Effect of cavity dissipation on the emission spectrum of an atom interacting with a field in the dispersive approximation*

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The emission spectrum of a two-level atom interacting dispersively with a single mode radiation field in the dissipative cavity is investigated. A general expression for the emission spectrum is derived. The numerical results for the initial field in coherent state are calculated. It is found that the spectrum structure is influenced significantly by the cavity damping constant κ, and the spectrum structure is dependent on the interaction time T when the cavity dissipation is present. Only one peak located at ω_a appears in the atomic spectra for larger T.

Keywords: emission spectrum, cavity dissipation, two-level atom

PACC: 4250, 3280

1. Introduction

Much attention has been paid to the atomic emission spectrum in the last decades, because it can provide much physical information about 1ight-matter interaction such as the energy 1evel structure of the system, the instabilities and the relative populations of the energy levels. The wide investigation of the emission spectrum[1−6] has revealed many nonclassical effects in the physical process of atom-field interaction. The successful operations of a single-photon micromaster[7] and a single-mode two-photon micromaster[8] have provided a new experimental tool for investigating the fundamental quantum nature of the atom-field interactions. The vacuum field Rabi splitting and the emission spectrum for a single atom in an optical cavity have already been observed.[9−11] With the advent of the two-mode two-photon micromaster, it has become possible to observe certain quantum effects of atoms interacting with a two mode field, such as the emission spectrum and squeezing.[12] A lot of theoretical work has been done on the emission spectrum of a single atom or a pair of atoms interacting with one mode or two modes of the cavity field.[13−20] For example, Zaheer and Zubairy[15] indicated that the emission spectrum for an atom interacting with a single-mode field exhibited phase sensitivity when the atom was initially prepared in a coherent superposition of its two states. Bužek and Jex[16] investigated the spectrum of light emitted from a single atom interacting with a single mode of the radiation field in a cavity with a Kerr-like medium. Zhou *et al.* studied the emission and the cavity-field spectrum for a system consisting of a cascade three-level atom interacting with a quantized radiation field.[17] Guo and Zhao[18] investigated the emission spectrum of a harmonically trapped two-level atom without making the Lamb–Dicke approximation. However, most of the studies were based on the ideal cavity, without taking cavity damping into account.

Nevertheless, the dissipation of the cavity is inevitable. Some recent studies[21−28] revealed that the cavity dissipation was not neglectably small in comparison with the value of the Rabi frequency under today's experimental conditions. So its contribution to the atomic emission spectrum should not be neglected. Peixoto and Nemes[28] presented the time evolution of a two-level atom interacting dispersively with an electromagnetic field in a dissipative cavity and investigated the influence of cavity dissipation on the entanglement of the two subsystems (atom and field). Based on their research results, we discuss the effect of cavity dissipation on the emission spectrum of a two-level atom in a dissipative cavity with large detuning and see the new characters of the spectrum in this paper.

The remainder of the present paper is organized as follows. In Section 2, we introduce the model of an atom interacting with a single-mode radiation field in a dissipative cavity with large detuning and derive

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some expressions for the emission spectra. In Section 3, we calculate the emission spectra and discuss the effects of the damping constant κ, the mean photon number $\bar{n}$ of the initial field and the interaction time T on the atomic emission spectra separately. Finally, some conclusions drawn from the present study are presented in Section 4.

2. Model and atomic emission spectrum

We consider a two-level atom interacting dispersively with a single-mode field in a dissipative cavity and derive the condition for the dispersive approximation to hold true for this system.

The Hamiltonian of the rotating-wave approximation of the system is given by ($\hbar = 1$)

$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm I}, \tag{1}$$

where

$$\hat{H}_0 = \omega_0 \left(\hat{a}^+\hat{a} + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z,$$
$$\hat{H}_{\rm I} = \Omega\left(\hat{a}^+\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right).$$

Here, ω_0 is the single mode field frequency; ω_a is the transition frequency between the two levels of the atom; $\hat{a}^+$ and $\hat{a}$ are the creation and the annihilation operators of the field; $\hat{\sigma}_z$, $\hat{\sigma}$ and $\hat{\sigma}^+$ are pseudospin operators of the atom; Ω is the Rabi frequency measuring the coupling between atom and field. Next, we define the detuning as $\delta = \omega_0 - \omega_a$, which measures how off resonance the two systems are.

We assume that $|\delta|/\Omega \gg \sqrt{n+1}$, for the value of any n (n is the initial number of photons in the field). Therefore, the effective Hamiltonian for this system is given by[28]

$$\hat{H}^{\rm eff} = \omega_0\hat{a}^+\hat{a} + \frac{\omega_a}{2}\hat{\sigma}_z + \frac{\Omega^2}{\delta}\left[\left(\hat{a}^+\hat{a}+1\right)|e\rangle\langle e| - \hat{a}^+\hat{a}\,|g\rangle\langle g|\right], \tag{2}$$

where the symbol $e(g)$ refers to the upper (lower) state for the atom.

In the dispersive approximation, we write the master equation that governs the dynamics of the two-level atom coupled with the electromagnetic field in a high Q cavity as

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = \frac{1}{\mathrm{i}}\left[\hat{H}^{\rm eff}, \hat{\rho}(t)\right] + D\hat{\rho}(t). \tag{3}$$

The losses in the cavity are phenomenologically represented by the superoperator D. At zero temperature, this superoperator can be written as

$$D = \kappa\left(2\hat{a}\cdot\hat{a} - \hat{a}^+\hat{a}\cdot - \cdot\hat{a}^+\hat{a}\right), \tag{4}$$

where κ is the damping constant of the cavity.

Suppose that the two-level atom is initially in the superposition of its two eigenstates: $(|e\rangle + |g\rangle)/\sqrt{2}$, and the cavity field is initially in a coherent state $|\alpha\rangle$ ($|\alpha\rangle = \exp\left(-|\alpha|^2/2\right)\sum_{n=0}^{\infty}(\alpha^n/\sqrt{n!})\,|n\rangle$). Assume that at $t = 0$ the two systems start to interact. At $t = 0$ the dissipative mechanism of the cavity is also turned on. The initial state of the atom-field coupling system gives

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)\otimes|\alpha\rangle. \tag{5}$$

According to expressions (2), (3), (4) and (5), we can obtain the density operator $\hat{\rho}(t)$ of this system as (see Ref. [28] for more details)

$$\hat{\rho}(t) = \lambda_+(t)\,|+\rangle\langle+| + \lambda_-(t)\,|-\rangle\langle-|, \tag{6}$$

where

$$|+\rangle = \frac{1}{\sqrt{2}}\left[\left|e, \alpha(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\right\rangle + \mathrm{e}^{-\mathrm{i}\Theta(t)}\left|g, \alpha(t)\,\mathrm{e}^{\mathrm{i}\omega t}\right\rangle\right], \tag{7}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left[\left|e, \alpha(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\right\rangle - \mathrm{e}^{-\mathrm{i}\Theta(t)}\left|g, \alpha(t)\,\mathrm{e}^{\mathrm{i}\omega t}\right\rangle\right], \tag{8}$$

$$\lambda_+(t) = \frac{1}{2}\left(1 + \mathrm{e}^{N(t)}\right), \tag{9}$$

$$\lambda_-(t) = \frac{1}{2}\left(1 - \mathrm{e}^{N(t)}\right) \tag{10}$$

with

$$\Theta(t) = -\omega t + \frac{|\alpha|^2\kappa}{\kappa^2+\omega^2}\left[\mathrm{e}^{-2\kappa t}\left(\kappa\sin(2\omega t) + \omega\cos(2\omega t)\right) - \omega\right], \tag{11}$$

$$N(t) = -|\alpha|^2\left(1 - \mathrm{e}^{-2\kappa t}\right) - \frac{|\alpha|^2\kappa}{\kappa^2+\omega^2}\left[\mathrm{e}^{-2\kappa t}\left(\kappa\cos(2\omega t) - \omega\sin(2\omega t)\right) - \kappa\right], \tag{12}$$

$$\omega = \frac{\Omega^2}{\delta}, \tag{13}$$

$$\alpha(t) = \alpha \exp(-\kappa t). \tag{14}$$

The physical spectrum $S(\nu)$ of the radiation field emitted by a two-level atom inside a dissipative cavity can be expressed as[29]

$$S(\nu) = 2\Gamma \int_0^T \mathrm{d}t' \exp\left[-(\Gamma - \mathrm{i}\nu)(T - t')\right] \times \int_0^T \mathrm{d}t \exp\left[-(\Gamma + \mathrm{i}\nu)(T - t)\right] \times \langle \Psi(t') | \sigma^+(t') \sigma(t) | \Psi(t) \rangle, \tag{15}$$

where T is the measurement time, the time that the measurement takes, Γ is the bandwidth of the filter, and $|\Psi(t)\rangle$ is the state vector of the system. By making use of expressions (7), (8), (9) and (10), we can carry out the integration over the interaction time, and obtain the following expression for the emission of the atom as

$$S(\nu) = \frac{1}{2} \sum_n \frac{|\alpha|^{2n}}{n!} \exp(-2\Gamma T) \left(\left| \int_0^T \sqrt{\lambda_+} A_n \mathrm{d}t \right|^2 + \left| \int_0^T \sqrt{\lambda_-} A_n \mathrm{d}t \right|^2 \right) \tag{16}$$

with

$$A_n = \exp\left[-\frac{|\alpha|^2}{2} \mathrm{e}^{-2\kappa t} \right] \exp\left[t(\Gamma - n\kappa) \right] \exp\left[\mathrm{i}t(\nu - n\omega - \omega_a) \right]. \tag{17}$$

3. Results and discussion

In our analysis about the atomic emission spectrum we start with the case where the field is initially prepared in the coherent state. In this case the spectrum is described by the function $S(\nu)$ defined in expression (16). The numerical results are shown in Figs. 1–4. We discuss the effects of the damping constant κ and the initial mean photon number $\bar{n}$ in the field. We turn our attention to the effect of the measurement time T on the emission spectrum when κ is not zero.

3.1. Influence of κ on the emission spectrum

In order to understand better the effect of the damping constant κ of the cavity on the atomic emission spectrum, we calculate the emission spectrum for various values of κ in Fig. 1. It can be observed that the cavity dissipation suppresses spectral peaks located on the right-hand side but enhances the peak on the left-hand side. When the damping constant κ is zero, the cavity is ideal: we obtain the four-peak structure. The relative heights of the right side peaks start to decrease while the central peak increases with κ increasing. When the damping constant κ is sufficiently large (about 0.1 ω), only the central single-peak spectrum appears. This result shows

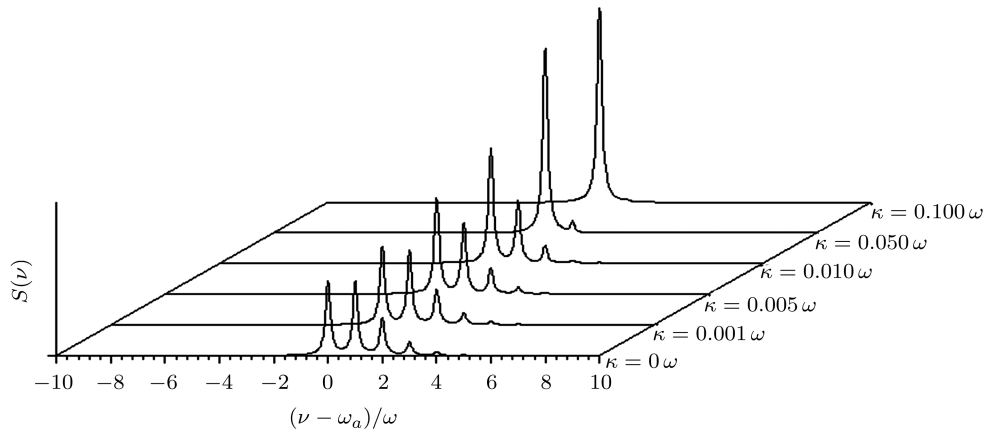


Fig. 1. Influence of the damping constant κ on the atomic emission spectra. The measurement time $T = 40\ \omega^{-1}$, the bandwidth of spectrometer $\Gamma = 0.1\ \omega$ and the average photon $\bar{n} = 1$.

that the non-classical character disappears when the cavity dissipation is about 0.1 ω or larger. The effect of cavity dissipation must be taken into account under the condition of advanced experimental techniques of cavity quantum electrodynamics. For example, Ref. [27] gives the cavity damping constant κ to be 0.19 ω, i.e. $\kappa \approx 0.19\ \omega$.

3.2. Relationship between $\bar{n}$ and the emission spectrum

We now discuss the structures of the emission spectrum for various values of mean photon number $\bar{n}$ of the initial field as shown in Fig. 2. Obviously, more and more spectral peaks appear on the right side of the spectrum and the relative height of the central peak diminishes as the mean photon number $\bar{n}$ increases. Meanwhile, the highest peak shifts toward the right side from the frequency ω_a. For comparison, we also depict the atomic emission spectrum in the case of an ideal cavity ($\kappa = 0$) and various values of the mean photon number $\bar{n}$ of the initial field as shown in Fig. 3. We can obtain that the variation trend of the atomic emission spectrum in the case of the ideal cavity ($\kappa = 0$) is similar to that in the case of a dissipative cavity ($\kappa \neq 0$) as $\bar{n}$ increases. But the numbers of spectral peaks are different for the same value of $\bar{n}$ in the two cases (except the case $\bar{n} = 0$). The presence of the dissipation of the cavity decelerates the change of atomic emission spectrum which is caused by the increase of $\bar{n}$. This is because the dissipation of the cavity reduces the number of photons in the cavity. When $\bar{n} = 0$, only one peak located at ω_a appears for both cases. This means that the vacuum Rabi splitting is not be observed under the large detuning condition.

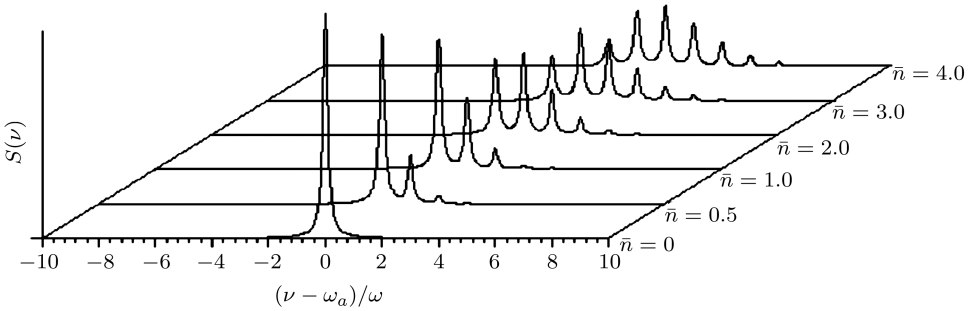


Fig. 2. Influence of mean photon number $\bar{n}$ on atomic emission spectra. The measurement time T is 40 ω^{-1}, the bandwidth of spectrometer $\Gamma = 0.1\ \omega$ and the damping constant $\kappa = 0.01\ \omega$.

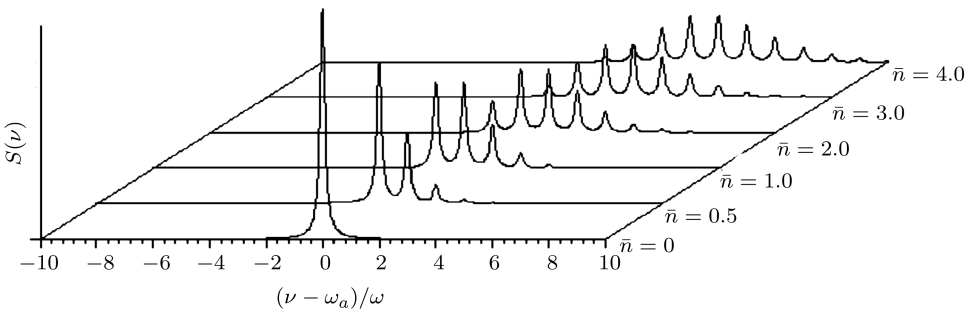


Fig. 3. Influence of mean photon number $\bar{n}$ on atomic emission spectra in ideal cavity ($\kappa = 0$). Other parameters are the same as those in Fig. 2.

3.3. Relationship between T and emission spectrum

In this section, we examine the effect of interaction time T on the atomic spectrum. As seen in Fig. 4, for $\kappa = 0.01\ \omega$ and $\bar{n} = 1.0$, when the measurement time T is 6 ω^{-1}, we can obtain one wide width peak. This phenomenon is understandable because the short measurement time decreases the spectral resolution. The emission spectra evolve into a complicated multipeak structure which is consistent with the result in Ref. [30] when T is 15 ω^{-1}. But when T is 40 ω^{-1}, the number of spectral peaks becomes less. With T increasing, the relative height of the central spectral peak will increase. In the case where the interaction time T is short, the

influence of the dissipation of cavity on the emission spectrum is not obvious. So the variation trend of emission spectrum in the case of the dissipative cavity $\kappa \neq 0$ is similar to that in the case of the ideal cavity ($\kappa = 0$) as T increases. The longer the interaction time T is, the more effective the influence of the cavity dissipation on the atomic emission spectrum will be. These results suggest that we must choose the interaction time in a suitable range for exploring the emission spectrum. Otherwise we obtain a wide-width single-peak for a short interaction time or a central single-peak for a long interaction time. We cannot see any quantum characteristics of the spectrum under these conditions.

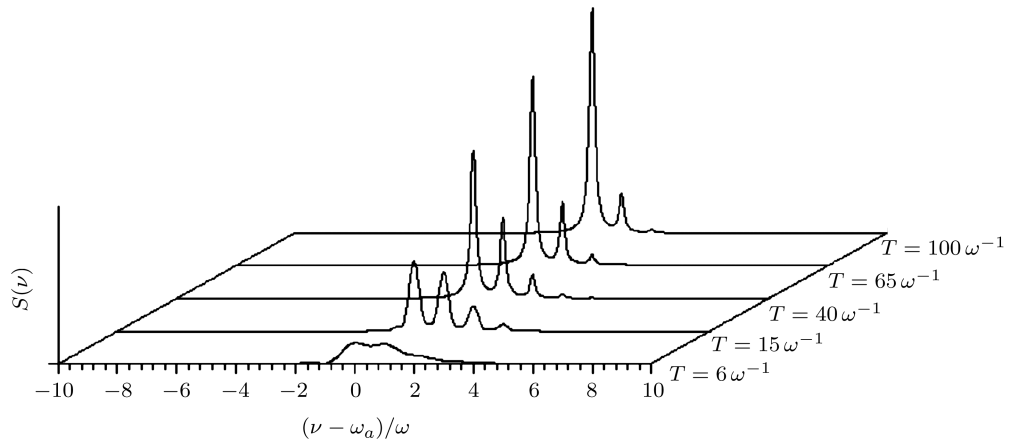


Fig. 4. Effects of the measurement time T on the atomic emission spectrum. The initial field average photon $\bar{n} = 1$, the bandwidth of spectrometer $\Gamma = 0.1\omega$ and the damping constant $\kappa = 0.01\ \omega$.

4. Conclusions

We investigate the emission spectrum of a two-level atom in an dissipative cavity with the field–atom coupled in large detuning. The numerical results for field in a coherent state initially are presented. We study the effects of the damping constant κ, the initial mean number of photons in the field $\bar{n}$ and the interaction time T on the spectrum structure separately. The main conclusions that can be drawn from the above results are as follows.

(i) In the case where other parameters are kept at certain values, the number of spectral peaks located on the right side of the atomic emission spectrum becomes smaller and smaller, and the central spectral peak is strengthened as κ increases. Only one central spectral peak remains in the spectrum when κ is sufficiently large. The effect of cavity dissipation must be taken into account when the cavity dissipation is about 0.1 ω or larger.

(ii) The presence of the dissipation of the cavity can reduce the influence of the increase of $\bar{n}$ on the atomic emission spectrum.

(iii) For the damping constant $\kappa \neq 0$, the spectral peak number becomes smaller and smaller as T increases. If the interaction time T is too short, only one wide width peak can be observed because of the low resolution. So if we want to obtain the quantum characteristics of the spectrum, we must make sure that the interaction time length is appropriate.

The analytical calculation that we have made shows that the influence of the dissipation of the cavity on the atomic emission spectrum should not be neglected under today's experimental technology.

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