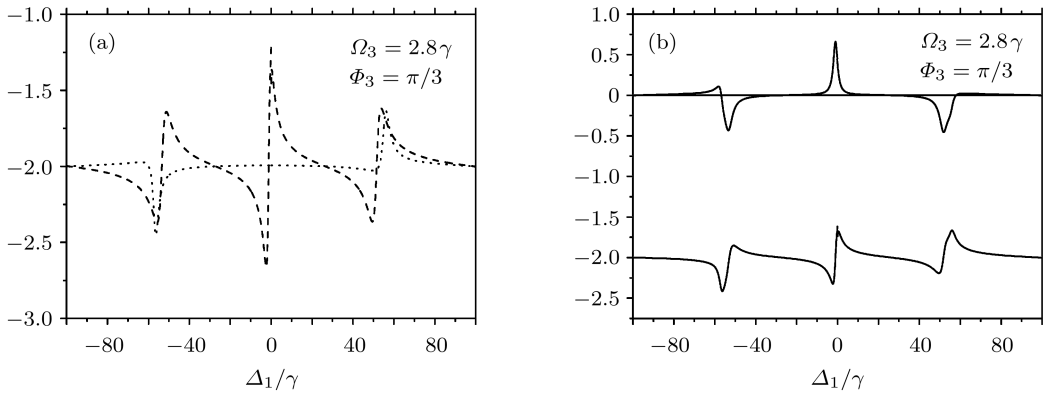


Fig. 2. (a) Real parts of the permittivity (dashed curve) and permeability (dotted curve), (b) real (solid curve) and imaginary (grey curve) parts of the refractive index as functions of Δ_1/γ.

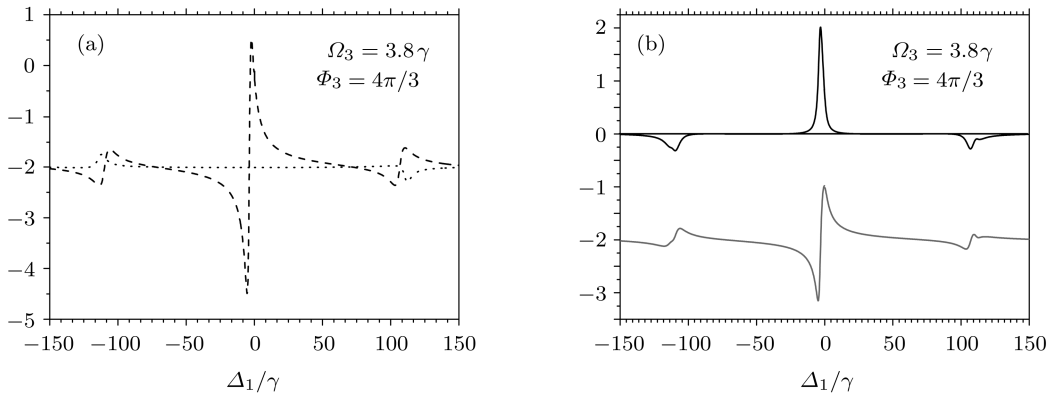


Fig. 3. (a) Real parts of the permittivity (dashed curve) and permeability (dotted curve), (b) real (solid curve) and imaginary (grey curve) parts of the refractive index as functions of Δ_1/γ; other parameters are the same as those in Fig. 2.

that the coherent atomic vapour becomes an isotropic left-handed medium with negative permittivity, permeability and refractive index. The absorption behaviours of the four-level atomic system are depicted by the imaginary part of the refractive index. From the profile of the solid curve, we can see an absorption peak approximately at the probe laser resonant location; the peak value is about 0.65. On both sides of the absorption peak, there are two intervals of the probe frequency detuning without absorption and gain; the intervals are [–39γ; –9γ] and [8γ; 36γ]. Beside the two zero absorption intervals appear two weak gain vales. The gain ranges are [–74γ; –39γ] and [36γ; 59γ]. The maximum values of the two gain vales are –0.42, –0.49, respectively.

In Fig. 3, the Rabi frequency and phase of the coupling field are set to $\Omega_3 = 3.8\ \gamma$ and $\Phi_3 = 4\pi/3$. From Fig. 3(a), we can see the relative dielectric permittivity $\varepsilon_{\rm r}$ exhibits a positive real part in the probe frequency detuning range [–2.5γ; 0]. A negative value appears in [–150γ; –2.5γ] and [0; 150γ]. The real part of $\mu_{\rm r}$ is negative in [–150γ; 150γ]. So in the ranges of [–150γ; –2.5γ] and [0; 150γ], the atomic system displays left-handedness. In Fig. 3(b), the profile of the solid curve shows an absorption peak in the interval of [–17γ; 12γ]; the peak value is 2. The zero absorption ranges are [–98γ; –17γ] and [12γ; 96γ]. Compared with Fig. 2, the absorption peak value and the zero absorption intervals are both enlarged in Fig. 3. In the ranges of [–138γ; –98γ] and [96γ; 128γ], two weak gain vales still appear. The maximum values of the two gain vales are about –0.3. In other words, increasing the intensity and varying the phase of the coupling beam can enhance the absorption peak value at the resonant position, extend the zero absorption ranges and reduce the maximum values of the gain.

From the results of Figs. 2 and 3, we can obtain zero absorption LHM. A more important fact is that we can manipulate the zero absorption property of the LHM by the variation of the intensity and phase of the coupling field. The zero absorption property may be used to amplify evanescent waves that have been lost in imaging by traditional lenses. Thus we have given an approach to realize LHM without absorption and a possibility of enhancing the quality of imaging.

4. Conclusion

In conclusion, we have investigated three external fields interacting with a four-level Y-type atomic system. Considering the Rabi frequency of the coupling field as a complex parameter, the atomic system can display left-handedness with simultaneous negative permittivity and negative permeability as well as a zero absorption property. Varying the intensity and phase of the coupling field can change the absorption properties at the probe light resonant position, extend the zero absorption ranges and reduce the gain values beside the zero absorption intervals. The variational properties manifest the controllable possibility. The zero absorption property may be used to amplify the evanescent waves that have been lost in imaging by traditional lenses. Our scheme has proposed a possibility of enhancing the imaging resolution in realizing 'superlenses'.

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